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PRINCIPLES

OF

ANIMAL MECHANICS.

Trinity College, Dublin,

21 April 1873

My dear Allman —

I send you a copy of my "Animal Mechanics" in memory of the old times when we used to — Letter with "dear old Prof" —

Yours sincerely

Sam'l Haughton

Prof Allman

Professor Allman
with the author's
kind regards &
best wishes

PRINCIPLES

OF

ANIMAL MECHANICS.

BY THE

REV. SAMUEL HAUGHTON, F.R.S.,

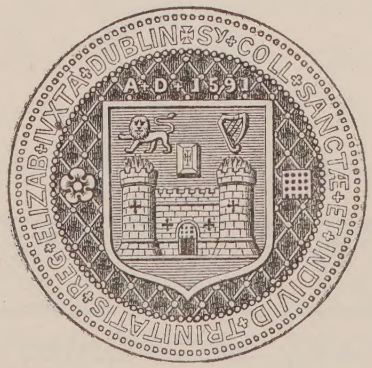
FELLOW OF TRINITY COLLEGE, DUBLIN;

M.D. DUBL., D.C.L. OXON.

ידעתי כי כל־אשר יעשה האלהים הוא יהיה לעולם
עליו אין להוסיף וממנו אין לגרוע

Ἔοικε ταῦτα σοφοῦ τινὸς δημιουργοῦ καὶ φιλοζώου τεχνήματι.

Nihil est enim, quod ratione et numero moveri possit sine consilio; in quo nihil est temerarium, nihil varium, nihil fortuitum.



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PREFACE.

THE observations and calculations contained in this Book have occupied my leisure hours during the past ten years; and they are now offered to the public, with the view of showing the mutual advantages obtainable by Anatomists and Geometers from a combination of the Sciences which they cultivate. Anatomists will gain by the increased precision which numerical statements must give to their observations, and Geometers will find in Anatomy a new field of problems opened out to their investigation. I may be allowed to call attention, from this point of view, to the problem of the equilibrium of an elliptical muscular dome, and to the use which I have made of the hyperboloid of one sheet, of Ptolemy's Theorem, and of some curves of the third order.

In the course of my investigations, I have met with numerous instances, in the muscular mechanism of the vertebrate animals, of the application of the principle of least action in Nature; by which I mean that the work to be done is effected by means of the existing arrangement of the muscles, bones, and joints,

with a less expenditure of force than would be possible under any other arrangement; so that any alteration would be a positive disadvantage to the animal. If, as I consider probable, this fact should prove to be of much wider occurrence in Nature than these instances show, it may serve to give us some slight glimpse of the mechanism by which the conservation of species in Nature is secured. In Astronomy, the conservation of the Solar System depends upon certain well-known conditions regulating the motions of the several bodies of which that system consists; and it is a matter of indifference whether these conditions were directly imposed by the Will of the Divine Contriver, or were the indirect result of some former condition of the System. In either case, these conditions are equally the foreseen result of the Contrivance. If the present state of the Solar System be the result, according to fixed laws, of some pre-existing state of that System, it may be said, in the language of Naturalists, to have been evolved out of its former state, but in such an Evolution there was nothing left to Chance; it was all foreseen, and the Evolution itself presided over by the Divine Mind that planned the whole. I cannot see why there may not be in Organic Life a similar process of Evolution of higher from lower forms of existence; but it is a Teleological Evolution, in which every step and every result was foreseen and planned beforehand. The Laws of such an Evolution appear to me, in the present state of our knowledge, to be entirely unknown.

I have explained the similarity of muscles and bones in the vertebrate animals on simple Teleological principles, without making use of the unproved hypothesis of their descent from a supposed common ancestor.

In the discussion of the Theory of Muscular Types, and of the Laws of Fatigue and Refreshment, I have succeeded in obtaining many new and interesting applications of exact science to the problems of Animal Mechanics.

I take this opportunity of returning my thanks to Dr. Alexander Macalister and to the Rev. Richard Townsend, who have assisted me materially in the Anatomical and Geometrical branches of my subject; and also, to the Provost and Senior Fellows of Trinity College, Dublin, for the assistance they have given me, by defraying a portion of the expenses of publication.

SAMUEL HAUGHTON.

TRINITY COLLEGE, DUBLIN,
1st January, 1873.

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ANIMAL MECHANICS.

1. **On the Nature of Muscular Fibres.**—If a portion of the muscle of any animal be macerated in water for a short time, and gently separated into its elements by means of dissecting needles, it will be found to consist of elementary fibres, admitting of no further subdivision by mechanical treatment. These elementary fibres extend through the whole length of the muscle from its origin to its insertion, and thus vary in length from several lines to upwards of two feet.* The thickness of the elementary fibres has been measured by various observers, among whom none is more worthy of confidence than Mr. W. Bowman, who has given the following Table as the result of his observations on various animals :†—

Diameter of the Elementary Fibres of Striped Muscle in Fractions of an English Inch.

	From	To		
1. Human, . . .	$\frac{1}{613}$ . .	$\frac{1}{192}$ . .	average of males, . .	$\frac{1}{352}$
			,, females, .	$\frac{1}{454}$
2. Other Mammals, .	$\frac{1}{1100}$ . .	$\frac{1}{192}$ . .	average	$\frac{1}{561}$

* In the *m. sartorius* of Man these fibres have been found upwards of two feet in length, and I have myself measured them in the *m. bicipiti accessorius* of the Lion exactly two feet long.

† "Cyclopædia of Anat. and Phys.," vol. iii., p. 507.

	From	To		
3. Birds,	$\frac{1}{1500}$	$\frac{1}{350}$	average,	$\frac{1}{807}$
4. Reptiles,	$\frac{1}{1000}$	$\frac{1}{100}$	„	$\frac{1}{484}$
5. Fishes,	$\frac{1}{755}$	$\frac{1}{65}$	„	$\frac{1}{222}$
6. Insects,	$\frac{1}{755}$	$\frac{1}{200}$	„	$\frac{1}{419}$

Professor Donders, of Utrecht, and Dr. Buys Ballot,* estimate the cross section of the fibres of the *m. biceps humeri* and *m. brachialis anticus*, in Man, as $\frac{1}{300}$th of a square millimetre. This estimate, converted into English measures, gives for the diameter

$$d = \sqrt{\frac{4}{500\,\pi}} \times \frac{39{\cdot}37}{1000} = 0.001986 = \frac{1}{503} \text{ in.}$$

In this formula, π denotes the ratio of the circumference to the diameter of a circle.

The same authors found for the cross sections of these muscles the following measurements:—

m. biceps (long head)	530$^{mm.}$	square.
„ (short head)	452	„
m. brachialis anticus	614	„
Total	1596$^{mm.}$	

From these results it follows that the total number of elementary fibres in these muscles amounts to 798,000. These important muscles are the only ones concerned in holding the forearm flexed at right angles to the arm, and we shall have occasion hereafter to make use of the results just given on the authority of Donders and Ballot.

The elementary fibres are not circular in their cross section, but polygonal, in consequence of their mutual pressure; and microscopical examination shows that the blood vessels which supply the fibres are interposed at the angles of the polygons: from this it follows that the muscles which have the smallest fibres are best provided with blood, and are therefore ca-

* "Over de Elasticiteit der Spiren," p. 47, 8vo., Utrecht, 1863.

pable of the longest endurance of work. This conclusion agrees with the Table of Mr. Bowman, from which it appears that of all animals the muscles of Birds possess the smallest fibres, and that the muscular fibres of women are smaller than those of men; for it is well known that no animals are more capable of long-continued muscular exertion and fatigue than Birds; and I have myself found, by direct experiment, that the muscles of women are capable of longer continued work than those of men, although inferior to them in force exerted for a short time.*

The elementary fibre is encased in a sarcolemma, or sheath, very tough and elastic, but not endowed with the vital properties of the muscular fibre itself; and, again, many such elementary fibres, each invested by its proper sheath, are united into greater or lesser bundles, separated from each other by areolar tissue, sometimes containing fat, which acts as a lubricating medium, to prevent friction between the muscular bundles during their rapid and varying action.

Before speaking of the varieties of origin, insertion, and action of the muscular bundles, it may be useful to say a few words on the distinction between striped and unstriped muscles, respecting which there seems to be much confusion of ideas. On examining most muscles with the microscope, they are found to be striped crossways with very delicate and close parallel lines, while other muscles are perfectly free from such an appearance. It is said by some that the distinction of muscles into voluntary and involuntary corresponds with the microscopical distinction into striped and unstriped muscles; and, if it were not for the all-important exception of the heart, this assertion might be maintained with much appearance of

* If any man wishes for a simple proof of the inferiority of the endurance of his muscles as compared with those of a woman, let him carry a child on his arm for the same time that his wife or nurse can do with ease, and he will find himself much fatigued.

truth; but since the heart is, of all the muscles of the body, the most essentially involuntary, and yet possesses striped muscles, we must seek for some other explanation of this remarkable difference between muscles; and indeed it must be granted that, even if the voluntary and involuntary muscles were invariably striped and unstriped, it would still remain as difficult of explanation as before why a voluntary muscle should differ from an involuntary muscle in so singular a manner.

On a careful examination of the stripes in muscular fibres, it is found that they arise from a tendency of the fibre to split crossways into thin disks, which give an appearance like that of cleavage in rocks to the whole of the fibres. Judging from the analogy of cleavage, I have come to the conclusion that the striped structure in muscles, or tendency to cleave into disks, is due to their repeated contraction between two fixed or nearly fixed points of origin and insertion. Mr. Sorby has shown, by examination of the microscopical structure of cleaved rocks, and I have shown by calculations founded on the distortion of their fossils, that the planes of cleavage are at right angles to the lines of maximum compression; Professor Tyndall also has proved, by direct experiments on compressed wax, that the formation of cleavage planes perpendicular to lines of pressure is an universal law of nature. Whenever, therefore, we find in muscular fibres a distinct origin and insertion, their contraction between these points will produce the pressure necessary for the developement of cleavage at right angles to the length of the fibres; and, in like manner, when a muscular fibre returns upon itself, as in the *sphincter ani* and *sphincter vaginæ*, its contraction produces the striped structure. In the case of the heart, as is now well established, the muscular fibres form closed circuits, coiling round each ventricle and round the whole heart in such a manner, that their contraction, like that of a twisted India rubber ring, developes

the conditions of pressure necessary for the production of a cleaved or striped structure. In the unstriped muscles, as those of the bladder and uterus, of the intestinal canal and scrotum, the contraction of the muscle is vermicular, and not a constant strain from end to end; hence the conditions of pressure requisite to produce cleavage are wanting, and the muscle and its fibres remain without this distinguishing mark of uniform and continued contraction from end to end.

It has been found, on careful microscopical examination of the elementary fibres, that they possess not only a transverse cleavage, dividing them into disks, but also a longitudinal cleavage or jointage, separating the fibres into *fibrillæ;* so that the ultimate subdivisions of muscles may be regarded as elementary molecules of muscular fibre, which are to be considered as units, or ultimate atoms of muscular tissue; and every muscle may be regarded as composed of a certain number of such atoms, variously distributed.

2. **On the Nature of Muscular Contraction.**—The muscular fibres already described are capable of contracting in a longitudinal direction, and in so contracting exert mechanical force on the points of origin and insertion. It was supposed by the older writers (as Borelli and others), that there was no change of volume during the contraction of muscles; and this conjecture has been confirmed in a satisfactory manner by recent experiments; and, as this law of constancy of volume during change of form holds true of bodies under the influence of magnetical and electrical forces, some writers have supposed that it furnishes a key to the explanation of muscular vital phenomena.

It is certain that muscular contraction is accompanied by developement of sensible heat (or loss of latent heat); by loss of electrical statical tension; and by the chemical transformation of fibrine, albumen, and the higher products of animal life, into urea, carbonic acid, and water, which are the main excretions of all animal tissues: and, as it is well known that

loss of latent heat, loss of statical electricity, and change of chemical products from higher to lower forms, are all representatives of force expended, or *work done*, it has been proposed by various authors to select one or other of these exponents of work, as the means of measuring its amount under various conditions. It seems to me that chemical transformations present the most certain and ready of all known methods of determining the *work done* by the various tissues of the body in the discharge of the several functions assigned to them in the economy of nature.

All these chemical, electrical, and calorifical changes are intimately connected with the supply of fresh arterial blood to the muscles; and any theory of muscular action that does not explain the necessity of such a fresh supply of blood must be erroneous.

On the electrical theory of muscular action, the contraction of the muscle is accompanied by a loss of electrical tension, caused by the partial discharge of the opposite electricities of the outer and inner portions of the muscular bundles, and a supply of fresh blood is admittedly necessary for the restoration of that tension; the contraction being supposed due to the molecular attraction of the muscular particles when freed from the control of their induced electricity. This theory of muscular action certainly commends itself by the explanation which it affords of *rigor mortis* on principles analogous to those of ordinary muscular contraction; but, inasmuch as no method has been as yet devised for measuring the total loss of electrical tension caused by a given amount of muscular action, it seems to me that the electrical theory of such action must be regarded, in the present state of science, as a theory incapable of yielding numerical results, which are the only tests to which true science can submit its decisions.

Of the other two exponents of *work done*, namely, the caloric developed, or the urea, carbonic acid, and water produced,

by a given amount of muscular action, I cannot treat further at present, as I propose to confine the scope of this work altogether to the mechanical effects of muscles. Of all the methods of estimation hitherto proposed, the chemical (as I have already stated) seems to me to be the most natural, and, in the existing state of science, the most certain.

That a fresh supply of arterial blood is necessary to enable the muscle to relax its contraction, is proved by many observations and experiments. Among these may be mentioned puerperal convulsions consequent on uterine hæmorrhage, the anæmic convulsions witnessed before death in horses and other animals destroyed by venesection, the phenomena of *rigor mortis*, which disappear when arterial blood is artificially pumped through the dying arteries, and the convulsions caused by simple hanging.

The curious reader will perhaps excuse the following digression on the art of hanging.*

* I am indebted to my friend Dr. J. K. Ingram, Fellow of Trinity College, Dublin, for the following interesting sketch of the History of the Art of Hanging in England :—

"Hanging was a mode of execution in use among the Anglo-Saxons. Indeed, in 'Beowulf'—which its able Editor, Kemble, believed to be a modernized form of a poem which the invaders of Britain had brought with them from their Continental homes—the gallows (*galga*) figures as an old-established institution of the Teutonic races of Northern Europe. But it is very difficult to get any definite information as to the history of hanging in England. It seems, however, quite certain that the idea of immediately extinguishing the life of the culprit by a sufficient fall never presented itself to our ancestors; their only notion was that of suspending him by the neck for what might seem an adequate time to insure (?) strangulation. It is noticed by Blackstone as a somewhat singular fact, that the only warrant the sheriff has for a capital execution is the signature of the judge to the calendar, or list of all the prisoners' names, with their respective judgments in the margin; 'as, for a capital felony it is written opposite to the prisoner's name, "hanged by the neck;" formerly, in the days of Latin and abbreviation, "*sus. per coll.*" for "suspendatur per collum.'" Originally, however, he states there was a formal precept to the sheriff under the hand of the judge; but in none of the law books have I found a copy or exact descrip-

Digression on the Art of Hanging.—Death is caused by hanging in one or other of the three following methods :—

1. By apoplexy, caused by pressure on the jugular veins;

2. By asphyxia, caused by stoppage of the windpipe ;

3. By shock of the medulla oblongata, caused by fracture of the vertebral column.

In the first two cases, death is preceded by convulsions, lasting from five to forty-five minutes, which are caused by the cessation of the supply of arterial blood to the muscles. In the third case, death is instantaneous and painless, and is unaccompanied by any convulsive movement whatever.

According to the original form of death punishment for treason in England, the hanging was used as an anæsthetic, preparatory to the disembowelling (or drawing) that always preceded the quartering of the criminal; and the present slow process of hanging, practised by Calcraft and others in England and Scotland, which consists in dropping the patient through two or three feet, and allowing him to hang until dead, is the faithful representation of the original process of hanging, which was intended to fulfil a purpose quite distinct from that of the speedy execution of the criminal.

It seems to me unworthy of the present state of science to continue a mode of execution which as at present used is

tion of this precept. I do not even find how long the sentence, to be 'hanged by the neck *till you are dead*,' has been in use.

"'It is clear,' says Blackstone, 'that, if upon judgment to be hanged by the neck till he is dead, the criminal be not thoroughly killed, but revives, the sheriff must hang him again, for the former hanging was no execution of the sentence.' But, strangely enough, we find in the 'Vision of Piers Plowman' a passage which seems to show that the opposite of this either was, or was believed to be, the established rule in his time :—

"'It is noght used on earthe
To hangen a felon
Ofter than ones,
Though he were a tretour.'"

extremely clumsy, and also painful to the criminal. Instead of the "short drop" generally used, we ought to employ the "long drop," which causes instantaneous death. It has been proved by me that the shock of a ton dropped through one foot is just sufficient to fracture the anterior articulating surfaces of the second vertebra at their contact with the atlas, and that this fracture allows the shock to fall upon the *medulla oblongata*, so as to produce instantaneous death. As the result of some consideration bestowed upon the subject, I would recommend the adoption of the following rule:—

RULE I. "Divide the weight of the patient in pounds into 2240, and the quotient will give the length of the long drop in feet."

For example, a criminal weighing 160 lbs. should be allowed 14 feet drop. If local circumstances will not allow of the long drop being employed, the requisite shock should be produced by strapping a shot to the feet, so as to secure the shock of 2240 ft. lbs. to the medulla.

Efforts have been made in the United States to give to hanging all the rapidity of death by the guillotine, without the painful spectacle of bloodshed. This method, which is borrowed from the mode of execution practised on board ship, consists in suddenly lifting the criminal into the air by means of a great weight attached to the other end of the rope fastened round his neck; the rope passes over two pulleys, one of which is placed vertically over the patient, and at a given signal the weight falls through a regulated height, lifting him suddenly into the air. Sufficient attention, however, has not been paid, even in that enlightened country, to the conditions necessary to be fulfilled in this mode of suspension; for in many of their executions the only care that seems to have been taken was to make the falling weight heavier than the criminal, so as to insure his permanent suspension by the neck until death terminated his sufferings.

The American method of hanging, if properly applied, seems to me to be capable of producing death by shock with even less suffering than the "long drop;" for, although by the latter method death is instantaneous when the shock actually occurs, yet the mental sufferings of the criminal during the second occupied by his fall may be very considerable. This painful interval is altogether avoided in the American method, provided the *initial* shock be sufficient to destroy the medulla oblongata. This important condition may be effected by the following calculations, which lead to an easy rule:—

The falling weight, acting through the intervention of the rope, produces its effect in a manner similar to that of the shock or collision of imperfectly elastic bodies.

Let m and m' denote the masses of the two bodies, and let v and v' denote their velocities previous to collision or shock, while e denotes the coefficient of elasticity of the rope.

Let u, u', denote the velocities of the masses m, m', after the shock; then it is well known* that

$$u = \frac{mv + m'v' - em'(v - v')}{m + m'}; \qquad (1)$$

$$u' = \frac{mv + m'v' - em(v' - v)}{m + m'}.$$

The *vis viva* lost during the shock is expended upon the neck of the criminal, and is represented by

$$mv^2 + m'v'^2 - mu^2 - m'u'^2.$$

After some reductions this is found to be

$$\textit{Vis viva} \text{ lost} = \frac{mm'}{m + m'}(1 - e^2)(v - v')^2. \qquad (2)$$

This result may be applied practically to the solution of the American problem of hanging, so as to cause instantaneous death, in the following manner:—

* Vide "Manual of Mechanics," p. 156.

Let P denote the weight employed, and Q the weight of the criminal; let e denote the coefficient of elasticity of the rope used, and V the velocity acquired by the weight Q in falling through the height h.

If we consider the problem of the weight Q moving with the velocity V, and causing the weight P to move through the intervention of the rope, whose elasticity is e; the shock produced on P at the moment when the "chuck" takes place is similar to that which occurs in the collision of bodies striking each other, and is measured by the *vis viva* lost during their collision. The *vis viva* lost during the shock is, by equation (2),

$$\frac{1}{g}\frac{PQ}{P+Q}(1-e^2)V^2;$$

but, since the *work lost* is half the *vis viva lost*, if we substitute for V^2 its value, $2gh$, we find

$$\text{Work lost} = \frac{PQ}{P+Q}(1-e^2)\,h. \qquad (3)$$

For the ropes usually employed, e may be considered as a very small fraction, and e^2 may be totally disregarded. The *work lost* is expended in causing shock to the neck, and should, as I have already stated, be equal to at least 2240 foot lbs.

In a case of hanging that came under my own observation, the criminal weighed 160 lbs., and was allowed to fall through 14 ft. 6 in., which, allowing for some elasticity in the rope, would correspond with 2240 ft. lbs. of shock; in this case, the anterior articulating surfaces of the second vertebra were fractured near their posterior border (the fracture of the bone extending to the foramina for the vertebral arteries), but the odontoid process of the axis was not broken, nor the transverse ligament of the atlas torn across. Death in this case was as instantaneous as it would have been had the transverse ligament given way, instead of the bone; for the shock reached the medulla, and its consequence was

immediate and painless death. In hanging, the rope supports the atlas, and presses it against the occipital articulations, while the second vertebra tends to fall with the body ; and it follows from this that either the odontoid process must give way, or the second vertebra be broken across at its superior articulating surfaces ; in either case death will be immediate. The height of the criminal just mentioned was 5 ft. $9\frac{1}{2}$ in. a few days before execution, and after death he was found to measure 5 ft. 11 in., having been elongated an inch and a half by the "long drop" of $14\frac{1}{2}$ ft.

From the foregoing, it therefore appears that a shock to the neck of 2240 ft. lbs. is just sufficient to cause immediate death. Substituting, therefore, 2240 ft. lbs. for the work lost in the preceding equation, we find

$$2240 = \frac{PQh}{P+Q};$$

or, solving for P,

$$P = \frac{2240\,Q}{Qh - 2240}. \qquad (4)$$

From this equation it follows that, unless Qh be greater than 2240, the value of P (the weight required) will be negative ; but Qh denotes the work produced by the criminal Q falling through the height h.

Let h, therefore, denote the "long drop" found by Rule I., and the following consequences may be inferred from (4)—

1. In the American mode of hanging, if the weight be let fall through the height h, sufficient to cause death instantaneously by the "long drop," it would require an infinite weight to cause immediate death ; for in this case

$$Qh - 2240 = 0,$$

and therefore P is infinite.

2. In the American mode of hanging, if the weight be let fall through twice the height of the "long drop," a weight

equal to that of the criminal will be sufficient to cause immediate death; for in this case,

$$Qh - 2240 = 2240;$$

and therefore, by equation (4)

$$P = Q.$$

For all heights intermediate between h and $2h$, the weight P must be found from equation (4), and it will always lie between Q and infinity.

In practice, twice the height of the "long drop" would always be found convenient, and therefore the following Rule for producing instantaneous death by the American method is confidently recommended :—

RULE II.—"Having found from Rule I. the height of the English long drop, use twice this height and a weight equal to that of the criminal in the American method."*

* I have searched in vain for well-authenticated instances of fracture of the cervical vertebræ produced by the usual method of hanging. Among the longest drops that I can find recorded by competent witnesses are two observed by Dr. Charles Croker King, when Professor of Anatomy in the Queen's College, Galway.

CASE I. (*a*)—A young man named Hurley was executed in Galway at 6·25, p.m., on the 27th August, 1853, for the murder of a young woman in Dunsandle Wood. The rope used was ten lines in diameter; the knot was large, formed of three turns of the rope; and, on the noose being tightened by the executioner, corresponded to the occipital protuberance. His weight was 10½ stone, and he was allowed to drop 7½ feet. These data give us as follows :—

$$\text{Work done} = 147 \times \frac{15}{2} = 1102 \text{ ft. lbs.}$$

In this case, as Dr. King remarks, "there was no dislocation or fracture of the vertebral column, or injury of the ligaments or of the spinal cord."

CASE II. (*b*)—On the 11th May, 1858, Patrick Lydon was hanged in Galway for the murder of his wife. Lydon was a small man, only 5 ft. 5 inches in height; the diameter of the rope was 10 lines; his weight was 9½ stone, and the drop 11 ft. Hence we find—

$$\text{Work done} = 133 \times 11 = 1463 \text{ ft. lbs.}$$

In this case, "that portion of the anterior common ligament of the spine which

(*a*) "Dublin Quarterly Journal of Medical Science," vol. xviii., 1854, p. 86, *et seq.*

(*b*) Ibid., August, 1863.

On the Nature of Muscular Contraction.—Whatever obscurity there may be as to the ultimate cause of muscular contraction, there need be none whatever as to the *modus operandi.*

When a stimulus is applied to a muscle, by a nerve, by a mechanical injury, or by electricity, there follows immediate contraction of the muscular fibres in the neighbourhood of the part affected, and this contraction is rapidly propagated by a wave-like motion to the other parts of the muscular fibres—each particle contracts for an instant of time, and then transmits the contracted phase to the neighbouring particles, so that the entire length of the muscle is never contracted together, but wave after wave of contraction is passed through it, so long as the action of the nerve or other stimulus is continued.

The rate at which the wave of contraction passes along the muscles of a frog has been shown by Prof. Aebe to be only 3 ft. per second, which is much less than the rate at which similar waves of action pass along the sensitive and motor nerves, which has been determined by several observers. The following may be given as examples:—

1. Rate of action of Motor nerves of Frog (Helmholz), . . . 88 ft. per sec.
2. Rate of action of Sensitive nerves of Man (Schelske), . . 97 „ „

The rate at which the wave of action runs along both sensitive and motor nerves may be approximately measured by noting what is well known to observers as the "personal equation."

An observer is obliged to watch with the eye, the ear, or any other sense, a certain phenomenon, and then to record with the hand, through some instrumentality, the instant of

passes from the body of the second to that of the third cervical vertebra was ruptured, so that the left halves of the bodies of the above-mentioned vertebræ were separated from each other by an interval of one-eighth of an inch, but there was no displacement."

These criminals were executed with the same rope; and death, in the second case, was not preceded by violent muscular convulsions, as in the first case; a fact which is readily accounted for by the excess of shock in the proportion 1463 to 1102.

the occurrence of that phenomenon. It is well known that different observers, equally competent, allow different but always constant intervals to elapse between the moment of the occurrence of the phenomenon and the moment of recording it, and the differences between different observers is called the "personal equation." In other words, the time occupied by the sensitive nerves in conveying the impression to the *optic thalamus*, and by the motor nerve in reconveying the order of the brain from the *corpus striatum*, is different in different persons—or, the velocity of nerve action differs in different persons, and *a fortiori* in different animals. *

Although the velocity of the wave of sensation, or of volition, may thus vary, according to original differences in individuals, or according to the state of health in the same individual, there is good reason to believe that the time occupied by each particle of the muscle or nerve fibre in going through all its changes of contraction and relaxation remains constant.

According to the well-known formula for wave motion—

$$\lambda = vt, \qquad (5)$$

where λ denotes the wave length, v the velocity of the wave, and t the time of molecular vibration or change of state.

From this formula it follows that, if the time of vibration be constant, the length of the wave will vary as the wave velocity; so that waves of different degrees of velocity may pass along the same nerve or muscle, without any change having taken place in the molecular conditions of contraction and relaxation of each portion of the muscle or nerve.

Nothing proves so conclusively as the observation of persons afflicted with softening of the brain, the fact that the

* The recent experiments of Prof. Donders show that the time occupied in the transmission of a sensation from the eye to the brain, the formation of a judgment, and the transmission of volition from the brain to the hand, is 0.15 second. When the ear is employed instead of the eye, the time is reduced to 0.09 second.

time between the observation and recording of a phenomenon may vary greatly in the same person according to the state of health; but, as the principal retardation in these cases may, and probably does, occur in the brain itself (between the reception of the sensation and the issuing of the volition), it would not be philosophical to adduce such cases as proofs of a difference of velocity in the nerves either of sensation or of motion.

Whatever be the rate of wave motion in the muscle or nerve, the time of its molecular vibration may be determined by the note of the *susurrus* caused by the muscles, and by the note of the *tinnitus* produced by the nerves.

In the Croonian Lecture, read before the Royal Society, on November 16th, 1809, Dr. Wollaston called attention to the sound, or susurrus, produced by the muscles when in a state of contraction. He states correctly that this sound is best produced by inserting gently the extremity of the finger into the ear, bringing at the same time the muscles of the hand and forearm into strong contraction. The muscular susurrus may also be heard easily by using a stethoscope upon a contracted muscle, either of the observer's or of any other person's body.

Having described the mode of obtaining the sound, Dr. Wollaston correctly adds, that it "resembles most nearly that of carriages at a great distance passing rapidly over a pavement;" to which admirable description of the sound I would add, that it bears a most striking resemblance to the deep hum produced by the blowing fan of a large foundry.

Dr. Wollaston attempted to estimate the frequency of the elementary muscular contraction that produces the susurrus by allowing his ear to rest on the ball of his thumb, while his elbow was supported by a horizontal board, in which he had cut a number of equal notches, about one-eighth of an inch asunder. Against these notches he rubbed a pencil with a

regular motion, until he made the sound so produced to coincide roughly with that of the muscular contraction; and he attempted to estimate the number of notches passed over in a second. His conclusion from these comparative experiments is the following:—

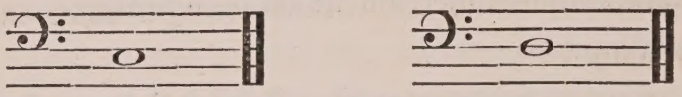
"The greatest frequency that I think I have observed was about 35 or 36 in a second, and the least was as low as 14 or 15 . . .; they appeared to be in general between 20 and 30 in a second; but it is possible that the method I employed may be found defective, and it is to be hoped that my estimate may be corrected by some means better adapted to the determination of intervals that cannot actually be measured."

An accidental observation made upon myself in the early part of 1862, enabled me to fix with the precision desired by Wollaston the rate of the muscular contraction that causes the susurrus. In December, 1859, on recovering from a fever, which I acquired from an English patient under the care of Dr. Jonathan Osborne, in Sir Patrick Dun's Hospital, I found that it had left the following trace of its visit in my system:— Occasionally since that period, when overworked by mental exertion, I have been subject to a singing in both ears, which is relieved by a drink of warm milk, and by sleep; sometimes, however, I have found the *tinnitus aurium* so great as to prevent sleep.

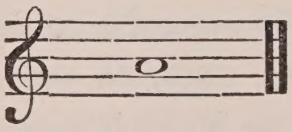
In June, 1862, while kept awake by this disagreeable noise, I amused myself with producing, by the contraction of the masseter muscles, their well-known susurrus. To my great surprise and pleasure, I observed that the tinnitus and susurrus were in unison, differing from each other by several octaves.

I followed up the clue thus found, and consulted my musical friends, whom I instructed in the mode of making Wollaston's experiment, without, however, informing them of the note that I myself believed to be the true sound of the susurrus and tinnitus. My friends have arranged themselves into

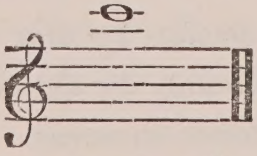
two groups, which have fixed upon CCC and DDD, respectively, as the note of their susurrus—that is to say, two octaves below bass C and D.



These notes are found on the new pianos only; and several of my friends informed me that the C or D, the lowest on their piano, was an octave above the susurrus; but, on inquiry, there was no difficulty in ascertaining what note was really indicated. The tuning of the pianos was carefully tested with a standard CC diapason, of 64 double vibrations per second, used for acoustical experiments, and corresponding with $426\frac{1}{3}$ vibrations in the second for the note, treble A.



Four of the observers, of whom two were ladies, found CCC, and five, of whom three were ladies, found DDD, which notes correspond, respectively, to thirty-two and thirty-six vibrations in the second.

To my own ear, the susurrus has constantly the sound of CCC; and the tinnitus had, on the morning of 30th June, 1862, the sound of the octave above treble C, that is to say, it was  ;* five octaves above the susurrus, and therefore corresponding to a rate of vibration thirty-two times faster than that of the muscle, or 1024 times in the second.†

* This is the note given out by a well-formed hand bell, of four inches diameter.

† Although the *tinnitus aurium* is caused by the molecular vibrations of the nerve particles, and takes its note from them, yet I believe it to be influenced by

The tinnitus is altogether independent of muscular or voluntary action of any kind, and is, I believe, a sign of the rate at which nervous action takes place in the brain.

The sound of the muscular susurrus, when fully heard, is ridiculously like the sound of the cab wheels of London heard in the silence of the night, when the absence of thoroughfare in the streets enables the cabmen to drive fats. I have measured the intervals of the Guernsey granite pavement, and found them to be about four inches, making therefore three impulses in a foot traversed by the cab wheels. If the cabs be supposed to drive eight miles an hour, the number of impulses per second will be

$$\frac{5280 \times 3 \times 8}{3600} = 35{\cdot}2.$$

Considering that the standard DDD of the susurrus is thirty-six vibrations per second, its resemblance to the sound of cab wheels ceases to be a matter of surprise.

In order to determine the sound of the susurrus precisely, I procured a long wooden tube, 3 inches by $2\frac{1}{2}$ inches inside, like an organ pipe, fitted with a moveable piston, so that on applying it to an acoustical bellows, on moving* the piston in or out, I could produce any note I pleased in the neighbourhood of CCC. This pipe is tuned, by means of the piston, to the susurrus of any observer, and then played in conjunc-

the resonance of the skull which contains the brain. The following observation throws light on this subject. On the 26th August, 1863, I received a violent blow from a racket ball in the back of the head, which partially stunned me; and I was conscious at the instant of receiving the blow of a loud singing sound in both ears, which I attributed to the vibration of the brain case, transmitted to the cochleæ aurium. This ringing sound seemed to me to have the same pitch as the singing that occasionally occurs in my ears.

* The piston is moved in the acoustical tube by means of an endless chain movement, made for me by Mr. Spencer, of Aungier-street.

tion with the standard CCC, and the beats counted if it be not in unison.

Let m denote the number of vibrations per second in the standard pipe, n the corresponding number in the tuned susurrus pipe, or *vice versâ*, so that m shall be greater than n, and let k be the number of beats per second counted on playing the two pipes together; then it is easily shown that

$$k = m - n. \qquad (6)$$

As examples of this principle I take the following observations:—

Dr. William Stokes, Regius Professor of Physic in the University of Dublin, kindly aided me in my investigations upon one occasion, by tuning the wooden pipe to the susurrus observed in himself, leaning both his elbows on a table, with the muscles of the fore-arm contracted, and making the sound of the pipe coincide with the susurrus when heard by both ears together.

On comparing the note he fixed on, with the standard CCC, it was found to be higher than it to such an extent as to produce 50 beats in 15 seconds.

Hence,

$$\frac{50}{15} = m - 32; \text{ or}$$

$$3\tfrac{1}{3} = m - 32;$$

and, finally,

$$m = 35\tfrac{1}{3} \text{ vibrations per second.}$$

This is DDD natural, a little flat, for DDD corresponds to thirty-six vibrations in the second.

In July, 1863, Professor Czermak, of Prague, assisted me at an acoustical experiment, and found his susurrus to correspond with a position of the moveable piston below CCC, such as to give 40 beats in 20 seconds.

Hence,

$$32 - \frac{40}{20} = k = 30;$$

showing that the pitch of his muscular susurrus corresponded to 30 vibrations per second.

The preceding investigation of the precise sound of the muscular susurrus was published by me in 1863, having formed part of a thesis for a medical degree in the University of Dublin in 1862; and shortly after its publication, Dr. Collongues, of Paris, forwarded to me a copy of a work published by him in 1862 ("Traité de Dynamoscopie," 8vo, Paris, 1862), in which he had solved the same problem, and found an identical result from his acoustical experiments. Dr. Collongues attempted to determine the pitch of the susurrus, with complete success, by means of a tuning fork, with sliding weights, which was put into unison with the susurrus, and then compared carefully with a standard tuning fork, whose time of vibration was determined graphically by the aid of a blackened revolving cylinder, which was marked with a zigzag line by a pin attached to one arm of the vibrating fork. The note fixed on by Dr. Collongues was Re_{-1}, or 72 vibrations per second, reckoned in the French fashion by half vibrations, and corresponding to 36 vibrations per second of English acousticians. I readily accord to Dr. Collongues the priority of publication of the exact determination of the pitch of Wollaston's susurrus, and believe it to be a matter of some interest to science that two independent observers, by different methods, one in Paris, and the other in Dublin, succeeded in arriving at a precise and almost identical value for this important physiological constant.

Dr. Collongues believes that the susurrus is caused by the nerves, and not by the muscles, and the greater part of his valuable book is taken up with an account of the variations of the susurrus in health and in disease. With this part of his inves-

tigations we have here no concern, as we are only interested, at present, in the rate of muscular action indicated by the susurrus. Dr. Collongues' tuning fork, which is made by M. Kœnig, of Paris, under the name of the Biometer, is an instrument more portable than my organ pipes, and in other respects also better suited for the investigation of variations in the susurrus; for this reason I have copied in the note* his description of the instrument.

* Le Biomètre est un instrument d'acoustique appliqué à l'homme bien portant ou malade, destiné à évaluer et à reproduire par l'unisson et à volonté les sons perçus ordinairement par l'auscultation des doigts.

A cet effet, le Biomètre fait entendre des sons de différente hauteur, les traduit en notes et en nombre de vibrations, ce qui permet la comparaison du rapport des intervalles qui existent des deux côtés du corps, soit dans l'état normal, soit dans l'état anormal, et d'exprimer en chiffres les différents degrés de santé et de maladie.

L'instrument se compose d'un manche, d'un diapason avec curseur et d'un dynamoscope.

Le manche est en caoutchouc : il comprend une poignée, une overture et une extrémité digitale. La poignée isole le bruit de la main qui supporte l'instrument, l'ouverture permet l'introduction de la tige du diapason, et l'extrémité digitale reçoit le dynamoscope.

Le diapason est formé de deux branches, longues de 30 centimètres; sur chacune de ces branches glisse un curseur qui peut adhérer sur tous les points de sa course, à l'aide d'une vis de pression. Chaque branche comprend huit divisions annotées de cette façon.

Notes et gamme du Biomètre,	*Re*$_{-2}$,	*Mi*$_{-2}$,	*Fa*$_{-2}$,	*Sol*$_{-2}$,	*La*$_{-2}$,	*Si*$_{-2}$,	*Do*$_{-1}$,	*Re*$_{-1}$.
Nombre absolu de vibrations,	36,	40,	42,	48,	54,	60,	64,	72.
Intervalles de *Re*$_{-2}$ à *Re*$_{-1}$,	1,	$\frac{10}{9}$	$\frac{7}{6}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{16}{9}$	2.
Intervalles de *Re*$_{-1}$ à *Re*$_{-2}$,	2,	$\frac{9}{5}$	$\frac{5}{3}$	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{6}{5}$	$\frac{9}{8}$	1.
Intervalle des notes,	1re	2e	3e	4e	5e	6e	7e	8e

L'acoustique a recherché quelle relation existe entre le nombre de vibrations de deux notes lorsqu'elles produisent tel ou tel accord.

L'accord entre deux notes peut être exprimé par le rapport de deux nombres de vibrations, et, si le rapport est égal à l'unité, les deux notes sont à l'unisson.

Accord parfait de la gamme du Biomètre : Re$_{-2}$, *Fa*$_{-2}$, *La*$_{-2}$, *Re*$_{-1}$, intervalle des vibrations en montant 1, $\frac{7}{6}$, $\frac{3}{2}$, 2.

The muscular susurrus varies in different persons from 30 to 36 vibrations per second, but in partially paralytic affections of the limbs it is always reduced in pitch. The following remarkable cases illustrative of this fact have come under my observation:—

On the 22nd May, 1863, I saw, in conjunction with Dr. Stokes, a gentleman engaged in the tobacco manufacture, in good health, but affected with paralysis agitans of both arms, probably produced by tobacco, but attributed by him to a slight injury of the left hand, by which he lost the nail of the thumb about three years previously, at which time, according to his account, the trembling motion of the hands and arms first appeared. We examined the muscles of the forearm and arm of both sides with a stethoscope, and found a "paddle wheel" susurrus in all the muscles, especially those of the forearm. Both Dr. Stokes and myself compared the sound of the susurrus estimated by the stethoscope with that of a

Re_{-1}, La_{-2}, Fa_{-2}, Re_{-2}.

Intervalle des vibrations en descendant:

1, $\frac{4}{5}$, $\frac{5}{8}$, 2.

Limite des sons perçus par l'auscultation des doigts:

Limite des sons graves: La_{-3}, 27 vibrations. Limite des sons aigus: Re_{-1}, 144 vibrations.

Manuel opératoire. On fait entrer la tige du Biomètre dans l'ouverture pratiquée dans le manche. Le dynamoscope est appliqué à son extrémité digitale, des deux branches du diapason sont tournées vers la terre. On les met en vibration en les rapprochant brusquement. Après avoir saisi la poignée du manche, on détruit les sons harmoniques en touchant légèrement les deux branches du diapason à leur extrémité fermée. On applique le dynamoscope dans l'oreille, on compare le son du Biomètre à celui produit par l'un des indicateurs, et si on ne les trouve pas semblables, on abaisse ou on élève les deux curseurs jusqu'à ce que les deux sons produisent l'unisson. Alors, grâce à cette loi de physique que deux sons de même hauteur ont toujours le même nombre de vibrations, on traduit en nombre le bourdonnement vital. On renouvelle l'expérience pour l'indicateur de l'autre main, et, après avoir obtenu sa valeur numérique, on compare les deux chiffres ou les deux notes, et l'on constate l'égalité ou la différence.

tuning fork properly weighted, and considered the number of vibrations to amount to only 6 per second, which is lower than two octaves below the normal CCC.

A friend of mine, aged forty-five years, who was a seven months' child, prematurely born while his mother was dangerously ill of typhus fever, has had slight congenital paralysis agitans of the right arm, forearm, and hand, all through his life; this gentleman was carefully examined by me in August, 1863, with the result that his muscular susurrus in the arm affected had only the pitch of 4 beats per second, although the susurrus of the other arm is of the usual note.

I have also ascertained, by over-fatiguing the forearm by racket playing, that the pitch of the muscular susurrus is lowered, and that a sensible paralysis agitans of a temporary character is set up by the over-exertion.

I have not been able to verify the continuance of the susurrus after death, except in cases of tetanus, nor have I succeeded in observing it in amputated limbs; but I have no doubt, from the published results of Dr. Collongues' careful observations, that my failure is simply the result of a deficiency in my own powers of observation, and that the susurrus may continue audible to more delicately organized ears than my own, long after death or amputation.

3. **On the Statical Work done by Muscles in continued Contraction.**—Muscles may be employed either in continuous statical work resisting pressure, or in dynamical work, lifting weights with intervals of repose. The work done by muscles in the latter case is easily measured by measuring the external resistances overcome; while in the estimation of statical work, there is much difficulty in obtaining numerical estimates of the work actually done: at the same time there is no doubt as to the reality of the work, for the muscles become rapidly fatigued when employed in doing statical work.

If both arms be extended horizontally, with the palm upward, either unloaded or loaded with equal weights in both hands, after a short time, much fatigue is experienced and the experimenter finds himself compelled to terminate the experiment—as the feeling of fatigue passes rapidly into pain, chiefly felt in the *supraspinatus* and central *deltoid* muscles. It is necessary to keep the arms quite horizontal, for if they be held either above or below the horizontal line the distressed muscles are aided by the action of other muscles.

The total work done in this case may be estimated as follows; let

w = weight held in hand;
a = weight of arm;
a = length of arm, measured from centre of glenoid cavity to centre of weight;
t = total time in seconds, until fatigue stops the experiment.

The muscles exert a force capable of sustaining the weight of the loaded arm at its centre of gravity, and if we suppose, this weight to be moved (with an unknown angular velocity) uniformly, the total work done is measured by the weight of the loaded arm multiplied into the arc through which the arm would move in the time during which the experiment lasts.

Let

x = distance of the centre of gravity from the centre of the glenoid cavity;
θ = the angle through which the arm would move in the time t;
ω = the unknown angular velocity.

The total work done is represented by

$$(w + a)\, x\theta = \text{Total work};$$

but

$$(w + a)\, x = a\left(w + \frac{a}{2}\right),$$

$$\theta = \omega t.$$

Hence we find

$$\text{Total work} = \omega a\left(w + \frac{a}{2}\right) t. \qquad (7)$$

The following experiments were made upon the unloaded arms:—

No. 1.—*Myself.*

a = 22 inches. w = 0.
a^* = 7.5 lbs. t = 420 secs.

Total work in ft. lbs. = 2887.5 ω.

No. 2.—*Dr. A. W. Foot.*

a = 24 inches. w = 0.
a = 7 lbs. t = 480 secs.

Total work in ft. lbs. = 3360 ω.

No. 3.—*Captain John Haughton, R. A.*

a = 22 inches. w = 0.
a = 5 lbs. 14 oz. t = 720 secs.

Total work in ft. lbs. = 3877.5 ω.

No. 4.—*Mr. M. H. Ormsby.*

a = 24 inches. w = 0.
a = 6.5 lbs. t = 390 secs.

Total work in ft. lbs. = 2535 ω.

* The approximate weight of the arm may be found by weighing it in a balance, whose scale is on a level with the centre of the glenoid cavity, and treating it as a lever, so that the weight of the arm shall be to the weight that supports any point of the arm inversely as the distances of the centre of gravity and of that point from the centre of the glenoid cavity.

No. 5.—*Mr. Samuel Haughton.*

a = 22 inches. w = 0.

a = 8.5 lbs. t = 525 secs.

Total work in ft. lbs. = 4090.6 ω.

If we collect together into one Table the preceding results, we obtain as follows :—

TABLE I.—*Work done by the Shoulder Muscles employed in holding the unloaded Arms in the horizontal Position, with Palms upward :—*

	Age.	Weight.	Height.	Work done.	Time of Exhaustion.
No. 1.	41 ys.	126 lbs.	67 inches.	2887.5 ω.	420 secs.
No. 2.	25 ,,	121 ,,	70 ,,	3360.0 ,,	480 ,,
No. 3.	30 ,,	120 ,,	65½ ,,	3877.5 ,,	720 ,,
No. 4.	24 ,,	144 ,,	69 ,,	2535.0 ,,	390 ,,
No. 5.	17 ,,	123 ,,	68 ,,	4090.6 ,,	525 ,,
			Means, . .	**3350·12** ω.	**507** secs.

The following experiments were then made on the same persons, holding the arms horizontally, as before, with two pounds weight in each hand, with the following results. The times of exhaustion were found to be as follows :—

1. Myself, ... 200 secs.
2. Dr. Foot, ... 210 ,,
3. Captain Haughton, . . 300 ,,
4. Mr. Ormsby, ... 180 ,,
5. Mr. Samuel Haughton, . 240 ,,

Using these numbers in equation (7), and substituting 2 lbs. for w, and half the weight of the arm for a, we obtain—

TABLE II.—*Work done by the Shoulder Muscles employed in holding the loaded Arms in the horizontal Position, with the Palms upwards.*

	Time of Exhaustion.	Work done.
No. 1.	200 secs.	2108.3 ω.
No. 2.	210 „	2310.0 „
No. 3.	300 „	2715.6 „
No. 4.	180 „	1890.0 „
No. 5.	240 „	2750.0 „
Means,	**226** secs.	**2354·78** ω.

I was led by several considerations to the conclusion, that it was probable that the total work done in exhausting muscles varies inversely as the rate of doing the work; from which it would follow that the square of the total work varies as the time of doing the work; this would give us the relation

$$\frac{(\text{Total work})^2}{\text{Time}} = \text{Constant.} \qquad (8)$$

Calculating this constant from Tables I. and II., I found the following results:—

Product of Total Work and Rate of Work.

	First set of Experiments.	Second set of Experiments.	Ratio.
No. 1.	19852	22224	1.12
No. 2.	23520	25410	1.08
No. 3.	20882	24581	1.17
No. 4.	16478	19845	1 20
No. 5.	31873	31510	0.99
		Mean,	**1·11**

From these results I obtained my first conception of the *Law of Fatigue*, which I shall fully develope further on, with the proofs of it, derived from more extensive and varied experiments ; and I shall also prove that the coefficient ω is equal to unity.

The *Law of Fatigue* may be thus stated :—

When the same muscle (or group of muscles) is kept in constant action until fatigue sets in, the total work done, multiplied by the rate of work, is constant.

Thus, if W denote the total work done, and T denote the time of doing it, the rate of work will be expressed by W divided by T, and

$$\text{Total Work} \times \text{Rate of Work} = W \times \frac{W}{T}.$$

If this product be constant, we see that the work and rate of work may be represented by the coordinates of an hyperbola referred to its asymptotes as axes of coordinates; while the relation of the total work to the time of doing it, may be represented, as in equation (8), by the coordinates of a parabola referred to a tangent and diameter as axes of coordinates.

As an illustration, let us suppose that a man walking at the usual rate does not become tired until he has walked 30 miles. On another occasion, let him walk twice as fast, then (neglecting the difference of action necessitated by the different rates of walking) we should expect, by the Law of Fatigue, that the man will be completely fatigued at the end of 15 miles, having done only half the total work, in a quarter of the original time. If the man walk three times as fast, he will become tired out at the end of 10 miles, in one-ninth part of the original time; and so on, the total work done always varying as the square root of the time necessary to produce fatigue.

In order to find the quantity of muscle that gives out the preceding quantity of work, it would be necessary to know the weights of the living muscles in each of the persons observed; but, as this, except in rare cases, is impossible, we must endeavour to obtain the result, indirectly, by observation of the muscles in the dead subject. With this object in view, I carefully dissected the muscles in four dead bodies, selecting those only that were well developed, and had not experienced much wasting of muscular tissue during their last illness. From these dissections, I obtained the following results :—

TABLE III.—*Weights of Muscles.*

	Supraspinatus.	Portion of Deltoid employed.	Deltoid.
A. Female,	1.01 oz. av.	1.82 oz. av.	7.10 oz. av.
B. Male,	1.76 ,,	2.98 ,,	10.16 ,,
C. Male,	2.18 ,,	3.99 ,,	12.97 ,,
D. Male,	1.75 ,,	2.53 ,,	10.25 ,,
Means,	1·675 oz.	2·830 oz.	10·12 oz.

The only difficulty in these observations consists in estimating the portion of the deltoid actually employed in holding the arm horizontal; this was done by holding the arm in that position while dissecting the deltoid, and estimating as well as I could the fibres actually in action. It will be seen that my estimate varies somewhat in the different cases, as follows :—

In A, the portion of Deltoid was $\frac{1}{3\cdot90}$th of the whole.
,, B, ,, ,, $\frac{1}{3\cdot41}$th ,,
,, C, ,, ,, $\frac{1}{3\cdot25}$th ,,
,, D, ,, ,, $\frac{1}{4\cdot05}$th ,,

Mean, . . $\frac{1}{3\cdot65}$th of the whole.

The portion of the deltoid employed, as found from the means, is $\frac{1}{3 \cdot 57}$th of the whole; so that, combining all the foregoing results together, it is very probable that the weight of deltoid used in holding the arm horizontal is $\frac{10}{36}$ths of the whole weight of the muscle.

If the muscle be divided into a clavicular, an acromial, and a scapular portion, the portion used in holding the arm horizontal is very nearly $\frac{3}{4}$ths of that taking origin from the whole tip and line of the acromion; but for the purpose of future experiments it may be found easier to weigh the whole muscle, and take $\frac{10}{36}$ths of that weight, than to separate it into its parts by direct dissection. The mean weight of the portion of the *deltoid* engaged in holding the arm in the horizontal position has been found to be 2.83 oz. av.; and to this must be added the mean weight of the *supraspinatus*, which may be found as follows:—

TABLE IV.—*Weight of Supraspinatus Muscle.*

A.	Female,	1.01 oz. av.
B.	Male,	1.76 ,,
C.	Male,	2.18 ,,
D.	Male,	1.75 ,,
E.	Male,	0.93 ,,
P. K.	Male,	2.40 ,,
	Mean, . .	1·671 oz. av.

This result is almost identical with that found in Table III., from four well-developed dead subjects. This result of Table IV. is interesting from the fact that it contains the actual weight of the *supraspinatus*, observed 40 minutes after death, in a criminal (P. K.) executed, by hanging, for murder. As I shall have occasion to refer to this case for other purposes, I here give the data, as observed by myself:—

P. K. was executed on the 20th of July, 1865, for murder:—

1. Weight in clothes, 160 lbs.
2. Length of drop, 14 feet 6 inches.
3. Height during life, 69½ inches.
4. Height after death, 71 inches.
5. From tip of acromion to metacarpo-phalangeal articulation, 28 inches.
6. From olecranon to metacarpo-phalangeal articulation, 15 inches.
7. Weight of *m. biceps humeri*, 5.9 oz. av.
8. „ *m. supraspinatus*, 2.4 „
9. „ *m. rectus femoris*, 9.2 „
10. „ *brain*, 53.3 oz. av.*

Combining together the results of Tables III. and IV., we have:—

1. Average weight of *supraspinatus*,	1.67	oz. av.
2. Average weight of portion of *deltoid* employed in holding the arm in the horizontal position,	2.83	„
Total Weight,	4.50	oz. av.

Hence, assuming the coefficient ω, to be equal to unity, we find, from Tables I. and II.:—

$$\left.\begin{array}{l}\text{Work done by Shoulder Muscles employed in holding the unloaded Arms horizontally, per ounce, per second,}\end{array}\right\} = \frac{3350.12}{507 \times 4.5} = 1.468 \text{ ft. lbs.}$$

* While these pages were passing through the press, I had an opportunity of weighing the brain of a distinguished mathematician, and found it to be 47 oz. I leave the deduction of suitable inferences to phrenologists.

$$\left.\begin{array}{l}\text{Work done by Shoulder Muscles, holding the loaded Arms horizontally, per ounce, per second.}\end{array}\right\} = \frac{2354.8}{226 \times 4.5} = 2.315 \text{ ft. lbs.}$$

Having thus found the work done by the muscles of the shoulder, I proceeded to check the result by observations made on the muscles of other parts of the body; and after trying many experiments on other muscles, such as the pronators of the forearm, and the openers of the jaw, &c., I finally selected as the most reliable the following experiments made on the flexors that hold the forearm in the horizontal position.

If the forearm be held, with the palm of the hand upwards, in the horizontal position, it is allowed by all authors on Animal Mechanics, that it is kept in that position by the action of the *biceps humeri*, and of the *brachiæus* or *brachialis anticus;* to these muscles must be added the *supinator radii longus*, which in the horizontal supine position of the forearm acts as a *flexor cubiti*, and has a resultant that passes above the axis of the elbow joint.

On holding a weight suspended from the wrist, with the forearm in the position described, it is found that the feeling of fatigue commences on the outer side of the lower half of the arm, with the *supinator*, then extends to the *biceps humeri*, and finally reaches the *brachiæus*, when the experimenter is forced to yield to the insupportable feeling of fatigue, and to confess that he can hold out no longer.

In the following experiments, I have made use of the same persons who so kindly aided me in the experiments on the muscles of the shoulder.

The total work done is expressed, as before, by the equation

$$\text{Total work} = \omega\,(w + a)\,xt, \qquad (9)$$

where

ω = coefficient of angular rotation.
w = weight.
a = weight of forearm.
x = distance of centre of gravity of loaded forearm from centre of elbow joint.
t = time in seconds, until fatigue sets in.

No. 1.*—*Myself.*

x = 8.2 in. w = 8 lbs.
a = 3.75 lbs. t = 480 secs.

Total work in ft. lbs. = 3854 ω.

No. 2.—*Dr. Foot.*

x = 8.2 in. w = 8 lbs.
a = 3.5 lbs. t = 360 secs.

Total work in ft. lbs. = 2829 ω.

No. 3.—*Captain Haughton.*

x = 8.38 in. w = 9 lbs.
a = 2.94 lbs. t = 720 secs.

Total work in ft. lbs. = 6003.3 ω.

No. 4.—*Mr. M. H. Ormsby.*

x = 8.27 in. w = 9 lbs.
a = 3.25 lbs. t = 555 secs.

Total work in ft. lbs. = 4685.4 ω.

* In all these experiments, the weight was suspended at a distance of 9 inches from the centre of the elbow joint.

No. 5.—*Mr. Samuel Haughton.*

x = 8.2 in. w = 9 lbs.

a = 4.25 lbs. t = 780 secs.

Total work in ft. lbs. = 7062.1 ω.

Combining together the previous results into one Table, we obtain—

TABLE V.—*Work done by the* Biceps humeri, Brachiæus (Brachialis anticus), *and* Supinator radii longus, *employed in holding the loaded Forearm horizontal, with the Palm upward :—*

	Time of Exhaustion.	Total work done.
No. 1.	480 secs.	3854.0 ω.
No. 2.	360 „	282.90 „
No. 3.	720 „	6003.3 „
No. 4.	555 „	4685.4 „
No. 5.	780 „	7062.1 „
Means,	**579** secs.	**4886·76** ω.

These experiments differ so much from those made on the shoulder, both as to rate of work and amount of muscles employed, that they are not directly comparable with them; still it may be useful to deduce from them a coefficient of work per ounce per second.

This coefficient, as well as those already given for the shoulder muscles, is not constant, but varies with the rate of work.

The weight of the muscles employed in giving out this quantity of work, previous to their exhaustion by fatigue, is estimated, as before, by observations on the dead subject, and

in making this estimate, we have no difficulty, as before, in finding the portions of muscles employed, for they are perfectly well defined; at the same time, it may be considered that the muscles now under investigation are not in themselves so well adapted as the muscles of the shoulder, to give an average result, as they differ more widely in one man as compared with another, than the shoulder muscles are found to do. This difference is caused by the more varied uses of the forearm, and of the muscles actuating it, than of the muscles of the shoulder that hold the entire arm horizontal.

The results obtained by me from well-selected subjects are contained in the following Table :—

TABLE VI.—*Weights of Muscles employed in holding the Fore arm horizontal, with the Palm upwards :—*

	Biceps.	Brachiæus.	Supinator radii longus.
A. Female,	2.30 oz. av.	2.74 oz. av.	0.75 oz. av.
B. Male,	3.91 ,,	4.01 ,,	1.40 ,,
C. Male,	3.24 ,,	4.30 ,,	2.30 ,,
D. Male,	4.00 ,,		
E. Male,	1.93 ,,	1.66 ,,	
P. K. Male,	5.90 ,,		
Means,	**3.54⅔** oz. av.	**3.17¾** oz. av.	**1.48⅓** oz. av.

Hence we find that the average total weight of the muscles employed in holding the forearm horizontal is **8.2075** oz. av.

From the preceding result as to the average weight of the muscles acting on the forearm, combined with Table V., and assuming ω to be equal to unity, we find—

$$\left.\begin{array}{l}\text{Work done by muscles holding the loaded Fore-arm horizontally, per ounce, per second,}\end{array}\right\} = \frac{4886.76}{579 \times 8.2075} = 1.028 \text{ ft. lbs.}$$

My first attempt to calculate a formula to represent the *Law of Fatigue*, in the case of the arms extended horizontally, was based upon the periodicity of muscular action, as proved by the muscular *susurrus*, explained in the preceding section, and may be thus stated :—

Each portion of the muscle contracts, relaxes, and goes through all its changes thirty-two times in each second, and does so in succession, so as to keep the ends of the fingers steady ; but the amount of work given out by the whole muscle is evidently the same as if all the particles went through their changes simultaneously, and not in succession.

If the whole muscle acted simultaneously, the arm would fall like a compound pendulum during the $\frac{1}{32}$nd part of a second, and then be raised to its original level by the renewed contraction of the muscle. It may be shown, as follows, that in this case the arc through which the arm falls is that which would correspond to the centre of oscillation falling freely for $\frac{1}{32}$nd of a second.

For, if W denote the weight of the arm, x the distance of its centre of gravity from the centre of the glenoid cavity, ω the angular velocity acquired after falling for any time, and I the moment of inertia of the whole arm ; we have, by D'Alembert's principle, the following equation :—

$$Id\omega = Wxdt. \qquad (10)$$

If θ denote the angle made with the horizontal line at any moment by the arm, and k denote its radius of gyration, and l the length of the equivalent simple pendulum ; we have at once from equation (10)—

$$\frac{d^2\theta}{dt} = \frac{gx}{k^2} = \frac{g}{l}; \qquad (11)$$

multiplying both sides by $2\frac{d\theta}{dt}$, and integrating, we obtain

$$\left(\frac{d\theta}{dt}\right)^2 = \omega^2 = \frac{2g\theta}{l} + Const.\ ;$$

and as ω and θ vanish together, we have

$$\omega^2 = \frac{2g\theta}{l},$$

or

$$(\omega l)^2 = 2g\ (\theta l),$$

or

$$v^2 = 2gs, \qquad (12)$$

where v denotes the velocity acquired by the centre of oscillation in traversing a small arc, such as would be described in $\frac{1}{32}$nd part of a second, and s denotes the space passed through by the same point.

The equation (12) is the same as that of falling bodies, and therefore for small arcs and times, the centre of oscillation of the arm falls through the same space as it would if the arm were released from its socket, and fell freely through space.

Let us now use the following notation :—

l = radius of oscillation;
x = distance of centre of gravity from centre of glenoid cavity;
δs = small space through which the centre of oscillation falls in $\frac{1}{32}$nd part of one second;
n = number of such falls during
t = whole time required to fatigue the muscles;
a = weight of arm.

The total work done by the muscles, on the foregoing supposition, in the time t, is evidently

$$(w + a)\frac{x}{l}\, n\delta s;$$

but $n\delta s$ varies as t; therefore

$$\text{Total Work varies as } (w + a)\frac{x}{l}t;$$

and the rate of work varies as

$$(w + a)\frac{x}{l}$$

Now, by the *Law of Fatigue*, the total work varies inversely as the rate of work, and hence we have

$$(w + a)^2 \frac{x^2}{l^2} t = \text{Constant};$$

but, supposing the arm to be a simple uniform cylinder, we have

$$\frac{x}{l} = \frac{3}{4} \frac{(2w + a)^2}{(w + a)(3w + a)}.$$

Squaring, and substituting in the preceding equation, we obtain

$$\frac{(2w + a)^4 t}{(3w + a)^2} = A,$$

or,

$$t = \frac{A(3w + a)^2}{(2w + a)^4}. \qquad (12a)$$

In order to compare this formula with observation, I made use of Mr. Stanley Jevons' experiments published in *Nature* (30th June, 1870), using the following constants, which I found to answer best :—

$$A = 22050,$$
$$a = 7.4 \text{ lbs.}$$

The comparison of observation and theory is shown in the following Table :—

Mr. Jevons' experiments (holding weights with arm horizontal).

w.	t (observed).	t (calculated).	Difference.
14 lbs.	32.5 secs.	34.2 secs.	− 1.7 secs.
10 ,,	60.3 ,,	54.7 ,,	+ 5.6 ,,
7 ,,	87.4 ,,	84.8 ,,	+ 2.6 ,,
4 ,,	147.9 ,,	147.6 ,,	+ 0.3 ,,
2 ,,	218.9 ,,	234.4 ,,	− 15.5 ,,
1 ,,	321.2 ,,	305.5 ,,	+ 15.7 ,,

The agreement here shown between observation and theory is quite satisfactory, and I remained quite contented with it, until I came to plot the curve (12*a*), and compare it with the observations, as shown in the accompanying diagram (page 41).

The relation between t and w represents a curve of the fifth degree, one branch of which is shown in the figure ; the other branch being hyperbolic in form, corresponding to positive values of t, and negative values of w greater than $\frac{a}{2}$. Mr. Jevons' observations are shown by the small circles corresponding to the weights used by him, and their agreement with the curve is evident.

Differentiating equation (12*a*) we find

$$\frac{dt}{dw} = -2A\,\frac{(6w + a)\,(3w + a)}{(2w + a)^5} \qquad (12b)$$

The preceding equations prove that—

1°. The values of t are positive, whether a weight be held in the hand, or whether the hand be supported by a force acting upwards.

2°. The axis of w is asymptotic to the curve at positive and negative infinity.

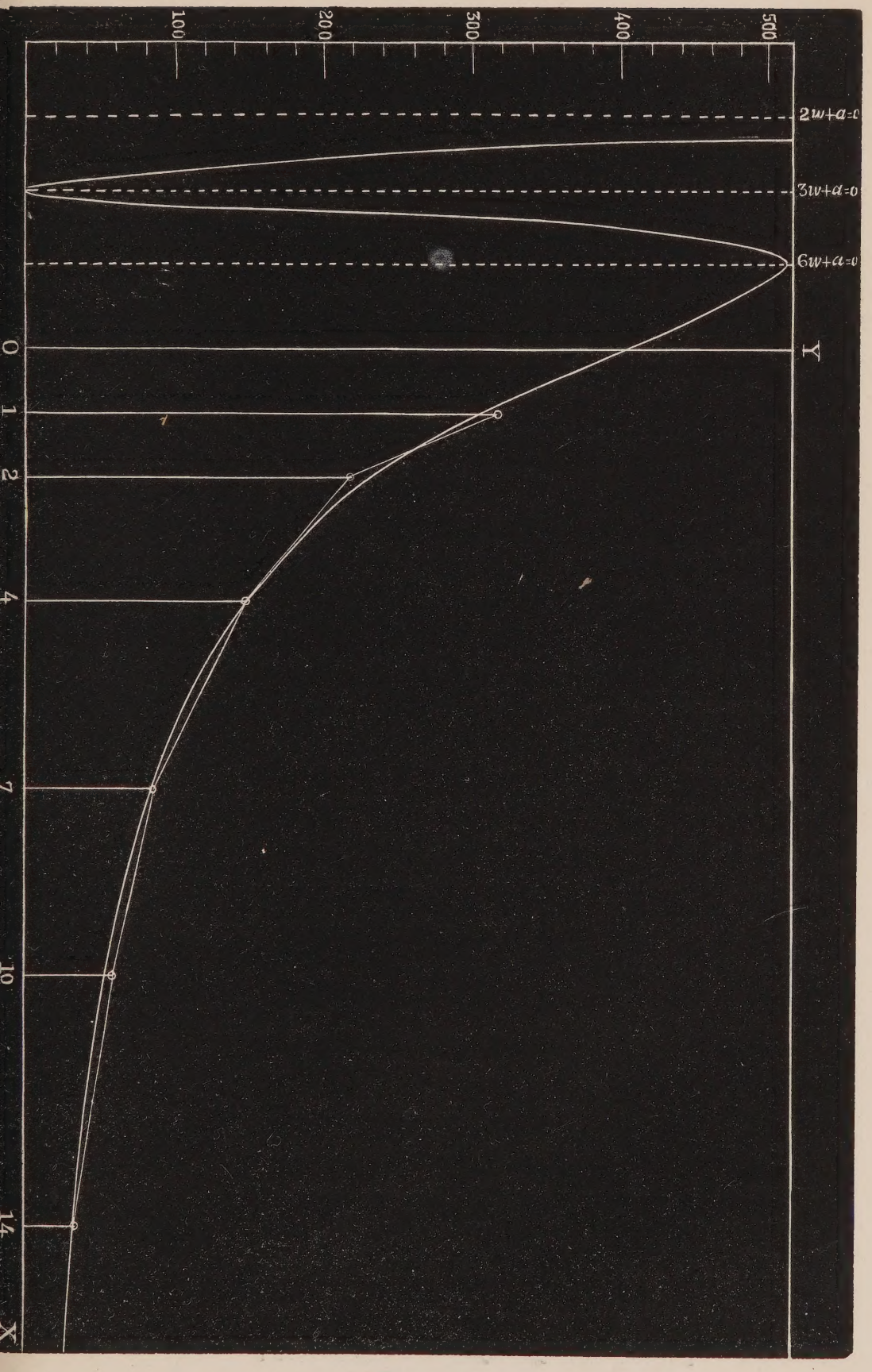


DIAGRAM.

3°. The line $2w + a = 0$, is also an asymptote to the curve.

4°. The time t has a *maximum* corresponding to the value of w given by the equation $6w + a = 0$.

5°. The value of t becomes equal to zero, and the curve is a tangent to the axis of w, when $3w + a = 0$.

The first three of these conditions are fulfilled by observation, for—

1°–2°. The time of holding up the arm must be always positive, and must become zero, when the weight is infinite.

3°. When $2w + a = 0$, the arm is supported by a force equal to half its weight applied at the hand, and being therefore in a condition of statical equilibrium, can be held up for an infinite time.

The last two conditions, however, are inconsistent with observation, for—

4°. The value of t, for all positive values of w, must increase continuously as w diminishes to zero; and for negative values of w can have no maximum but infinity, corresponding to the condition for statical equilibrium, $2w + a = 0$.

5°. The value of t can never be zero, except when w is infinitely great.

From the preceding investigation, I became satisfied that my first idea, viz., that the arm dropped and was raised again during repeated short intervals, was erroneous, and that a simpler form of curve would represent better the relation between t and w. This curve I afterwards found to be a cubical hyperbola, which represents the observations equally well, and is free from all the preceding objections.

So far as the observations themselves are concerned, they can be represented by any curve of an hyperbolic form, and

our difficulty is to find what kind of curve is best. Having found that a curve of the fifth degree will not answer, I shall now show that the relation between w and t may be represented by a cubical hyperbola.

By equation (7),

$$\text{Total work} = \omega a \left(w + \frac{a}{2} \right) t;$$

and

$$\text{Rate of work} = \omega a \left(w + \frac{a}{2} \right).$$

But, assuming the truth of the Law of Fatigue, the product of these two expressions will be constant. Hence

$$\omega^2 a^2 \left(w + \frac{a}{2} \right)^2 t = \text{constant},$$

or,

$$\left(w + \frac{a}{2} \right)^2 t = A. \qquad (12c)$$

This equation represents a cubical hyperbola, whose asymptotes are

$$t = 0,$$

$$w + \frac{a}{2} = 0.$$

This curve is free from the objections made to the curve of the fifth degree, for t becomes zero when w is infinite, and increases continuously, as w diminishes, and becomes infinite when $w + \frac{a}{2} = 0$, which corresponds to the case of statical equilibrium; and t has no intermediate maximum between zero and positive infinity. In fact, if we differentiate the equation (12c), we find

$$\frac{dt}{dw} = -\frac{4A}{(2w + a)^3}. \qquad 12d)$$

This equation shows that the tangent to the curve does not become horizontal for any value of w, except infinity.

An interesting consequence may be drawn from equation (12*c*) by seeking the relation between the *useful work done* and the weight w. The useful work may be represented by the product of w and t; hence if u denote the useful work, we find

$$u = wt,$$

but

$$t = \frac{A}{\left(w + \frac{a}{2}\right)^2}.$$

Hence

$$u = \frac{Aw}{\left(w + \frac{a}{2}\right)^2}. \qquad (12e)$$

From this equation, which denotes a cuspidal cubic, I shall hereafter show that the useful work is a maximum when the weight held in the hand is half the weight of the arm.

4. **On the Dynamical Work done by Muscles alternately contracted and relaxed.**—In the preceding section, I have discussed the work done by muscles kept in a state of continued contraction until tired out; and I have called this work "Statical," because the muscles appear to be at rest, and really do undergo no movement of translation in space, although a very rapid and active molecular movement takes place within their substance, to which the work done is altogether due.

In the present section, I shall consider the more usual kind of work done by muscles, to which the name "Dynamical" is given, the amount of which is measured by external movements of various parts of the body, and by the removal of external resistances through spaces easily measured.

In most kinds of labour which are recognised as effective, and which are usually employed, almost all the muscles of the upper and lower extremity are employed, together with the

muscles of the lumbar and dorsal vertebræ; and previous to instituting a comparison of the work done in this kind of labour with that done by muscles in continued contraction (which has been already discussed), it is necessary to have an approximate estimate of the total weight of all the muscles of the upper and lower extremity in a healthy man of average weight.

In order to obtain this estimate, I made a series of observations on the weights of the muscles of two well-developed male subjects, of 71 inches and 67 inches in height, respectively, and who had died of acute diseases (nephritis and pneumonia), that terminated life without much loss of substance by wasting.

The results thus obtained were the following:—

Muscles of Lower Limb.		*Muscles of Upper Limb.*	
1. Hip Joint, . .	77.33 oz.	1. Shoulder Joint, .	47.30 oz.
2. Knee Joint, . .	55.28 „	2. Elbow Joint, .	22.94 „
3. Ankle and Foot,	32.91 „	3. Wrist and Hand,	17.33 „
4. Lumbar, . .	14.81 „	4. Trunco-scapular,	19.41 „
	180.33 oz. av.		106.98 oz. av.

Doubling these results we find

Muscles of Lower Limbs =	360.66 oz.
Muscles of Upper Limbs =	213.96 „
Total	**574.62** oz.

If we compare the muscles of these two subjects with those of P. K. (p. 32), dissected in the prime of life, we find reason to believe that 50 per cent. should be added to the muscles of even the best selected subjects, in order to bring them up to the standard of life.

Thus,

1. *m. rectus femoris* (selected subject), . . . 6.1 oz.
m. rectus femoris (P. K.), ... 9.2 „
Percentage to be added,. . 50.8

2. *m. biceps humeri* (selected subject), . . 3.24 oz.
m. supraspinatus „ . . . 2.18 „
5.42 oz.

m. biceps humeri (P. K.), ... 5.9 oz.
m. supraspinatus „ . . . . . 2.4 „
8 3 oz.
Percentage to be added, . 53.1

We may therefore safely assume 50 per cent. to be added to the muscles of a carefully selected subject in order to bring them to the standard of life.

Hence,

Total muscles of upper and lower limbs, . 574.62 oz.
Add 50 per cent. on ditto, . . . 287.31
Total, ... **861.93** oz. av.

The work done by the muscles of the human body, in exertion that calls into play nearly all the muscles of the body, may be divided into excessive work and ordinary work. The first kind, or excessive work, produces an injurious effect upon the respiratory and circulatory systems, rather than upon the muscles themselves; so much so, indeed, that death (which sometimes is the consequence of excessive muscular work) is found to be caused, in this kind of over-exertion, by pulmonary apoplexy, or by rupture of an artery, produced by the effort made by the lungs and heart to supply fresh blood to the over-worked muscles.

Example No. 1.—As a first example of excessive muscular work, I shall take the case of an eight-oared outrigger propelled at the rate of one knot in seven minutes. The mea-

surements by means of which the following calculation is made, were kindly furnished to me by Dr. E. A. Parkes, and by Mr. Maclaren, of Oxford.

The Resistance offered by the water to the motion of the boat is divisible into the following parts, due respectively to,

1. The Distortion of the particles of water;
2. The production of Currents;
3. The production of Waves;
4. The production of Frictional Eddies.

In the case under consideration, the first three causes of Resistance may be neglected, in consequence of the "fair" form of the boat, and of the limited speed at which she is driven; and the whole resistance may be regarded as due to the production of Frictional Eddies.

In discussing the amount of Resistance due to this cause, I shall adopt the principles laid down by Professor Rankine,* from which it appears that

$$\text{The Eddy Resistance} = f\, w\, \frac{c^2}{2g} \int q^3 ds\,;$$

where ds denotes the element of the boat's skin;

q, the ratio which the velocity of gliding of the water over that element bears to the speed of the boat;

c, the speed of the boat;

g, gravity;

w, the specific gravity of the water, or weight of one cubic foot;

f, the coefficient of friction (= 0.0036).

In this equation, $\int q^3 ds$ is the *augmented surface* of the boat's skin, and is supposed to sum up together the skin resistance and that due to the excess of water in front, and to the deficiency of water behind.

* "Treatise on Shipbuilding," Fol., Mackenzie, London, 1866, pp. 78 *et seq.*

From the value of the coefficient of friction employed (which is deduced from Professor Weisbach's experiments on the flow of water in iron pipes), it follows from the preceding equation that *at ten knots the Eddy Resistance is nearly equal to one pound avoirdupois per square foot of Augmented Surface ;* and varies, for other speeds, as the square of the speed.*

The whole difficulty of the calculation of the eddy resistance turns upon the calculation of the augmented surface, which is effected by Professor Rankine, on the assumption (conformable to repeated experiments), that the augmented surface and its resistance are the same as those of a Trochoidal ribbon, whose length is the length of the boat on the plane of floatation, whose breadth is the mean immersed girth of the boat, and whose coefficient of augmentation is

$$1 + 4 \sin^2\omega + \sin^4\omega,$$

where ω is the angle of the greatest obliquity to the horizon formed by a tangent to the Trochoid.

Applying the foregoing principles to the sections of the Oxford eight-oar when loaded with its crew,† I have found the following results :—

1. Length of plane of floatation = 52 ft.
2. Girth of central immersed section = 31.5 in.
   (Mean immersed girth) = $\frac{2}{3}$rds = 21 in. = 1 75 ft.
3. Sine of obliquity = $\frac{1}{4}$.

Hence the coefficient of augmentation is

$$1 + 4\,(\tfrac{1}{4})^2 + (\tfrac{1}{4})^4 = 1.254\,;$$

and the

Augmented surface = 52 × 1.75 × 1.254 = 114.11 sq. ft.

The speed of the boat is assumed to be one knot in seven

* It is exactly $\frac{6}{6}\frac{2}{4}\frac{5}{4}$ths of a pound.

† The weight of the boat is 350 lbs., and the average weight of the crew is 11 st. 4 lbs. each, and the coxwain is 8 stone.

minutes, or $\frac{60}{7}$ knots per hour; hence, by the rule already laid down, the resistance per square foot of augmented surface is

$$\left(\frac{60}{7}\right)^2 \times \frac{1}{100} \times \frac{625}{644} = \frac{36}{49} \times \frac{625}{644} \text{ lb. av.}$$

and finally, the total resistance is

$$114.11 \times \frac{36}{49} \times \frac{625}{644} = 81.36 \text{ lbs.}$$

This resistance is overcome through the space of one knot (1000 fathoms) in seven minutes; and therefore

$$\text{The Total Work done} = \frac{81.36 \times 6000}{2240} = 218 \text{ ft. tons.}$$

The work done per man is 27.25 ft. tons in seven minutes, or nearly 4 ft. tons each minute.

A good idea may be formed of the rate in which the muscles give out work in a boat-race, from comparing this work with the average daily work of a labourer. At many kinds of labour there are 400 ft. tons of work accomplished in ten hours. The oarsman therefore performs, in one minute, the hundredth part of his day's labour; and, if he could continue to work at the same rate, he would finish his day's task in $1^h.\ 40^m.$, instead of the customary 10 hours.

The work done, therefore, in rowing one knot in seven minutes, is, while it lasts, performed at a rate equal to *six times* that of a hard-worked labourer.

The rate of work in this case is four tons per minute; and if we assume (p. 45) 575 oz. av. as the weight of muscles employed, we find the rate per ounce per minute to be

$$\frac{81.36 \times 6000}{8 \times 7 \times 575} = 15.169 \text{ ft. lbs. per oz. per minute.}$$

The experiment detailed in the note,* made under the direction of Mr. Maclaren, was communicated to me after I had made the preceding calculation, and seems to confirm the result arrived at as to the resistance overcome in this kind of labour.

* *Mr. Maclaren's Experiment.*—"An eight-oared racing boat, weighted with sandbags to represent an 11 st. 3 lb. crew (the weight for which she was built), and steered by an 8 st. coxwain, was towed over part of the Oxford course, where the water is straight, broad, and deep.

"The four-oared boat by which she was towed was itself towed by men on the bank, and kept in a straight course by a coxwain. The eight-oar was kept as nearly as possible in a line with the four-oar by the coxwain placed on board for that purpose.

"The towline from the four to the eight was fastened to the bow-oar's thwart in the eight, exactly on a line with the keel, and the strain measured by a dynamometer (a Salter's spring-balance), interposed between the end of the tow line and the four-oar.

The distance travelled, ...	560 yards.
The time occupied, ...	6^{m}. 20^{s}.
The average strain on the dynamometer, . .	7 lbs.

"There was a light side wind, but not enough to ruffle the water, or seriously interfere with the experiment. The strain was measured when the keels of the two boats were as nearly as possible in the same straight line. The course was against the stream, which is very slight."

In this case, the velocity expressed in knots per hour is

$$\frac{560 \times 60 \times 3}{19 \times 2000} = 2.65 \text{ knots per hour.}$$

In the case already considered, the velocity was assumed to be one knot in seven minutes, or

$$\frac{60}{7} = 8.57 \text{ knots per hour.}$$

Hence it follows (assuming the resistance to increase as the *square* of the velocity) that the resistance at one knot per seven minutes would be

$$\left(\frac{857}{265}\right)^2 \times 7 \text{ lbs.} = 73.21 \text{ lbs.}$$

Professor Rankine, in a letter addressed to me, 12th September, 1866, suggests that I ought to have reduced Weisbach's coefficient from 0.0036 to 0.0030,

Example, No. 2.—The most effective mode in which the labour of man can be employed is in lifting his own weight through a height for a number of consecutive hours. Captain Coignet, a French officer of engineers, invented (1835) a hoist or box-lift, applied to the lifting of barrows of earth from an excavation of about forty feet deep, which was intended to be used in the formation of earthworks for fortifications. This box-lift consists of a strong rope passing over a large pulley, and having suspended at each end a cage, or enclosed platform. The barrow load of earth is brought by an assistant to the foot of the hoist, and placed in the cage, which is supposed to have just descended to the lower level. At the same moment, the labourer, with an empty barrow, steps

because the skin of the boat is smoother than the skin of iron steam-boats, and he has found that the friction in iron pipes is diminished to about five-sixths, by lining them with smooth pitch. This would reduce the result of my calculation from 81.36 to 67.80 lbs.

In consequence of my publication of the foregoing results, Mr. J. Scott Russell published in "The Practical Mechanic's Journal," November, 1866, an additional calculation of the resistance of the Oxford eight-oar, the result of which is 74.23 lbs.

The preceding calculations and observations give—

Haughton, ...	81.36 lbs.
Rankine, ...	67.80 ,,
Scott Russell	74.23 ,,
Maclaren, ...	73.21 ,,
Mean, . . .	**74.15** lbs.

From the preceding results, we may obtain the following rates per ounce per minute of the muscles engaged :—

Rate per ounce per Minute of Muscle employed in rowing a Boat race.

Foot pounds per ounce per minute.	
15.169	Haughton,
12.640	Rankine.
13.840	Scott Russell.
13.650	Maclaren.
Mean, . .	**13.825**

into the upper cage, and descending by his weight, lifts the full barrow through forty feet, to the higher stage, where it is at once removed by a second assistant. The labourer then leaves the cage in which he has descended, and returns to the higher stage, by mounting a ladder; and again steps into the upper cage, which is now ready to commence the descent afresh. It has been found, by repeated trials, that a labourer, aided by two assistants, to bring and remove the loaded and empty barrows, is able to work for eight hours a day, during which he lifts his own weight (70 kilos.) through a height of 13 metres, 310 times.

From these data we find

$$\text{Work done in one day} = \frac{70 \times 13 \times 310}{311} = 910 \text{ ft. tons.}$$

Converting this result, as before, into work done, expressed in ft. lbs. per ounce of muscle, per minute, we obtain

$$\left.\begin{array}{l}\text{Rate of work, per ounce, per minute, of muscles employed in climbing, . . .}\end{array}\right\} = \frac{910 \times 2240}{60 \times 8 \times 575} = 7{\cdot}389 \text{ ft lbs.}$$

The example just given may be confirmed by the following:—A friend has informed me that he performed the ascent of Monte Rosa, from Zermatt, 9000ft., in nine hours, his weight being 14 stone. Here we have

$$\left.\begin{array}{l}\text{Rate of work, per ounce, per minute, of muscles employed in climbing, . . .}\end{array}\right\} = \frac{14 \times 14 \times 9000}{9 \times 60 \times 575} = 5{\cdot}683 \text{ ft.lbs.}$$

Example, No. 3.—It is recorded in the Memoirs of the incomparable Martinus Scriblerus, how, in his eager pursuit of knowledge, he met with an extraordinary misadventure, through the ignorance of his assistant, Crambe. Having secured the body of a malefactor, he hired a room for its dissec-

tion near the Pest Fields of St. Giles, at a little distance from Tyburn Road. The body was carried by night, without much difficulty, on Crambe's back, who found it easy (being both young and lusty) to travel along the level road, carrying on his legs the double weight, of the malefactor, and of his proper self. As soon, however, as he commenced to ascend the staircase, the wonderfully increased exertion (20 to 25 times as great as before) began to tell upon Crambe, and, as the accurate narrator records, upon the corpse also; in consequence of which Crambe dropped his burden in disgust and fright, and allowed it to roll down the staircase, while he himself ascended breathless into the upper room, where Martin, scalpel in hand, eagerly awaited the arrival of his expected subject.*

Next to the exertion of lifting his own weight, the simplest mode in which the force of man can be employed is in transporting the weight of his body, loaded or unloaded, along a horizontal road; and the work done in horizontal transport of loads may be readily shown to be about $\frac{1}{25}$th part of the work done in lifting the same loads vertically through heights equal to the horizontal distances.

In order to obtain the proportion between the work done in the horizontal and in the vertical transport of weights, we must examine the mechanism of simple walking.

In the act of walking, it is found that the leg which rests upon the ground supports the centre of gravity, which is first lifted by the thrust of the other leg against the ground, and afterwards falls forward, while the suspended leg swings as a pendulum, and becomes in its turn the support of the centre

* It is true that Mr. Pope gives a different account of the cause of this celebrated disaster, and one which acquits Crambe of the charge of having proved unable to carry the malefactor up a flight of stairs; but I feel no hesitation in asserting that my own explanation of the misadventure is preferable.

of gravity, which is made to ascend and descend, as before, like an inverted pendulum, by the thrust and subsequent fall of the other leg. The forward movement of the body, or its *horizontal transport*, is effected by the pendulous movement of the legs, caused by the successive raising of the centre of gravity; and, consequently, the work done in walking one step, is simply the work of lifting the man's weight through the height, to which his body is lifted at the commencement of each step.

It has been ascertained, by means of an apparatus invented by Messrs. William and Edward Weber,* that in ordinary walking, when the length of the step is 28.740 inches, the mean elevation and depression of the centre of gravity of the whole body is 1.248 inch; and it was also found that this proportion remained the same for different lengths of steps.

From this observation we find,

$$\frac{\text{Vertical elevation}}{\text{Horizontal transport}} = \frac{1248}{28740} = \frac{1}{23.03}$$

Hence it follows that a man walking 23.03 miles along a horizontal road has done as much work as if he had lifted his body up a vertical ladder through a height of one mile.

Under such unfavourable circumstances, it is not to be wondered at, that poor Crambe's courage failed him, when he commenced the ascent of the staircase, with the corpse upon his back; and that, finding himself unable to ascend with the alacrity required by the occasion, he quietly let go his hold, and rushed breathless into the presence of his master, being completely overcome by his unwonted exertions.

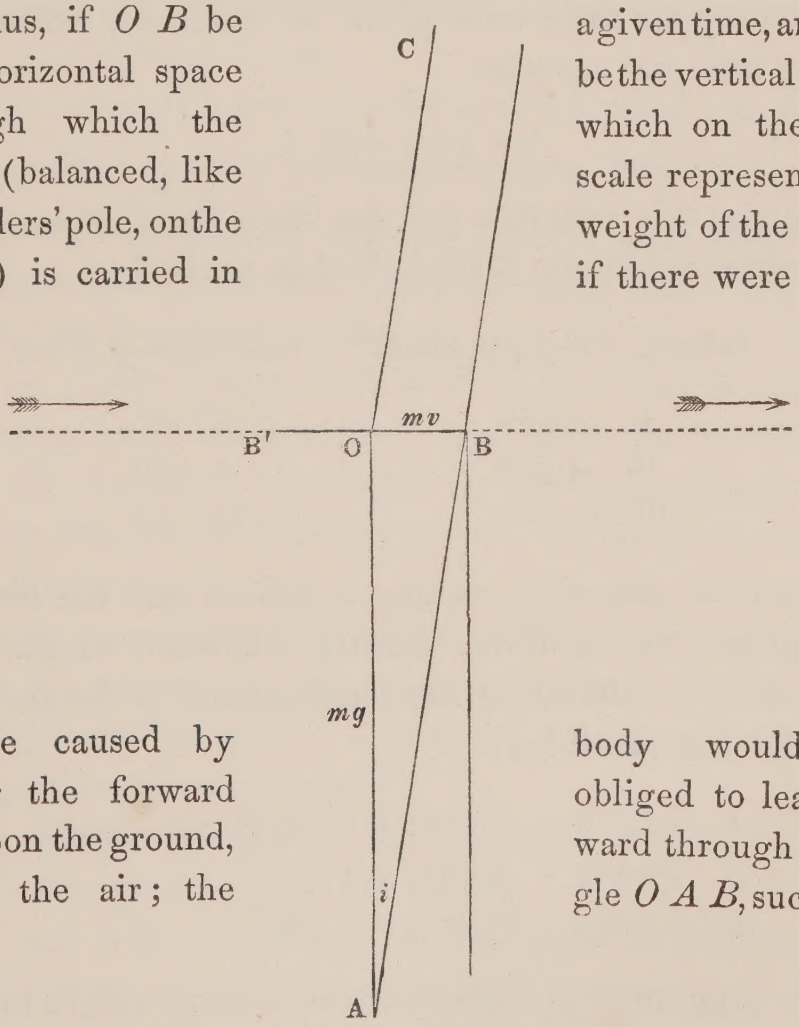
The work done in the horizontal transport of burdens may

* Mechanik der menschlichen Gewerkzeuge, p. 238. Göttingen, 1836.

be calculated also from a totally different set of observations made by the Messrs. Weber.*

When the body is at rest, in the standing posture, the line drawn from the centre of gravity of the trunk to the centre of the horizontal line joining the centres of the acetabula of the pelvis must be vertical; and when, in walking or running, the body is carried horizontally with a velocity v, this line must incline forwards, with an inclination such that it shall be in the direction of the resultant of the momentum of the horizontal velocity and the weight of the body.

Thus, if $O\ B$ be the horizontal space through which the trunk (balanced, like a jugglers' pole, on the pelvis) is carried in a given time, and $O\ A$ be the vertical space, which on the same scale represents the weight of the body; if there were no resistance caused by setting the forward foot upon the ground, or by the air; the body would be obliged to lean forward through an angle $O\ A\ B$, such that



$$\tan i = \tan O\ A\ B = \frac{mv}{mg} = \frac{v}{32.2} \qquad (13)$$

* Mechanik der menschlichen Gewerkzeuge, pp. 233, 4.

The angle i has been carefully measured for three different velocities by Messrs. Weber, with the following results :—

	Velocity.		*Inclination of Trunk.*
	Feet per Second.	Miles per Hour.	
I.	2.667	1.818	2° 43′
II.	6.385	4.353	7 40
III.	15.513	10.577	19 13

If there were no resistance to horizontal motion, the slope of the body would be found by calculation from equation (13); or, assuming the slope to be given, as measured, we find the velocity from the equation

$$v = 32.2 \times \tan i \qquad (14)$$

Solving this equation for the three inclinations of the trunk measured by the Messrs. Weber, we find—

	Velocity of Feet per Second.	*Inclination of Trunk.*
I.	1.528	2° 43′
II.	4.334	7 40
III.	11.224	19 13

From the preceding results, it follows that the observed velocity is in excess of the velocity calculated on the hypothesis of no resistance for fixed inclinations of the trunk, by the following quantities :—

I. (2.667 − 1.528) = 1.139 ft. per second.
II. (6.385 − 4.334) = 2.051 ,,
III. (15.513 − 11.224) = 4.289 ,,

The proportion of resistance overcome to weight lifted, in the three cases, is therefore—

	Velocity. Miles per Hour.	*Proportion of Resistance to Weight.*
I.	1.818	$\frac{1.139}{32.2} = \frac{1}{28.27}$
II.	4.353	$\frac{2.051}{32.2} = \frac{1}{15.70}$
III.	10.577	$\frac{4.289}{32.2} = \frac{1}{7.51}$

From these results it follows that a man walking at the rate of 1.818 miles per hour should travel 28.27 miles before he has done the work represented by lifting his body through one mile of vertical height.

If we interpolate between the results I. and II., to find the resistance at the pace of 3 miles per hour, we find it to be $\frac{1}{20.65}$, a result that agrees well with that already found from direct measurement of the vertical elevation of the centre of gravity, at each step in ordinary walking.

As an example of the rate of work that may be attained in carrying loads horizontally, I shall give the following, to which my attention was called by Professor Rankine, and which is founded on data obtained by his father, Lieutenant David Rankine :—

A labourer, supposed to weigh 150 lbs., carrying a load of 126 lbs. for 30 seconds, can travel at the rate of 11.7 ft. per second (8 miles an hour) ; what is the rate of work done, per ounce of muscle, per minute ?

At 6.385 ft. per second, the resistance is measured by a loss of velocity of 2.051 ft. per second.

At 15.513 ft. per second, the resistance is measured by a loss of velocity of 4.289 ft. per second.

Therefore,

$$15.513 - 6.385 : 11.700 - 6.385 : : 4.289 - 2.051 : x$$

or,

$$9.128 : 5.315 : : 2.238 : x = 1.303$$

And, finally,

$$\frac{1.303 + 2.051}{32.2} = \frac{1}{9.60}$$

In other words, at the pace of 11.7 ft. per second, the resistance to horizontal motion is $\frac{1}{9.60}$th of the weight, according to the results of the experiments of Messrs. Weber. Hence the work done per ounce, per minute, in Lieutenant Rankine's experiment, is—

$$\text{Work} = \frac{276 \times 11.7 \times 60}{9.6 \times 575} = 35.119 \text{ ft. lbs.}$$

From the foregoing example, it appears that the rate of work done in carrying half one's weight at 8 miles an hour is more than double the work done in a boat race, although it is considerably less than the work done in *statical* work, by muscles sustaining weights, as already described.

The following examples, among many others, may be given of the daily amount of work given out, in ordinary labour, by the muscles of man :—

Example, No. 4.—The *useful* work, measured by the bricks lifted per day, done by a Dublin hodman, amounts to 181 ft. tons ; to which must be added the *fatigue* work done in lifting his own weight and that of his hod up and down the ladders. I have observed the hodmen in Dublin to make 40 ascents per day of ladders 40 ft. high ; if, therefore, we assume the weight of the labourer and his hod to be 170 lbs., we have,

Work done in mounting = 170 × 40 × 40 =	272,000	ft. lbs.
,, descending = one-half, . .	136,000	,,
	408,000	,,

Hence,

$$\text{Fatigue work} = \frac{408000}{2240} = 182 \text{ ft. tons.}$$

Useful work, . .	181	,,
Total, . . .	**363**	ft. tons.

Example, No. 5.—The paviours of Dublin make 78 blows of the rammer in $2^{m.}\ 45^{s.}$, and then rest for $3^{m.}\ 30^{s.}$; their rammer weighs 5 st. 9 lbs., and is lifted through 16 inches each stroke; and their labour is continued each day for 10 hours. Let us calculate from these data the work done per day.

The time of labour is to the time of rest as 165 to 210; hence, the time of labour is to the whole time as 165 to 375; therefore, in the day of 10 hours the number of minutes of labour is

$$\frac{10 \times 60 \times 165}{375} = 264 \text{ minutes};$$

but in $2\frac{3}{4}$ minutes 79 lbs. are lifted 78 times through 16 inches. Hence the work done in $2\frac{3}{4}$ minutes is

$$79 \times 78 \times 1\tfrac{1}{3} = 8216 \text{ lbs.};$$

and the work done in a whole day of 10 hours, is

$$8216 \times \frac{264}{2\frac{3}{4}} = 788736 \text{ ft. lbs.};$$

which, divided by 2240, gives us

Daily work of paviour, . . . **352** ft. tons.

Example, No. 6.—In the construction of the Oxford and Birmingham railway, a "box-lift" was employed in the cuttings, identical in principle with that used by Captain Coignet (*Example, No.* 2), and the average result of more than a year's observation on average workmen, in all weathers, was taken, with the following results:—

Work done, ... 1510 ft. lbs. per minute.
Daily work (10 hours), . **404** ft. tons.

On the same railway a "swing-lift" was used, in which part of the work of the labourers was employed, not in raising materials, but in shifting the position of the "lift." Ob-

servations made for more than a year, in all weathers, on
daily labour of average workmen, gave the following results

Work done, ... 1040 ft. lbs. per minute.
Work, ... 300 ft. tons per day (10 hours

The mean result of an ordinary day labourer, employ
with the box or swing-lift, is therefore,

"Box or swing-lift," . . . . 312 ft. tons.

Example, No. 7.—A very accurate estimate may be form
of the average day's labour, from observations formed on
horizontal transport of weights, interpreted by the rules
ready laid down for the conversion of horizontal into vert
transport.

(a.) If a pedestrian be employed in walking only, with
load, it is found that 20 miles a day, with rest on Sund
give full employment to his strength, and that this amo
cannot be exceeded with prudence. From this assumpti
if we suppose the man's weight, including clothes, to
160 lbs., we find—

a. Horizontal transport of man walking without load, $= \frac{160 \times 20 \times 5280}{2240}$

$= 7040$ ft. tons.

(b.) Combined observed the work done by persons emplo
to carry goods 2000 metres, returning unloaded, and fo
that each carried, in the course of the day, 348 kilos. in
journeys, loaded with 58 kilos. at a time. If we supp
each person to weigh 70 kilos., we find the following v
for the horizontal transport:—

Useful work, $2000 \times 348 = 696000$ kil. me
Fatigue work, $2000 \times 2 \times 70 \times 6 = 1,680000$ "

Total, ... 2,376000 kil. me

Dividing this result by 300, in order to reduce it to hours, we find

b. Work done by porters, alternately loaded and unloaded, ... 7560 ft.-tons.

(c.) On questioning a number of porters who travelled, were loaded with their packs, Coolumbi found that they were 80, with a load of 66 kilos., to travel 19200 metres per day, assuming their weight at 70 kilos., as before, we find—

Useful work = 19200 × 66 = 1267200 kil. mets.
Fatigue work = 19200 × 70 = 1344000 "

Total, . . . 2,611200 kil. mets.

Converting this, as before, into foot tons, we obtain—

c. Work done by loaded porters = 7560 ft.-tons.

Collecting together the preceding estimates, we find—

Man walking without load, ... 7563 ft.-tons.
Man alternately loaded and without load, . 7560 "
Man loaded, ... 6576 "

Mean, ... 7233 ft.-tons.

If the horizontal transport be effected at the usual rate of miles an hour, we must divide the preceding result by 20·65 (as already explained), in order to convert it into weight lifted through height. Hence we obtain, as the mean of the work done in the horizontal transport of weights,—

Work done in horizontal transport of loads, . $\frac{7233}{20{\cdot}65}$ = 350 ft.-tons.

From Examples 4, 5, 6, 7, a very close approximation may be made to the daily work of a good labourer.

Day Labourer.	Foot tons.	Kilogrammeters.
1. Hodman, . . .	363 . . .	112413
2. Paviour, . . .	352 . . .	109008
3. Navvy, . . .	342 . . .	105910
4. Pedlar, . . .	358 . . .	110866
Mean, . .	**353.75**	**109549**

If the day's labour be estimated at 10 hours, and converted, as before, into work done in ft. lbs. per ounce, per minute, we find—

$$\text{Day Labourer} = \frac{353{\cdot}75 \times 2240}{575 \times 600} = 2.297 \text{ ft. lbs. per oz. per min.}$$

Collecting together into one Table the rates of Statical and Dynamical work of muscles already described, we obtain—

TABLE VII.—*Statical Work done by Muscles.*

Muscles Used.	Quantity of Work in foot pounds.	Duration of Labour in minutes.	Weight of Muscles in oz. av.	Rate of Work in foot pounds per oz. per min.	Rate of Work in Kilogram-meters per Hectogram per minute.
1. Unloaded muscles of shoulder, .	1206	8.29	4.50	32.3	15.7
2. Loaded muscles of shoulder, . .	842	4.05	4.50	46.2	22.5
3. Loaded muscles of forearm, . . .	3170	9.29	8.21	41.5	20.2

TABLE VIII.—*Dynamical Work done by Muscles of the whole Body.*

Kind of Work.	Quantity of Work in foot pounds.	Duration of Labour in minutes.	Weight of Muscles in oz. av.	Rate of Work in foot pounds per oz. per min.	Rate of Work in Kilogram-meters per Hectogram per minute.
1. Boat race, ...	61,040	7	575	15.17	7.4
2. Coignet's lift, . . .	2,038,400	480	575	7.39	3.6
3. Running with load,	10,091	½	575	35.11	17.1
4. Day labour, . . .	792,400	600	575	2.30	1.1

The last two columns of Table VIII. measure the respiratory distress produced by different kinds of labour; and show that Coignet's lift is more than *thrice* as severe as ordinary labour; and that running at 8 miles an hour with a load equal to half the weight of the body is more than *twice* as severe as pulling a boat race.

5. On the Absolute Force of Muscles.—We have already considered the amount of work given out by muscles, when employed either statically or dynamically; and it now remains to investigate their absolute force when employed for an instant, or very short space of time.

Example, No. 1.—*Flexors of the Forearm.* In order to determine the force of the flexors of the forearm, I held my arm vertically, and the forearm horizontally, with the fist shut and in supination, and found that 39 lbs. was the limit of the weight I could just raise, when suspended at $12\frac{1}{4}$ inches from the axis of the elbow joint; this gives

$$\textit{The moment of Force applied} = 41 \times 12\tfrac{1}{4} = 502\tfrac{1}{4};$$

estimating the forearm itself as equivalent to 2 lbs. suspended at $12\frac{1}{4}$ inches from the joint.

I endeavoured to ascertain the *moment of the muscles* by dissecting a very finely-developed subject,* who had been a blacksmith by trade, with the following results:—

Cross Sections of Muscles.

1. *Biceps humeri,*	1.914 sq. in.	= 12.346 sq. cent.
2. *Brachiæus,*	1.276 ,,	= 8.230 ,,

In order to find the perpendiculars let fall upon the directions of the muscles in my own arm, I selected another sub-

* This was the same subject on which I made the measurements of the leg muscles described subsequently.

ject, in which the distance from the axis of the elbow joint to the ungual articulation of the middle finger was the same as in my own arm, viz. 16.55 in., and found as follows:—

1. Perpendicular on *Biceps humeri*, 2.06 in.
2. Perpendicular on *Brachiæus*, . 1.07 ,,

Hence, if K denote the coefficient of muscular force, per square inch of cross section, we have

$$\textit{The moment of the muscles} = K\,(1.91 \times 2.06 + 1.28 \times 1.07)$$
$$= 5.304 \times K;$$

and, equating this to the moment of force applied, we obtain

$$502\tfrac{1}{4} = 5.304 \times K,$$

and,

$K = 94.7$ lbs. per sq. in. = 6.658 Kilos. per sq. cent.

Considering that the arm examined after death was that of a blacksmith, it is possible that I have somewhat over-estimated the cross section of the muscles, as compared with my own; but, on the other hand, it must be remembered, as I have shown in pp. 45, 6, that a large percentage should be added to dead muscles, in order to bring them up to the standard of life.

Other writers, especially Donders and Mansfelt, have given measures of the lengths and cross sections of the *biceps* and *brachiæus*, which I have added in a note,* partly on account

* Over de Elasticiteit der Spieren. Utrecht, 1863.

	Av. of Eight.	One.
1. Distance from axis of elbow to *insertion* of biceps, .	39 m.m.	45m.m.
2. Length of fibres of long head, ...	158	160
3. Length of fibres of short head, ...	202	204
4. Length of the whole long head, ...	388	400
5. Length of the whole short head, ...	355	366
6. Distance from axis of elbow to middle of *insertion* of brachiæus, ...	32	30
7. Length of fibres of *brachiæus*, ...	(51–143)	(48–134)
8. Length of whole *brachiæus*, ...		237

of their intrinsic value, and partly for the purpose of comparison with my own.

It will be observed that their measurements of the cross sections of these muscles are smaller than those given by me, and would give, therefore, a larger value for the coefficient *K*.

MEASUREMENTS OF DONDERS AND MANSFELT.

Cross Sections of Biceps humeri *and* Brachiæus.

	Millims.	Sq. in.
1. *Biceps humeri* (long head), . . .	530	0.821
,, (short head), . . .	452	0.701
2. *Brachiæus*, ...	614	0.952
	1596	2.474

Example, No. 2.—*Flexors of the Leg.* In order to measure the force of the muscles flexing the leg, I placed the observer lying upon his face upon a table, with the legs extended over its edge, and having fastened down the thighs, I observed the maximum weights suspended from the heel that could be conveniently lifted, and found that 34 lbs. was the limit; to this must be added 3 lbs. for the weight of the leg, supposed suspended at the heel, which was measured as $16\frac{1}{2}$ inches from the axis of rotation of the knee-joint. The perpendiculars let fall upon the directions of the several muscles flexing the leg were then measured:—

9. Distance from *origin* of brachiæus to axis of elbow, .	(183-31)	(190-37)
10. Length of radius, ...	235	238
11. Length of humerus, ...	342	336

12. Section of long head (*biceps*), ...	530 sq. m. m.
13. Section of short head, ,, ...	452 ,,
14. Section of *brachiæus*, ,, ...	614 ,,

Cross section of an elementary bundle

$$= \frac{1}{500} \text{ sq. m. m.}$$

Total number of elementary bundles in the cross section of the flexors of forearm

= 798,000.

	Perpendicular.
1. *Biceps femoris* (long head), . .	0.95 in.
,, (short head), . .	0.56 ,,
2. *Semitendinosus*, ...	0.40 ,,
3. *Semimembranosus*, ...	0.65 ,,
4. *Gracilis*, ...	0.25 ,,
5. *Sartorius*, ...	0.00 ,,

The cross sections of the muscles that bend the leg were found to be in the same subject*—

1. *Biceps femoris* (long head), . .	2.59 sq. in.
,, (short head), . .	1.14 ,,
2. *Semitendinosus*, ...	1.87 ,,
3. *Semimembranosus*, ...	2.25 ,,
4. *Gracilis*, ...	0.89 ,,
5. *Sartorius*, ...	0.59 ,,
	9.33 sq. in.

Hence we find, for the determination of K (the coefficient of muscular contraction per square inch of cross section),

$$37 \times 16\tfrac{1}{2} = K \times \begin{cases} 0.95 \times 2.59 \\ + 0.56 \times 1.14 \\ + 0.40 \times 1.87 \\ + 0.65 \times 2.25 \\ + 0.25 \times 0.89 \\ + 0.00 \times 0.59 \end{cases}$$

or,

$$610.5 = K \times \begin{cases} 2.460 \\ + 0.638 \\ + 0.748 \\ + 1.462 \\ + 0.222 \\ + 0.000 \end{cases}$$

$$5.530$$

* This was also the subject in which I had measured the muscles of the forearm, described in *Example*, No. 1.

and, finally,

$$K = \frac{610.5}{5.53} = 110.4 \text{ lbs.}*$$

The cross sections of the muscles were found, by cutting them across with a sharp scalpel, and marking out their section on cardboard, and afterwards weighing the marked portions, the weights of which were then compared with the weight of a known number of square inches of the same cardboard, and so the cross sections in square inches calculated.

I give here, for the purpose of illustration, the actual sections of two of the muscles of the leg:—

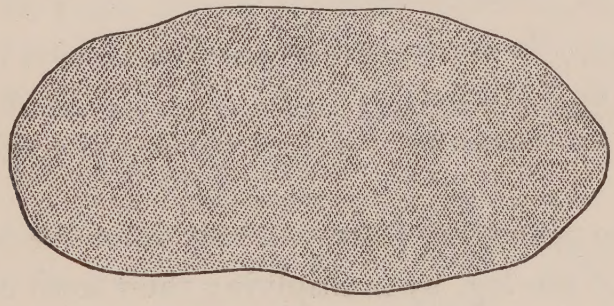


FIG. 2.—*Semitendinosus.*

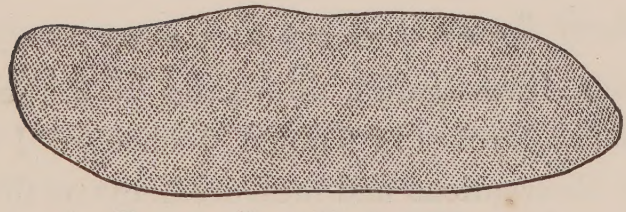


FIG. 3.—Short Head of *Biceps.*

* As it was not convenient to procure a good subject destroyed by a violent death, I made use of a powerful man who had died of cholera, and who had been a blacksmith by profession. Now, it is natural to suppose that the muscles of the arm of a blacksmith are more developed than those of his leg, so that their cross section would be relatively too great, and the coefficient derived from that cross section, therefore, probably too small; a circumstance which may help to account for the smaller coefficient, 94.7 lbs., found from the cross section of the muscles flexing the forearm.

It is well known that after death by cholera, life continues in the muscles, and manifests itself for some hours by movements, and by the existence of the muscular susurrus. Both of these facts I have repeatedly verified. It appeared to me, therefore, that such a subject as I selected was one well suited to the purpose of my observations.

The perpendiculars let fall upon the directions of the muscles were measured by stretching strings from the origin to the insertion of the muscles, and measuring, by means of a compass, the perpendiculars let fall upon these strings from the axis of the joint.

The weights of the muscles themselves were as follows :—

	oz.
1. *Biceps humeri*, ...	4.22
2. *Brachiæus*, ...	5.04
3. *Biceps femoris*, ...	10.74
4. *Semitendinosus*, ...	5.17
5. *Semimembranosus*, ...	7.25
6. *Gracilis*, ...	2.98
7. *Sartorius*, ...	5.66

From the preceding facts, we may infer the total force of the flexors of the leg, by multiplying their total cross section, 9.33 sq. inches, by the coefficient of contraction, 110.4 lbs.

Total force of flexors of leg = 9.33 × 110.4 = 1030.03 lbs.

The whole investigation illustrates well the difficulties that surround researches in Animal Mechanics. I was obliged to make the observations and measurements on three different subjects :—

(*a*). The maximum weights were observed by trials made upon myself and a friend of a similar height and strength, with results closely agreeing.

(*b*). The measurements of the arms of the levers were made upon the skeleton of a Frenchman, of the same height and length of bones as myself.

(*c*). The cross sections of the muscles of the arms and legs were measured on a blacksmith who had died of cholera, and who, when living, was a much more muscular man than myself, with probably, also, a greater developement of muscles

of arm than of leg. It is very difficult to obtain exact results from observations necessarily involving so many sources of error; and yet I venture to think that the mean of my two sets of observations cannot be far from the true contractile force of the muscles:—

Coefficient of flexors of arm, . .	94.7 lbs.
Coefficient of flexors of leg, . .	110.4 ,,
Mean, . . .	**102.55** lbs.

It has been already stated, on the authority of Donders (p. 2), that the cross section of the flexors of the forearm contains 798000 muscular fibres, having each a diameter of $\frac{1}{300}$th of an inch; from which number, and the preceding data, we may readily find in grains, the

$$\text{Muscular force of contraction of a single muscular fibre,} = \frac{3.190 \times 102.55 \times 7000}{798000} = 2.87 \text{ grs.}$$

Other observers have found coefficients for the force of human muscles, referred to the unit of cross section, which differ somewhat from the coefficient deduced above.

Weber made some experiments on this subject, from which he deduced the force of *gastrocnemius* and *solæus* to be

1 kilogram per 1 sq. centimeter.

Henke and *Knorz* corrected the calculation of Weber, and found that his result should have been

4 kilograms per sq. centimeter.

Their own experiments gave for the *biceps*, *brachialis internus*, and *supinator longus*, of the right arm—

8.991 kilograms per sq. centimeter;

and, for the mean of the two arms,

8.187 kilograms per sq. centimeter.

Similar experiments made on the *gastrocnemius* and *solæus* gave them

5.9 kilograms per sq. centimeter.

Koster has recently published (in the *Nederlandsch Archief voor Genees en Natuurkunde*) an account of similar experiments, from which he deduces, from the *gastrocnemius* and *solæus*,

11.6 kilograms per centimeter,

which number he afterwards reduces to

9.5 kilograms per centimeter.

The preceding observations, collected together, give us the following results:—

1. *Flexors of arm*, . .	8.991 kilos. per centr.	(Henke).
2. „ „ . .	8.187 „	
Mean, . .	**8.589**	

1. *Extensors of foot*, .	5.90 kilos. per centr.	(Henke)
2. „ „ . .	11.60 „	(Koster).
Mean, . .	**8.75**	

My own results, converted into French measures, are,

Flexors of arm, . .	6.67 kilos. per centimeter.
Flexors of leg, . . .	7.78 „ „
Mean, . .	**7.225**

The mean of all the observations made by Henke and Koster, on the muscles of the arm and foot, gives—

Coefficient of contraction } = **8.669** kil. per centr. = **123.04** lbs. per sq. in.

I believe that the difference between this result and that found by me, viz. **102.55** lbs. per inch, is a real difference arising from the fact that these observers used the cross sections of muscles as they occur in ordinary dead bodies, whereas I took care to use the cross sections of muscles in a strong man, who had died quickly by cholera, and in whom I supposed the muscles would be found in a state more nearly approaching that in which they occur in the living body.

If we compare the cross sections of the *biceps* and *brachiæus* found in ordinary dead bodies by Donders and Mansfelt (p. 65) with those employed by me (p. 63), we find—

Cross Sections of Muscles of Arm.

	Donders and Mansfelt.	Haughton.
Biceps humeri, . .	1.522 sq. in.	1.914 sq. in.
Brachiæus, . . .	0.952 „	1.276 „
	2.474	3.190

If we wish to ascertain the coefficient of muscular contraction per square inch of cross section of living muscle, we must alter the coefficient of force per square inch of cross section of dead and wasted muscle in the proportion of 2474 to 3190. This will give us, as the coefficient of living muscle, from the experiments of Henke and Koster,

$$\text{Coefficient of contraction of living muscle,} \left.\right\} \frac{123.04 \times 2474}{3190} = \mathbf{95.42} \text{ lbs. per sq. inch.}$$

This result comes very near that found by me as the coefficient of the flexors of the forearm, viz. **94.7** lbs.

I have proved (pp. 42, 43) that the same muscles, when moderately worked, give out equal quantities of work before becoming fatigued; but this statement requires to be modified, when applied to the amount of work performed suddenly

by a muscle, during a single contraction. When human muscles are moderately used, their maximum force per square inch of cross section is found to be :

102.55 *av. lbs. per sq. in.*, or 7.225 *kilos. per sq. centimeter ;*

but the muscles of all animals are capable of being stimulated for a time to perform an amount of work much greater than that done under ordinary circumstances.

The following results, obtained by Matteucci and Helmholz, from experiments made upon the muscles of the calf of the frog, exhibit this principle very clearly. It will be observed that the total *work done* in a single contraction of the muscle increases with the weight it is compelled to lift.

TABLE IX.—*Work done in a Single Contraction by the Gastrocnemic Muscles of the Leg of a Frog.*

	Weight lifted.	Height.	Work done.	Observer.
1°.	10.32 grms.	1.412 mm.	14.57	Matteucci.
2°.	40.32 „	1.270 „	51.21	„
3°.	70.32 „	1.130 „	79.46	„
4°.	100.32 „	0.840 „	84.27	„
5°.	180.32 „	0.765 „	137.94	Helmholz.

The most complete series of experiments on the *gastrocnemius* of the frog are those made by Mr. Baxter, who found, as the result of the examination of 60 frogs, of whom 26 were females—

Total weight of muscles, . 262 grs.
Total weight lifted, . . . 159360 grs.

From these experiments we find the average force of the frog's *gastrocnemius*—

$$\frac{159360}{60} = 2656 \text{ grs.} = 172.11 \text{ grms.}$$

Other observers (Rosenthal) have found, as the maximum force of the gastrocnemic muscles of the frog, from 1000 to 1200 grms., corresponding to a coefficient of contraction of muscle, per centimeter of cross section, of from 1.8 to 3 kilograms per square centimeter.

Mr. Baxter's average result given above corresponds to a similar coefficient per square centimeter of only 371.51 grms.

Borelli has given* many estimates of the force of the flexors of the forearm, leg, jaw, and other muscles; but as these estimates are open to several objections, as well as his measurements of the lengths of the arms of the levers engaged, I must refer the curious reader to his original Propositions, which may be justly regarded as a remarkable attempt, considering the imperfections of the mechanical philosophy of his time, to anticipate the more precise results demanded of animal mechanics in our own day. The following remarks may be made upon Borelli's observations:—

1. He has not been sufficiently careful to avoid the action of muscles different from those supposed to be exclusively employed.

2. He has not measured, with sufficient care, the arms of his levers.

3. He has committed the common error of doubling the

* De Motu Animalium Pars prima, Props. xxii. . . . xxvii. Props. xl. xli. liii. lxxxvii. lxxxviii.—Rome: 1680. 2 vols. 8vo.

I assume the pound used by Borelli to be 5000 grs., for the following reasons:—

The *Cantaro piccolo* is 106 lbs. av., or 150 lbs. of 12 oz. each; this would give 4957 grs. to the Neapolitan pound.

Gold and silver, in the time of Borelli, were weighed by a pound of 4950 grs.

The Roman pound was 5234 grs.

There is some difficulty in ascertaining what pound was used by Borelli, but it is clear that it did not differ much from 5000 grs., and was therefore $\frac{5}{7}$ths of the pound avoirdupois.

tension of the muscle, which resembles the tension of a stretched cord.

4. He has misunderstood the mechanical conditions of equilibrium of a many-jointed lever, or funicular polygon.

5. From not being acquainted with the composition of forces, he has misunderstood the action of penniform and radiating muscular fibres.

Errors 1 and 2 may be regarded as errors of observation; but 3, 4, and 5, are mechanical blunders;—and I have therefore calculated the following forces from Borelli's experiments, correcting all his erroneous mechanical inferences. These forces may be regarded as Borelli's results, including errors of observation only:—

TABLE X.—*Borelli's Observation of Force of Muscles.*

1. *Biceps humeri and Brachiæus* (Prop. xxii.), . . .	400 lbs. av.
2. *Biceps humeri* (Prop. xxiv.), ...	214 ,,
3. *Brachiæus* (Prop. xxiv.), ...	186 ,,
4 *Biceps femoris, Gracilis, Semimembranosus, and Semitendinosus* (Prop. xxvii.), ...	678 ,,
5. *Vastus externus et internus, Cruræus*, (Prop. xl.), .	814 ,,
6. *Solæus* (Prop. xli.), ...	407 ,,
Solæus (horse) (Prop. xli. *scholium*), ...	3000 ,,
Solæus (swan) do. do., ...	80 ,,
7. *Glutæi* (Prop. liii.), ...	691 ,,
8. *Solæus* (Prop. liii.), ...	261 ,,
9. *Flexor pollicis longus* (Prop. lxxxvi.),.	43 ,,
10. *Temporales et masseteres* (Prop. lxxxviii.), . . .	180 ,,

6. On the Comparative Anatomy of the Tendons of the Hand and Foot, and their Mechanical Uses.—The maximum force exerted by muscles has been shown in the preceding section to be proportional to the cross sections of those muscles, and to amount to upwards of 100 lbs. av. per square inch of section. The relative values of the maximum efforts resisted by the tendons of the various muscles may be more readily ascertained by comparing together the cross sections of their several tendons

instead of the cross sections of the muscles themselves. As the cross sections of both muscles and tendons must be measured after death, and as it is not always convenient to kill subjects suitable for observation, it is obvious that the tendons, which are less liable to be wasted by natural disease than the muscles, are capable of affording most valuable relative measurements, if we are allowed to assume that the cross sections of the tendons are proportional to the forces resisted by them.

It may be readily shown, *a priori*, that this must be the case, by means of the following considerations:—

The principle of economy of force, or of material, in nature, would lead necessarily to the principle that each tendon conveying the effect of a force to a distant point should have the exact strength required, and neither more nor less; for, according to the doctrine of Final Causes, it was originally contrived by a perfect Architect, and according to Lamarckian views it must have perfectly accommodated itself to the uses to which it is applied. According, therefore, to either view, if the tendon be too strong, it will become atrophied down to the proper limit; and if too weak, it must either break, or be nourished up to the requisite degree of strength. It seemed to me desirable to prove this fundamental proposition in animal mechanics by direct observation; and I selected for this purpose the tendons in the leg of several of the large running birds (*Struthionidæ*), with the result, that the cross sections of any two muscles tending to produce a similar effect, and subjected to similar friction in their tendons, are directly proportional to the cross sections of those tendons.

I shall select as an example the case of the *Flexor hallucis longus* and *Flexor digitorum communis perforans* of the Rhea, whose tendons unite into a common tendon half way down the posterior side of the cannon bone of the bird.

The areas of the cross sections of these muscles were found

to be as 245 to 160; or the lesser was 65 per cent. of the greater.

Two equal lengths of the dried tendons were then weighed, and found to be in the proportion of 845 to 495, which was assumed to be the proportion of their cross sections. The lesser of these numbers is 59 per cent. of the greater; a result that seems to be as near to the former result derived from the muscles as can be expected in this class of experiments.

A. *Comparative Anatomy of the Tendons of the Foot.*

The flexor muscles of the toes which we have to compare are three in number, viz.:

Flexor hallucis longus.
Flexor digitorum longus.
Flexori longo accessorius.

This latter muscle, when it exists, always acts as a guy or force directing the line of force of the long flexor of the toes; but in many animals it is absent, if the tendon of the long flexor does not require to be assisted to keep in the proper direction. The *accessory* muscle seems also to serve as an addition to the force of the *Flexor hallucis*, whenever the tendon of this muscle unites with that of the long flexor of the toes; for the fibres of the *accessory* are parallel to the tendon of the *Flexor hallucis*, and pass into it, uniting it with the tendon of the *Flexor longus* by means of broad slender sheets of fibres. I have found the following arrangement of tendons in various animals:—

(*a.*) *Man.*—In the human foot, the *Flexor hallucis*, generally, has a tendon that divides into two branches, the largest of which forms the flexor tendon of the great toe, while the smaller branch proceeds to form a portion of the flexor tendon of the second toe. The *long flexor* of the toes has a tendon branching into four parts, and crosses the flexor tendon of

the *hallux* at an angle of 24°. The four branches of the long flexor tendon are distributed to the second, third, fourth, and fifth toes; that which goes to the second toe uniting with the lesser branch of the *hallux* tendon to form the flexor of the second toe.

This arrangement, which is subject to many variations, is shown in Fig. 4, which represents the tendons seen from below, in the sole of the foot, but displaced from their natural positions in order to exhibit their mechanical arrangement.

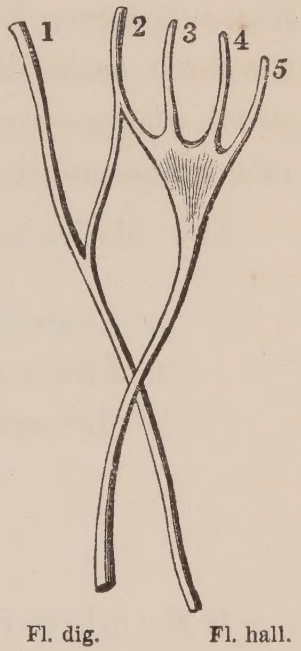


Fig. 4.

Example, No. 1.—In order to obtain the relative strengths of the various tendons, I weighed equal lengths of those tendons, with the following results:

In the sole of foot.

Flexor hallucis longus, ...	27
Flexor digitorum longus, ...	26
Total, . .	53

In the toes.

1. First toe, ...	27
2. Second toe, ...	10
3. Third toe, ...	12
4. Fourth toe, ...	$8\frac{1}{2}$
5. Fifth toe, ...	$8\frac{1}{2}$
Total, . .	66

It will be observed that the sum of the cross sections of the tendons actually distributed to the toes exceeds the

sum of the cross sections of the two great flexor tendons at the heel, the chief increase taking place in the tendons of the *flexor digitorum longus*. This increase is due to and represents the force exerted by the *accessory* muscle, which joins the *flexor digitorum* at its point of subdivision into the tendons of the separate toes.

The relative strengths of the tendons are, therefore,

1. *Flexor hallucis longus*, . . .	41	per cent.
2. *Flexor digitorum longus*, . .	40	,,
3. *Accessorius*, ...	19	,,
Total, . .	100	

If the whole force employed in flexion be called 100, as above, it will be distributed to the toes in the following proportions :—

1. First toe, ...	41	per cent.
2. Second toe, ...	15	,,
3. Third toe, ...	18	,,
4. Fourth toe, ...	13	,,
5. Fifth toe, ...	13	,,
Total, . . .	100	

Example, No. 2.—In this case, also, the *Flexor hallucis* was found to supply the whole tendon of the first toe and part of the tendon of the second toe; while the *Flexor digitorum* supplied the remaining portion of the second toe and the whole tendons of the third, fourth, and fifth toes.

The relative strengths of the several tendons were ascertained to be—

1. First toe, ...	41 per cent.
2. Second toe, ...	18 „
3. Third toe, ...	17 „
4. Fourth toe, ...	15 „
5. Fifth toe, ...	9 „
Total, . . .	100

1. *Flexor hallucis longus*, . .	47 per cent.
2. *Flexor digitorum longus*, . .	36 „
3. *Accessorius*, ...	17 „
Total, . . .	100

Example, No. 3.—This case resembles the two preceding in the distribution of tendons, the relative strengths of which were found to be as follows,

1. First toe, ...	37 per cent.
2. Second toe, ...	17 „
3. Third toe, ...	17 „
4. Fourth toe, ...	15 „
5. Fifth toe, ...	14 „
Total, . . .	100

1. *Flexor hallucis longus*, . .	41 per cent.
2. *Flexor digitorum communis*, .	38 „
3. *Accessorius*, ...	21 „
Total, . . .	100

Example, No. 4.—In this case the *flexor hallucis* supplies the whole of the tendon of the first toe, and part of the

tendons of the second and fifth toes (Fig. 5); while the *flexor digitorum longus* supplies the whole tendons of the third and fourth toes, and the remaining parts of the tendons of the second and fifth toes.

This arrangement is shown in the partially displaced tendons of Fig. 5.

The relative strengths of the several tendons were found to be as follows :—

1. First toe,	. .	47 per cent.
2. Second toe,	.	14 ,,
3. Third toe,	. .	11 ,,
4. Fourth toe,	.	11 ,,
5. Fifth toe,	. .	17 ,,
Total,	. .	100

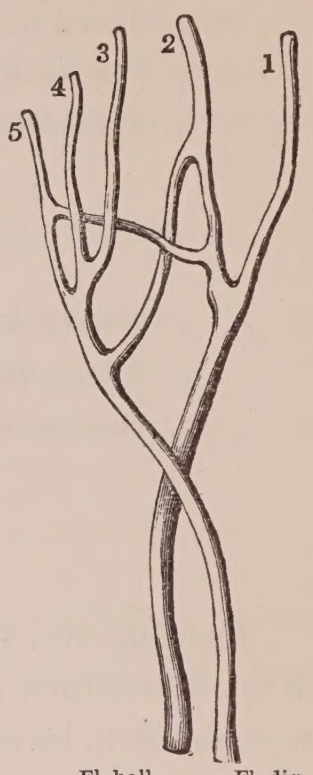


Fig. 5.

1. *Flexor hallucis longus*,	. .	42 per cent.
2. *Flexor digitorum communis*,	.	41 ,,
3. *Accessorius*,	. . . . .	17 ,,
Total,	. .	100

Example, No. 5.—In this case the *flexor hallucis* supplies the first toe, the second toe, and part of the tendons of the third and fourth toes; while the *flexor digitorum communis* supplies the remaining parts of the tendons of the third and fourth toes, and the whole tendon of the fifth toe; sending also a small branch, previous to its own trifurcation, to the tendon proper of the first toe. This arrangement of tendons is shown in Fig. 6, and is a deviation from the *human* type, in which the whole tendon of the first toe is usually supplied by a single muscle, viz., the *flexor hallucis longus.*

The relative strengths of the several tendons in this case were found to be as follows :—

1. First toe,	39	per cent.
2. Second toe,	15	,,
3. Third toe,	15	,,
4. Fourth toe,	19	,,
5. Fifth toe,	12	,,
	100	

1. *Flexor hallucis longus*,	46	per cent.
2. *Flexor digitorum longus*,	47	,,
3. *Accessorius, &c.*,	7	,,
	100	

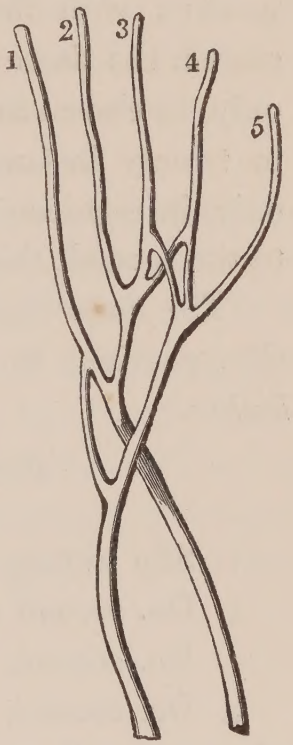


Fig. 6.

If we collect together (disregarding the differences of distribution of tendons) the foregoing examples, we find—

TABLE XI.—*Long Flexors in Foot of Man.**

	No. 1. per cent.	No. 2. per cent.	No. 3. per cent.	No. 4. per cent.	No. 5. per cent.	Mean per cent.
1. First toe,	41	41	37	47	39	41.0
2. Second toe,	15	18	17	14	15	15.8
3. Third toe,	18	17	17	11	15	15.6
4. Fourth toe,	13	15	15	11	19	14.6
5. Fifth toe,	13	9	14	17	12	13.0
	100	100	100	100	100	**100**
1. Fl. hall.,	41	47	41	42	46	43.4
2. Fl. dig.,	40	36	38	41	47	40.4
3. Access., &c.,	19	17	21	17	7	16.2
	100	100	100	100	100	**100**

* It is well known that the prairie hunters, on soft ground, always turn their feet well inwards, in order to compel the smaller toes to take their due

The distribution of the flexor tendons of the foot has been carefully studied by Mr. Turner ("Trans. R. S. Edin." vol. xxiv. p. 181), with the following results:—Out of fifty feet dissected, the *flexor hallucis* gave off a slip; to the second toe only, in eleven instances; to the second and third toes only, in twenty instances; to the second, third, and fourth toes only, in eighteen instances; and in one instance it gave slips to the second, third, fourth, and fifth toes.

The *flexor digitorum longus*, in nine instances, gave off a slip, previous to its subdivision, to the flexor tendon of the *hallux*.

Flexor hallucis of Man (Turner).

	Instances.	Per cent.
1. Slip to long flexor of second toe only,	11	22
2. Do. second and third toes only,	20	40
3. Do. second, third, and fourth toes only,	18	36
4. Do. second, third, fourth, and fifth toes,	1	2
Total,	50	100

Prof. Schultze, of Rostock, also (Zeitschrift für wissenschaftlichen Zoologie XVII., 1.) has published the results of a similar examination of 100 human feet, as follows—

Flexor hallucis of Man (Schultze).

1. Slip to long flexor of second toe only,	32 per cent.
2. Do. second and third toes only,	58 ,,
3. Do. second, third, and fourth toes only,	10 ,,
4. Do. second, third, fourth, and fifth toes,	0 ,,
Total,	100

share of the work of grasping the ground in walking; if this precaution be neglected, it is found that great fatigue is caused, on a long march, in the flexor muscle of the hallux, which, when the toes are turned outwards, is compelled to do nearly the whole work required.

He found also, that in twenty-nine instances the *flexor digitorum longus* sent a slip to the tendon of the *hallux*, and that this occurred most frequently in the cases in which the *flexor hallucis* sent slips to the second, third, and fourth toes, as if to compensate for the loss of power thus occasioned in the *hallux*.

If P' denote the force applied by a muscle to one extremity of a tendon, and P denote the force conveyed to its other extremity, or insertion, the forces P' and P will be equal, if the tendon experiences no friction in passing from the muscle to the insertion of the tendon; but P' will exceed, or fall short of P, by a force equal to the friction F, according as the muscle overcomes an external resistance, or external forces overcome the resistance of the muscle. We shall, therefore, have

$$P' = P \pm F \qquad (15)$$

in the two cases supposed.

In both cases P' is proportional to the cross section of the muscle, or inherent force of muscular contraction; and P, which represents the external force applied, must always be less than the force sufficient to break the tendon. Solving the foregoing equation for P, we find

$$P = P' \mp F;$$

from which it follows, that the safety of the system requires the tendon to be made always strong enough to resist the force $P' + F$, or the sum of the muscular force and friction combined.

Hence it follows that the cross section of the tendon will always indicate a strength, sufficient not only to resist the force of the muscle, but the sum of that force and the friction experienced by the tendon.

In order to ascertain the proportion of the cross section (or force) of a given muscle, to the cross section (or strength) of its tendon, I obtained permission from Professors Morgan and Bevan, of the Royal College of Surgeons in Ireland, to

make use of the right arm and hand of a well-developed subject (male) brought into the dissecting room of that College, in March, 1868.

I first ascertained the specific gravities of muscles and tendons, with the following results:—

Muscles.

	Sp. Gr.
Biceps humeri, ...	1.050
,, ,, ...	1.054
Brachiæus, ...	1.053
Mean, . . .	**1·0523**

Tendons.

Scapular tendon of biceps, . .	1.112
Radial tendon of biceps, . .	1.119
Mean, . . .	**1.1155**

From these data it was easy to determine the cross section of either muscles or tendons, by weighing a known length of either one or other. The following Table was thus readily constructed:—

TABLE XII.—*Cross Sections of Muscles in an Adult Human Male Subject.*

Muscle.	Weight of Muscle.	Length of Muscle used.	Weight of Muscle used.	Cross Section.
1. Biceps humeri, . .	3.48 oz. av.	3 inches.	713.8 grs.	0.895 sq. in.
2. Palmaris longus, . .	0.22 ,,	1 ,,	39.4 ,,	0.148 ,,
3. Ext. carp. rad. longr.,	1.26 ,,	2 ,,	310.6 ,,	0.584 ,,
4. Ext. carp. rad. brevr.,	0.81 ,,	2 ,,	215.3 ,,	0.405 ,,
5. Biceps humeri (long head), ...	}	5 ,,	503.9 ,,	0.379 ,,
6. Fl. pollicis long., . .	0.44 ,,	1 ,,	60.6 ,,	0.228 ,,
7. Fl. carp. rad., . . .	0.53 ,,	2 ,,	124.6 ,,	0.234 ,,
8. Ext. carp. uln., . .	0.58 ,,	1 ,,	56.5 ,,	0.212 ,,
9. Fl. digit. subl., . .	2.21 ,,	2 ,,	328.4 ,,	0.618 ,,
10. Fl. dig. prof., . . .	2.74 ,,	2 ,,	408.2 ,,	0.768 ,,
11. Ext. oss. met. poll., .	0.52 ,,	2 ,,	118.7 ,,	0.223 ,,
12. F. carp. uln., . . .	0.83 ,,	2 ,,	97.0 ,,	0.182 ,,
13. Brachialis anticus, .	3.13 ,,	3 ,,	736.0 ,,	0.923 ,,

TABLE XIII.—*Cross Sections of Tendons in an Adult Human Male Subject.*

Tendon.	Length used.	Weight.	Cross Section.
1. Biceps humeri (radial tendon), ...	2 inches.	17.87 grs.	0.0317 sq. in.
2. Palmaris longus, . . .	4 ,,	6.3 ,,	0.0056 ,,
3. Ext. carp. rad. longr., .	5 ,,	31.4 ,,	0.0223 ,,
4. Ext. carp. rad. brevr., .	3 ,,	18.6 ,,	0.0220 ,,
5. Biceps humeri (scapular tendon), ...	3 ,, ,,	17.90 ,,	0.0212 ,,
6. Fl. pollicis long., . . .	2 ,,	8.2 ,,	0.0145 ,,
7. Fl. carp. rad., ...	3 ,,	13.1 ,,	0.0155 ,,
8. Ext. carp. uln., . . .	1 ,,	5.6 ,,	0.0199 ,,
9. Fl. digit. subl., . . .	2 ,,	37.5 ,,	0.0665 ,,
10. Fl. digit. prof., . . .	2 ,,	52.3 ,,	0.0928 ,,
11. Ext. oss. met. poll., . .	2 ,,	16.3 ,,	0.0289 ,,
12. Fl. carp. uln., ...	2 ,,	14.3 ,,	0.0254 ,,
13. Fl. dig. long. ped., . .	. . ,	. . . .	0.0240 ,,
14. Fl. hallucis long., . . .	. . . .	. . . .	0.0260 ,,

The cross sections of the tendons of the foot, Nos. 13, 14, are estimated from the dried tendons of the subject No. 1, Table XI. (figured in page 77), by a process sometimes usefully employed in these investigations. The tendon is dried at 212° F. and weighed, and from that weight the original cross section, when quite fresh, estimated as follows:

A portion of the tendon of the *gastrocnemius* muscle of a large Pyrenean mastiff weighed, when quite fresh, 21.25 grs., and when dried, at 212° F. weighed 9.1 grs. From this experiment we may infer the quantity of solid matter and water in fresh tendon—

Solid matter, ...	42.8 per cent.
Water, ...	57.2 ,,
	100.0

I found also, by careful measurement of the cross sections of perfectly fresh tendons, that a tendon originally one inch in

length, and weighing, when dried at 212° F., five grains, had an original cross section of 0.0401 sq. inches.

If we unite Tables XII. and XIII. into one Table, showing the proportion between the cross sections of the muscles and their tendons, we obtain the following results :—

TABLE XIV.—*Ratio of Cross Section of Muscle to that of Tendon in Man.*

Muscle.	Ratio of Cross Section of Muscle to that of Tendon.	Force of Muscle.
1. Biceps humeri, ...	28.2	91.78 lbs. av.
2. Palmaris longus, . . .	26.4	15.18 ,,
3. Ext. carp. rad. long., . .	26.2	59.89 ,,
4. Ext. carp. rad. brev., . .	18.4	41.53 ,,
5. Biceps humeri (long head),	18.0	38.86 ,,
6. Fl. poll. long., ...	15.7	23.38 ,,
7. Fl. carp. rad., ...	15.1	24.00 ,,
8. Ext. carp. uln., ...	10.7	21.74 ,,
9. Fl. dig. subl., ...	9.3	63.38 ,,
10. Fl. dig. prof., ...	8.3	78.76 ,,
11. Ext. oss. met. poll., . . .	7.7	22.87 ,,
12. Fl. carp. uln., ...	7.2	18.66 ,,
13. Brachiæus, ...	..	94.65 ,,

From an examination of the foregoing Table, it is evident that the muscles which experience least resistance have the largest coefficient of cross section as compared with their respective tendons. Thus the *biceps humeri* has a coefficient of 28.2 as compared with its *radial* tendon, and a coefficient of 18.0 only, as compared with its *scapular* tendon, which is exposed to the friction of the head of the humerus. Again, the *ext. oss. met. poll.*, whose tendon winds round the *radius*, and has the duty imposed on it of binding down the tendons of the radial extensors of the wrist, has the coefficient of 7.7, as compared with 26.2 and 18.4, the coefficients of the comparatively free tendons of those extensors.

If we solve equation (15) for F, the friction experienced, we shall have

$$F = P - P',$$

where P denotes the strength of the tendon necessary for safety, and P' denotes the actual force of the muscle.

Assuming, for the present, that the cross section of the free muscle is to the cross section of the free tendon in the proportion of 28.2 : 1, as in the case of the *biceps humeri* and its radial tendon, we find the following frictions for the several muscles :—

TABLE XV.—*Comparison of the Strength of Tendon, the Force of Muscle, and the Force consumed in Friction, in several Muscles of Man.*

Muscle.	P Strength of Tendon.	P' Force of Muscle.	F Friction, &c.	Friction, &c., per cent.
1. Biceps, ...	91.78 lbs.	91.78 lbs.	0.00 lbs.	0.0 per cent.
2. Palmaris long., . .	16.19 ,,	15.18 ,,	1.01 ,,	6.2 ,,
3. Ext. carp. rad. long.,	64.49 ,,	59.89 ,,	4.60 ,,	7.1 ,,
4. Ext. carp. rad. brev.,	63.62 ,,	41.53 ,,	22.09 ,,	34.7 ,,
5. Biceps humeri (long head), ...	61.31 ,,	38.86 ,,	22.45 ,,	36.6 ,,
6. Fl. poll. long., . .	41.93 ,,	23.38 ,,	18.55 ,,	44.2 ,,
7. Fl. carp. rad., . .	44.82 ,,	24.00 ,,	20.82 ,,	46.4 ,,
8. Ext. carp. uln., . .	57.55 ,,	21.74 ,,	35.81 ,,	62.2 ,,
9. Fl. dig. subl., . .	192.31 ,,	63.38 ,,	128.93 ,,	67.0 ,,
10. Fl. dig. prof., . .	268.37 ,,	78.76 ,,	189.61 ,,	70.7 ,,
11. Ext. oss. met. poll.,	83.56 ,,	22.87 ,,	60.69 ,,	72.6 ,,
12. Fl. carp. uln., . .	73.45 ,,	18.66 ,,	54.79 ,,	74.5 ,,
13. Fl. dig. long. ped., .	69.40 ,,	..	..	..
14. Fl. hall. long, . .	75.19 ,,	..	..	..

In order to compare the relative cross sections of the muscles and tendons of other animals with those of man, given in Table XIV., I destroyed, on the 21st March, 1868, a large Pyrenean mastiff bitch, by strychnia, and proceeded to inves-

tigate the relation between the cross sections of her muscles and their respective tendons, with the following results :—

A portion of the *gastrocnemius* muscle, two inches in length, weighed in air, 1382 grs., and in water, 53·7 grs., giving a specific gravity of 1.040.

A portion of the tendon of the same muscle was found to weigh in air, 21.25 grs., and in water, 2.80 grs., giving a specific gravity of 1.152.

From these data, the following cross sections of muscles and tendons were readily calculated.

TABLE XVI.—*Cross Sections of Muscles in Pyrenean Mastiff.*

Muscle.	Weight of Muscle.	Length of Muscle used.	Weight of Muscle used.	Cross Sections.
1. Gastrocnemius, . .	6.22 oz. av.	2 inches.	1382.0 grs.	2.631 sq. in.
2. Fl. carp. rad., . . .	0.35 ,,	1 ,,	74·3 ,,	0.283 ,,
3. Fl. dig. long. (pedis),	0.28 ,,	1 ,,	51·3 ,,	0.195 ,,
4. Ext. carp. rad., . .	1.40 ,,	2 ,,	332.0 ,,	0.632 ,,
5. Fl. carp. uln.,* . .	0.38 ,,	2 ,,	92.5 ,,	0.176 ,,
6. Fl. hall. long., . .	1.59 ,,	2 ,,	357·5 ,,	0.680 ,,
7. Biceps humeri, . .	2.11 ,,	2 ,,	477·3 ,,	0.909 ,,
8. Fl. dig. subl., . . .	1.06 ,,	2 ,,	167.6 ,,	0.319 ,,
9. Fl. dig. prof., . . .	2.75 ,,	1½ ,,	355·3 ,,	0.902 ,,
10. Ext. carp. uln., . .	0.47 ,,	2 ,,	95·3 ,,	0.181 ,,

The cross sections of the corresponding tendons were found to be—

* The *flexor carpi ulnaris* consists of two muscles, of which one takes its origin from the front of the inner condyle of the *humerus*, and the other from the *olecranon* of the *ulna*.

TABLE XVII.—*Cross Sections of Tendons in Pyrenean Mastiff.*

Tendon.	Length used.	Weight of do.	Cross Section.
1. Gastrocnemius, . . .	2 inches.	30.27 grs.	0.0520 sq. in.
2. Fl. carp. rad., ...	5 ,,	8.6 ,,	0.0059 ,,
3. Fl. dig. long. (pedis), .	5 ,,	6.5 ,,	0.0045 ,,
4. Ext. carp. rad., . . .	3 ,,	14.0 ,,	0.0160 ,,
5. Fl. carp. uln., ...	3 ,,	4.9 ,,	0.0056 ,,
6. Fl. hall. long., ...	2 ,,	13.3 ,,	0.0228 ,,
7. Biceps humeri (scapular tendon), ...	$1\frac{1}{2}$,,	19.6 ,,	0.0449 ,,
8. Fl. dig. subl., ...	2 ,,	14.6 ,,	0.0251 ,,
9. Fl. dig. prof., ...	$1\frac{1}{2}$,,	36.2 ,,	0.0830 ,,
10. Ext. carp. uln., . . .	2 ,,	11.5 ,,	0.0197 ,,

From the preceding Tables we may easily construct the following :—

TABLE XVIII.—*Ratio of Cross Section of Muscle to that of Tendon in Mastiff.*

Muscle.	Ratio of Cross Sections.
1. Gastrocnemius, ...	50.6
2. Flexor carpi radialis, ...	48.0
3. Flexor digitorum longus (pedis), ...	43.3
4. Extensor carpi radialis (long. et brev.), . . .	39.5
5. Flexor carpi ulnaris (olecranalis), ...	31.4
6. Flexor hallucis longus, ...	29.8
7. Biceps humeri (scapular tendon), ...	20.2
8. Flexor digitorum sublimis, ...	12.7
9. Flexor digitorum profundus, ...	10.9
10. Extensor carpi ulnaris, ...	9.2

If we suppose, as we have already done in the case of human muscles, that the strength of the tendon is intended to be divided between the pull exerted upon it by the muscle and the strain caused by external forces; and also suppose that 50.6* represents in the mastiff the proportion of cross

* This coefficient is taken from the *gastrocnemius* muscle, and exceeds that taken from the radial tendon of the *biceps humeri* in man; this difference is, no

sections of free muscle and free tendon, we may readily obtain the following Table :—

TABLE XIX.—*Comparison of the Strength of Tendon and Force of Muscle in the Pyrenean Mastiff.*

Muscle.	Strength of Tendon.	Force of Muscle.	Friction, &c. &c.
1. Gastrocnemius, . . .	100	100	0
2. Fl. carp. rad., ...	100	94.8	5.2
3. Fl. dig. longus, . . .	100	83.6	16.4
4. Ext. carp. rad., . . .	100	78.1	21.9
5. Fl. carp. uln., ...	100	62.1	37.9
6. Fl. hall. long., ...	100	58.9	41.1
7. Biceps humeri (scapular tendon), ...	100	39.9	60.1
8. Fl. dig. subl., ...	100	25.1	74.9
9. Fl. dig. prof., ...	100	21.2	78.8
10. Ext. carp. uln., . . .	100	18.2	81 8

The results exhibited by the preceding Table are similar to those shown in Table XV., for human muscles. In both cases, the muscles whose tendons are exposed to the greatest amount of friction, are those which transmit the least force to their points of application. Thus, in the mastiff, the *flexor sublimis* and *flexor profundus* lose, by friction, a much greater portion of their original force than the comparatively free tendons of the *radial* and *ulnar flexors* of the wrist; and more than the *radial extensor* of the wrist, while the *ulnar extensor* of the same joint resembles the flexors of the fingers in its proportionately large loss of force transmitted.

The following observations were made on the muscles and tendons of a Wallaby Kangaroo :—

The *gastrocnemius* muscle weighed 2.93 oz. av., and one inch of its central portion weighed in air, 356.9 grs. ; in water,

doubt, real, for the dog's tendon is stronger than that of man; at the same time it is worth while to notice the similarity of the coefficients of the scapular head of the biceps.

25.3 grs. Three inches of its tendon weighed in air, 31.4 grs., in water, 4.4 grs.

The *flexor digitorum longus* (*pedis*) weighed 0.83 oz. av.; one inch of its central portion weighed 96.3 grs., and three inches of its tendon weighed 21.7 grs.

From these data we obtain—

Specific gravity of kangaroo muscle, . .	1.076
Specific gravity of kangaroo tendon, . .	1.163

From the preceding data we obtain—

Kangaroo Muscles.	Cross Section of Muscle.	Cross Section of Tendon.	Ratio.
1. Gastrocnemius, . .	1.313 sq. in.	0.0356 sq. in.	36.9
2. Fl. dig. longus, . .	0.354 „	0.0246 „	14.4

Assuming, as in the case of Man and the Mastiff, that the friction is zero in the muscle that has the highest coefficient of cross section as compared with its tendon; we find in the kangaroo—

Fl. dig. long.—Strength of tendon, . .		100
	Force of Muscle, . . .	39
	Friction, &c., ...	61

It appears from the whole of the preceding investigation that the cross section of a muscle does not bear a constant ratio to the cross section of its tendon, unless the friction exerted upon the muscle and tendon be also constant; and it is also possible that the strength of the tendon may be somewhat greater than what would be absolutely necessary to counterbalance the force of the muscle and friction; this additional strength forming, in fact, a surplusage difficult to calculate, but one which cannot be supposed very large, if the principle of *economy of material* in nature be admitted.

On examining Tables XV. and XIX., and the result just stated for the *long flexor* of the foot of the kangaroo, it appears that the friction of the tendons and muscles, and the surplusage of strength provided against any sudden accident, may consume as much as 75 or 80 per cent. of the strength of the tendon, leaving only *one-quarter* or *one-fifth* of that tendon to be strained by the contraction of the muscular fibres. This, however, can only occur in very extreme cases, and the force expended in such cases reappears as heat, and so assists the general purposes of the body, which requires its temperature to be maintained at 100° F. in order to perform its functions. In the great majority of cases the muscles, in performing their usual duties, act in such a position of the tendons as to cause the minimum amount of friction, and to produce the maximum of external effect; the surplus strength of the tendons being provided as a safeguard against strains that rarely occur, and then only as accidents.

(*b*). *Chimpanzee.*—In this animal the *flexor hallucis* supplies the whole tendons of the first and fourth toes, and part of the tendons of the second and third toes; while the *flexor digitorum* supplies the remaining parts of the tendons of the second and third toes, and the whole tendon of the fifth toe. This arrangement would seem to indicate a concurrence in action between the first and fourth toes, which must correspond to some peculiarity in the animal's mode of grasping objects; while the fifth or outer toe retains its independence.

(*c.*) *Monkeys of the Old Continent.*—The *Macaques* of Asia, and the *long-tailed Monkeys* of Africa, such as the *Mangabeys*, are characterized by an arrangement of flexor tendons of the foot, which is shown in Fig. 7, representing the flexor tendons of the foot of the Bruh, or Pig-tailed Macaque (*Macacus Nemestrinus*), seen from below, and drawn aside from their natural positions, so as to exhibit their mechanical arrangement.

Flexor Tendons of the Foot in Asiatic and African Monkeys.

In these monkeys, the *flexor hallucis* supplies part of the tendon of the first toe, and the whole tendons of the third and fourth toes; while the *flexor digitorum longus* supplies the remainder of the tendon of the first toe, and the whole tendons of the second and fifth toes. This arrangement produces a remarkable concurrence of action in the following combinations of toes:—

a. First . Third . Fourth.
b. First . Second . Fifth.

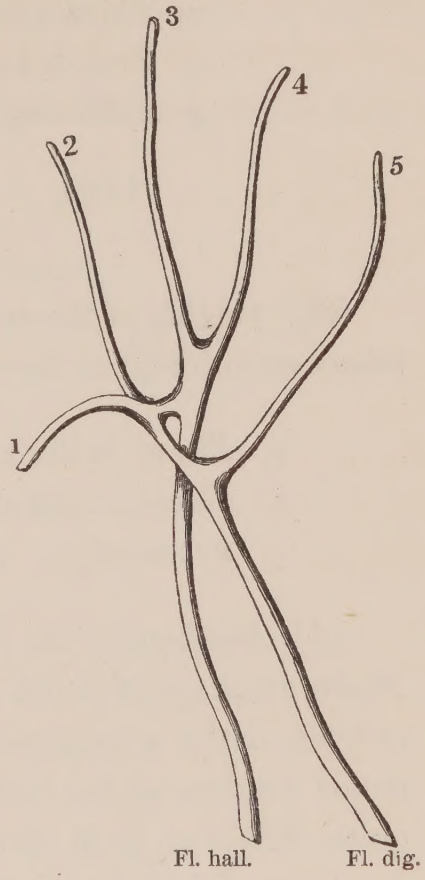


Fig. 7.

I have often seen the Rhesus Macaque avail himself of the first of these combinations to catch a flea, with a dexterity that showed how superior his foot is, considered as a prehensile organ, when compared with that of Man.

The relative strengths of the several tendons in the foot of the *Macacus nemestrinus*, measured by the weights of equal lengths, were found to be as follows:—

Macacus Nemestrinus.

1. *Flexor hallucis longus*, . .	54 per cent.
2. *Flexor digitorum longus*, . .	44 ,,
3. *Accessorius, &c.*, ...	2 ,,
Total, ...	100

1. First toe,	. . . .	25 per cent.
2. Second toe,	. . .	15 ,,
3. Third toe,	. . .	21 ,,
4. Fourth toe,	. . .	25 ,,
5. Fifth toe,	. . .	14 ,,
Total,	. . . .	100

The weights of the several muscles in the *Macacus Nemestrinus* were found to be—

1. *Flexor hallucis longus*,	. . .	0.66 oz. av.
2. *Flexor digitorum longus*,	. .	0.36 ,,
3. *Accessorius*,	. . . . .	0.05 ,,

(*d.*) *Monkeys of the New Continent.*—The *Capuchin* and *Negro* monkeys of South America exhibit, in the arrangement of the flexor tendons of the foot, a type quite different from that of the monkeys of the Old World. This arrangement is shown in Fig. 8, which represents the tendons of the foot of *Lagothrix Humboldtii*, or Negro monkey of South America.

FIG. 8.

In the monkeys of the New World, the *flexor hallucis longus* supplies part of the first toe, and the whole of the long flexors of the second, third, and fourth toes; while the *flexor digitorum longus* supplies the remainder of the flexor tendon of the first toe, and the whole long flexor tendon of the fifth toe.

In these animals we may observe a concurrence in action of the first and fifth toes, like that which takes place between the first, third, and fourth toes, in the monkeys of the Old Continent.

The relative strengths of the flexor tendons in the Negro monkey were found to be—

Lagothrix Humboldtii.

1. *Flexor hallucis longus*, . .	49	per cent.
2. *Flexor digitorum longus*, . .	43	,,
3. *Accessorius, &c.*, ...	8	,,
Total, ...	100	

1. First toe, ...	18.5	per cent.
2. Second toe, ...	18.5	,,
3. Third toe, ...	25.0	,,
4. Fourth toe, ...	19.5	,,
5. Fifth toe, ...	18.5	,,
Total, . . .	100.0	

In the *Hapale* or *Jacchus*, the *flexor hallucis* supplies the third and fourth toes only, so that its human name becomes inappropriate; and the *flexor digitorum* supplies the second and fifth toes, with a small slip to the *hallux*.

(*e.*) *The Carnivores.*—In most of the Carnivores, the *hallux* is wanting, and in all of them the *flexor hallucis*, and *flexor digitorum*, instead of being distributed separately to the different toes, are united into one tendon (into which also the *accessorius* is inserted), which is afterwards distributed to all the toes, generally four in number, but which may be five or occasionally six in number.

The Carnivores, of the flexor tendons of whose feet I have kept a record, are the following:—

1. The Lioness (young).
2. The Jaguar (female).
3. The Leopard (male).
4. The Indian Jackall (male).
5. The Bengal Fox (male).
6. The Dingo (female).
7. The Dog (female).
8. The Otter (female).
9. The Virginian Bear (female).

1. In the *Lioness* I found the following relative strengths of the long flexor tendons distributed to the four toes :—

Lioness (*young*), (*long flexor tendons of toes*).

1. First toe, . . .	27.2	per cent.
2. Second toe, . .	27.0	,,
3. Third toe, . .	23.9	,,
4. Fourth toe, . .	21.9	,,
5. Retractor of pad, . .		
	100.0	

Lioness (*flexor tendons of foot*).

			Weight.
1. *Flexor hallucis longus*, .	31.8	per ct.	. 0.93 oz. av.
2. *Flexor digitorum longus*, .	9.2	,,	. 0.27 ,,
3. *Flexor accessorius*, . .	. .	,,	. . . ,,
4. *Friction, &c.*, ...	59.0	,,	. . . ,,
Total, . . .	100.0		

The angle between the *fl. hall.* and the *fl. dig. long.* was 21°.

I was unable to determine satisfactorily the absolute strengths of the tendon of the *accessorius* muscle, and of that which retracts the pad of the foot, but I found that equal lengths of these tendons had equal weights, so that they mutually eliminate each other from the Tables.

2. The tendons of the sole of the foot in the *Jaguar* are shown in Fig. 9, which represents the *pad-retractor* (*a*) and *accessorius* (*a'*) tendons in their natural positions.

The relative strengths of the tendons are as follows:—

Jaguar (*long flexor tendons of toes*).

1. First toe,	. .	30.9 per cent.
2. Second toe,	.	24.2 ,,
3. Third toe,	. .	23.3 ,,
4. Fourth toe,	. .	21.6 ,,
5. Retractor of pad,	. .	
Total,	. .	100.0

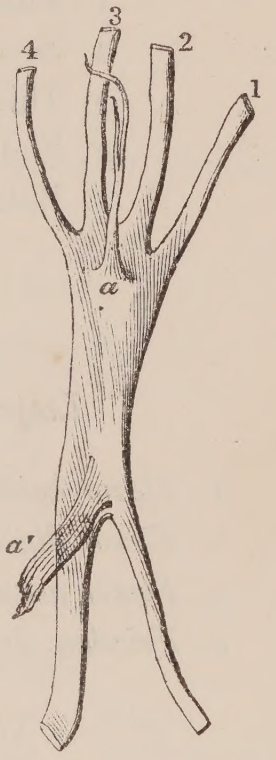


FIG. 9.

Jaguar (*long flexor tendons of foot*).

			Weight.
1. *Flexor hallucis longus*,	.	35.8 per cent.	. 0.83 oz.
2. *Flexor digitorum longus*,	.	15.0 ,,	. 0.26 ,,
3. *Flexor accessorius*,	. .	. . ,,	
4. *Friction, &c.*,	. . . .	49.2 ,,	
Total,	. . .	100.0	

The angle between the *fl. hall. long.* and the *fl. dig. long.* was 32°.

3. In the *Leopard* the relative strengths of the tendons were found to be as follows:—

Leopard (*long flexor tendons of toes*).

1. First toe, ...	21.9	per cent.
2. Second toe, ...	23.6	,,
3. Third toe, ...	27.0	,,
4. Fourth toe, ...	27.5	,,
5. Retractor of pad, ...		,,
Total, . . .	100.0	

Leopard (*long flexor tendons of foot*).

			Weight.
1. *Flexor hallucis longus*, .	39.2	per cent. .	1.63 oz. av.
2. *Flexor digitorum longus*, .	15.3	,, .	0.50 ,,
3. *Flexor accessorius*, ...		,,	
4. *Friction, &c.*, ...	45.5	,,	
Total, . . .	100.0		

The angle between the *fl. hall. long.* and the *fl. dig. long.* was 20°.

4. In the *Indian Jackall* I found the following relative strengths of the tendons of the foot:—

Indian Jackall (*long flexor tendons of toes*).

1. First toe, ...	23.8	per cent.
2. Second toe, ...	28.2	,,
3. Third toe, ...	26.5	,,
4. Fourth toe, ...	21.5	,,
Total, . . .	100.0	

Indian Jackall (*long flexor tendons of foot*).

			Weight.
1.	*Flexor hallucis longus*,	40.3 per cent.	0.28 oz. av.
2.	*Flexor digitorum longus*,	10.5 „	0.05 „
3.	*Friction, &c.*,	49.2 „	
	Total,	100.0	

5. In the *Bengal Fox*, the relative strengths of tendons were found to be :—

Bengal Fox (*long flexor tendons of toes*).

1.	First toe,	25.0 per cent.
2.	Second toe,	27.0 „
3.	Third toe,	22.5 „
4.	Fourth toe,	25.5 „
	Total,	100.0

Bengal Fox (*long flexor tendons of foot.*)

			Weight.
1.	*Flexor hallucis longus*,	32.9 per cent.	0.08 oz. av.
2.	*Flexor digitorum longus*,	9.5 „	0.02 „
3.	*Friction, &c.*,	57.6 „	
	Total,	100.0	

6. In the *Australian Dingo* I found the following results :—

Australian Dingo (*long flexor tendons of toes*).

1.	First toe,	24.6 per cent.
2.	Second toe,	28.9 „
3.	Third toe,	25.3 „
4.	Fourth toe,	21.2 „
	Total,	100.0

Australian Dingo (long flexor tendons of foot).

		Weight.
1. *Flexor hallucis longus*, .	53·5 per cent.	. 0.59 oz. av.
2. *Flexor digitorum longus*,	12.7 ,,	. 0.10 ,,
3. *Friction, &c. &c.*, . . .	33.8 ,,	
Total, . . .	100.0	

7. In the *Pyrenean Mastiff*, two "dew claws" or spurious toes are found, in addition to the regular four toes characteristic of the digitigrade carnivores. The relative strengths of the tendons supplying the six toes were found to be—

Pyrenean Mastiff (long flexor tendon of toes).

1. First toe, ...	4.1 per cent.
2. Second toe, . . .	10.7 ,,
3. Third toe, ...	20.9 ,,
4. Fourth toe, . . .	25.0 ,,
5. Fifth toe, ...	22.8 ,,
6. Sixth toe, ...	16.5 ,,
Total, . .	100.0

Fig. 10.

Pyrenean Mastiff (long flexor tendons of foot).

		Weight.
1. *Fl. hall. long*,	28.1 per cent.	. 1.59 oz. av.
2. *Fl. dig. long.*,	6.5 ,,	. 0.28 ,,
3. *Friction, &c.*,	65.4 ,,	
Total, .	100.0	

8. The *Otter* possesses regularly five toes, supplied with long flexor tendons nearly equal in strength. If the united

cross section of these tendons be called 100, we find for the cross sections of the flexor tendons of the foot the following values :—

Otter (*long flexor tendons of foot*).

1. *Flexor hallucis longus*, . } 2. *Flexor digitorum longus*, . }	80.2 per cent.
3. *Friction, &c.*, ...	19.8 „
Total, ...	100.0

9. The *Bear*, like other plantigrade Carnivores, possesses five toes, the tendons of which gave me the following results, in the Virginian and Japanese Bears :—

Bear (*long flexor tendons of toes*).

	Virginian Bear.	Japanese Bear.
1. First toe, ...	11.6 per cent.	16.3 per cent.
2. Second toe, ...	24.5 „	22.0 „
3. Third toe, ...	24.7 „	24.4 „
4. Fourth toe, ...	22.4 „	22.0 „
5. Fifth toe, ...	16.8 „	15.3 „
Total, . . .	100.0	100.0

	Virginian Bear.	Japanese Bear.
1. *Flexor hallucis longus*, .	57.5 per cent.	53.7 per ct.
2. *Flexor digitorum longus*,	18.6 „	14.6 „
3. *Friction, &c.*, ...	23.9 „	31.7 „
Total, . . .	100.0	100.0

If we compare together the amount of friction experienced by the long flexor tendons of the foot of the various carnivores, we find the following Table :—

TABLE XX.*—*Amount of Friction, &c., in the Long Flexor Tendons of the Toes of Carnivores.*

Carnivores.	Friction, &c.
1. Mastiff, ...	65.4 per cent.
2. Lioness, ...	59.0 ,,
3. Fox, ...	57.6 ,,
4. Jaguar, ...	49.2 ,,
5. Jackall, ...	49.2 ,,
6. Leopard, ...	45.5 ,,
7. Dingo, ...	33.8 ,,
8. Japanese Bear, . .	31.7 ,,
9. Virginian Bear, . .	23.9 ,,
10. Otter, ...	19.8 ,,

* Since writing the preceding Table, I have had an opportunity of examining the tendons of the foot of a male European Wolf, and of a magnificent Bengal Tiger, with the following results :—

European Wolf (long flexors of toes).

1. First toe, ...	24.6 per cent.
2. Second toe, ...	25.8 ,,
3. Third toe, ...	25.3 ,,
4. Fourth toe, ...	24.3 ,,
Total, ...	100.0

European Wolf (long flexor tendons of foot).

		Weight.
1. *Flexor hallucis longus*, . .	13.5 per cent. .	0.30 oz. av.
2. *Flexor digitorum longus*, .	52.5 ,, .	0.79 ,,
3. *Friction, &c.*, ...	34.0 ,, .	
Total, ...	14.40	

It will be seen, on comparing these results with those already given for Man and the Quadrumans, that the Otter's foot, used for swimming, resembles that of Man; and that the feet of the Carnivores differ widely from those of Man and the Quadrumans, being characterized by an enormous strengthening of the flexor tendons of the toes. The scratching and digging actions of the hind claws of these Carnivores are essentially distinct from the grasping actions of the feet of the monkeys, as well as from the swimming motions of the feet of Man and the Otter.

(*f.*) *The Ungulates.*—Of the even-toed ungulates, I have examined the flexor tendons of the Ox and Goat; and of the odd-toed, those of the Rhinoceros.

1. The tendons of the foot of the Ox (*Zebu Cow*) are shown in Fig. 11, and their relative cross sections are as follows:—

Bengal Tiger (*long flexors of toes*).

1. First toe,	28.3	per cent.
2. Second toe,	24.6	,,
3. Third toe,	21.8	,,
4. Fourth toe,	18 0	,,
5. Retractor of pad,	7.3	,,
Total,	100.0	

Bengal Tiger (*long flexor tendons of foot*).

			Weight.
1. *Flexor digitorum longus*,	12.2	per cent.	1.57 oz. av.
2. *Flexor hallucis longus*,	35.8	,,	6.52 ,,
3. *Accessorius*,	6.0	,,	
Friction, &c.,	46.0	,,	
Total,	100.0		

Zebu Cow (*long flexors of toes.*)

1. First toe,	. . .	49.1 per cent.
2. Second toe,	. . .	50.9 ,,
Total,	. . .	100.0

			Weight.
1. *Fl. dig. long.*,	74.6	per c.	0.90 oz. av.
2. *Fl. hall. long.*,	18.6	,,	4.43 ,,
3. *Friction, &c.*,	6.8	,,	
Total, . . .	100.0		

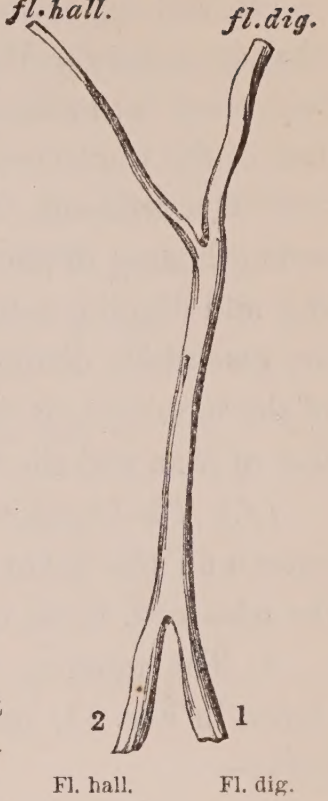


Fig. 11.

2. The flexor tendons of the Goat (*Bombay*) resemble those of the Cow very closely; and gave the following results :—

Bombay Goat (*long flexors of toes*).

1. First toe,	. . . . .	47.6 per cent.
2. Second toe,	. . . . .	52.4 ,,
Total,	. . .	100.0

			Weight.
1. *Flexor digitorum longus*,	69.05	per cent.	0.04 oz. av.
2. *Flexor hallucis longus*,	21.43	,,	0.25 ,,
3. *Friction, &c.*, . . .	9.52	,,	
	100.00		

3. The arrangement of the flexor tendons of the toes in the foot of the Rhinoceros (Indian) is somewhat peculiar, as is shown in Fig. 12.

The *plantaris* muscle takes an origin from the back of the outer condyle of the femur, and is inserted into the plantar fascia that passes over the *os calcis*, and gives origin to the perforate tendons of the toes. The tendon of *plantaris*, flowing freely over the heel, becomes partly inserted into the tendon of the long flexor of the toes, in the centre of the foot, and partly terminates in the three perforate flexors that correspond with the *flexor brevis* of other animals. The long flexor muscles arise from the entire back of the fibula, and lower half of the tibia, and unite into a tendon that represents *fl. dig. long.* and *fl. hall. long.* united; this tendon is joined, as already described, in the sole of the foot, by the tendon of the *plantaris* muscle and *flexor brevis* united.

Pl. Tendon of *plantaris.*
Fl. long. Joint tendon of *fl. hall. long* and *fl. dig. long.*
1, 2, 3, deep flexors of toes.
1′. 2′. 3′. perforate flexors of toes.

Fig. 12.

The relative strengths of the foregoing tendons, measured by their cross sections, are as follows, representing the total cross section of the flexors of the leg by 100:—

Rhinoceros (long flexor tendons of toes).

		Weight.
1. *Plantaris*, ...	44·3 per cent.	3·5 oz. av.
2. *Flexores longi dig. et hall.*,	55·7 ,,	21·0 ,,
Total, . . .	100.0	

1. *Long flexors of toes,*	61.1	per cent.
2. *Short flexors of toes,*	18.4	,,
3 *Friction, &c.,*	20.5	,,
Total,	100.0	

Long Flexors of Toes.

1. First toe,	18.4	per cent.
2. Second toe,	24.6	,,
3. Third toe,	18.1	,,
Total,	61.1	

Perforate Flexors of Toes.

1. First toe,	5.6	per cent.
2. Second toe,	6.6	,,
3. Third toe,	6.2	,,
Total,	18.4	

The arrangement of the flexor tendons in the foot of the Rhinoceros differs from that of any of the animals already described—for the total strength of the tendons of the toes is 20½ per cent. short of that of the tendons of the muscles, instead of exceeding it, as in the former cases.

If we consider, however, only the deep tendons of the toes, and tendon of the long flexors, as in the former cases, neglecting the *plantaris* and short flexor, and perforate tendons of the toes, we find a result similar to those already recorded, viz., counting the total cross section of the long flexors of the toes as 100 :—

Long Flexors of Toes.

1. First toe, ...	30	per cent.
2. Second toe, ...	40	„
3. Third toe, ...	30	„
Total, . . .	100	

1. *Flexor dig. long.*, 2. *Flexor hall. long.*, . . .	91	per cent.
3. *Friction, &c.*, . . .	9	„
Total, . . .	100	

(*g.*) *The Insectivores.* I shall take the flexor tendons of the foot of the common Hedgehog as the type of the Insectivores. They belong to a degraded type, as the *fl. hall. long.* and *fl. dig. long.* are fused into one muscle, and possess a common tendon distributed by slips to each of the five toes.

Equal lengths of the common tendon and of the five slips gave the following comparative results, counting the cross section of the five tendons of the toes as 100 :—

Hedgehog (long flexor tendons of toes).

			Weight.
1. *Flexor hall. longus*, 2. *Flexor dig. longus*, .	75	per cent.	0.01 oz. av.
3. *Friction, &c.*, ...	25	„	
	100		

(*h.*) *The Edentates.*—The six-banded Armadillo, and three-toed Sloth of South America, are the best living types of this remarkable group of animals.

1. In the foot of the *Armadillo*, the *fl. hall. long.* of other animals constitutes the whole of the deep flexor of the toes,

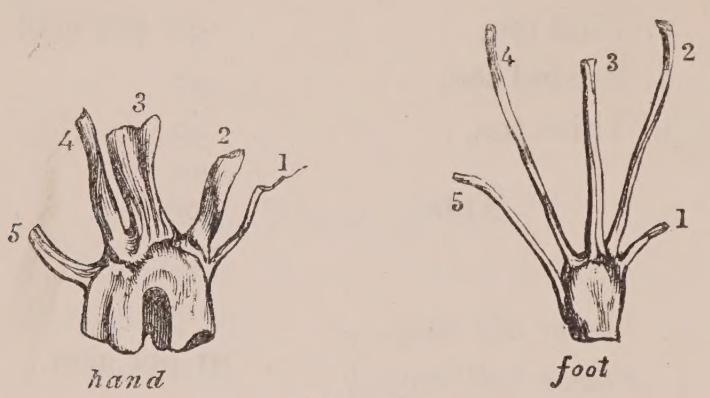


Fig. 13.

and is inserted, Fig. 13, into a plantar sesamoid bone, from which radiate five tendons, distributed to the five toes.

The muscle, which corresponds to the *fl. dig. long.*, is inserted into the near end of the first metatarsal.

The relative strengths of the *fl. hall. long.* and of the tendons of the toes are as follows :—

Six-banded Armadillo (*long flexor tendons of foot.*)

		Weight.
1. *Fl. hall. long.*,	55.6 per cent.	0.11 oz. av.
2. *Friction, &c.*,	44.4 ,,	0.02 ,,
	100.0	

1. Flexor of first toe,		13.2 per cent.
2. ,,	second toe,	22.2 ,,
3. ,,	third toe,	24.4 ,,
4. ,,	fourth toe,	22.2 ,,
5. ,,	fifth toe,	18.0 ,,
	Total,	100.0

2. In the three-toed Sloth, the arrangement of the tendons

of the foot is as follows: the *fl. hall. long.* and *fl. dig. long.* unite into a common tendon, Fig. 14; this is joined in the sole of the foot by the tendon of the large *plantaris* (*pl.*), and the united tendons diverge to form the tendons of the three toes. The tendon of the *accessorius* muscle (*acc.*) is inserted into the outer side of the tendon of the third toe.

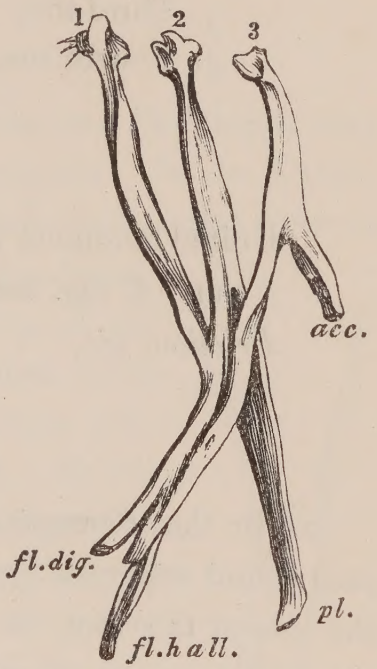


Fig. 14.

Equal lengths of these tendons, when weighed, gave the following comparative results:—

Foot of Three-toed Sloth (*long flexors*).

		Weight.
1. *Accessorius*,	14.2 per c.	0.06 oz.av.
2. *Plantaris*, .	10.0 ,,	0.13 ,,
3. *Fl. hall. long.*,	12.0 ,,	0.24 ,,
4. *Fl. dig. long.*,	21.3 ,,	
5. *Friction, &c.*,	42.5 ,,	
Total, . .	100.0	

1. First toe, ...	26.0 per cent.
2. Second toe, . . .	43.8 ,,
3. Third toe, ...	30.2 ,,
Total, . . .	100.0

(*i.*) *The Rodents.*—The following observations on the Hare and Porcupine, will suffice to show the general character of the deep flexor tendons of the feet of this important Order:—

1. In the *Hare*, the *fl. hall. long.* and *fl. dig. long.* unite, as in the Hedgehog, into a single tendon, before reaching the heel, and this tendon is afterwards subdivided into four, distributed to the four toes. Equal lengths, when weighed, gave the following results:—

Hare (long flexors of foot).

1. First toe,	28.4	per cent.
2. Second toe,	26.5	,,
3. Third toe,	23.8	,,
4. Fourth toe,	21.3	,,
	100.0	

United tendon of *fl. hall. long.* and *fl. dig. long.*,	64	per cent.
Friction, &c.,	36	,,
	100	

2. In the *Porcupine*, the *fl. dig. long.* and *fl. hall. long.* pass round separate grooves on the inner ankle, and unite in the sole of the foot, to give origin to tendons distributed to the five toes, as shown in Fig. 15. The relative cross sections of the tendons are as follows, and differ from those previously found for other animals, by the flexor tendons of the muscles being in excess of the flexors of the toes, as in the case of the *Rhinoceros*.

Porcupine—Deep flexors of Toes.

First toe,	8.0	per cent.
Second toe,	19.2	,,
Third toe,	21.3	,,
Fourth toe,	20.0	,,
Fifth toe,	11.5	,,
Deficiency,	20.0	,,
	100.0	

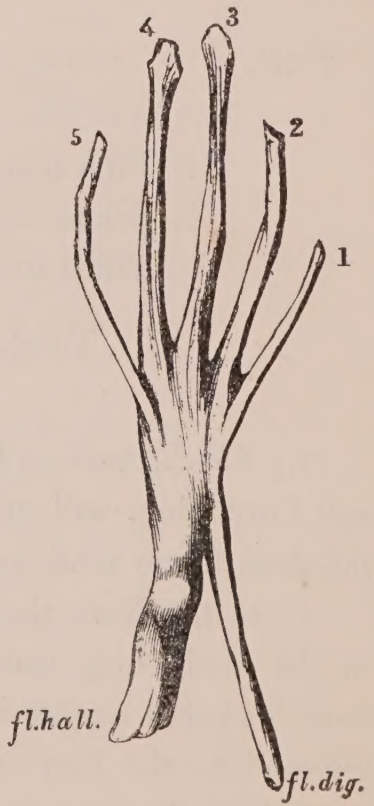


Fig. 15.

		Weight.
Flexor hall. long., . .	81.6 per cent. .	0.32 oz. av.
Flexor dig. long., . .	18.4 „ .	0.10 „
	100.0	

(*k.*) *The Marsupials.* The Kangaroos and Phalangers may be selected as good specimens of Marsupials. In the Kangaroos, the *plantaris* muscle takes an origin from the outer condyle of the femur, and from the crescent-shaped sesamoid bone lying beneath it at the top of the fibula; it is inserted into a tendon that flows freely over the calcaneum, and becomes ultimately the perforate tendon of the toes. At the outer side of the heel, a tendon (*a*) is given off, Fig. 16, which runs direct to the outer toe, and forms half of its perforate tendon, the other half being formed of a slip given off by the main tendon near the toes.

Fig. 16.

Perforate flexor tendons of Wallaby Kangaroo.

Plantaris tendon, ...	100.
1. Perforate tendon of third toe,	71 per cent.
2. „ „ fourth toe,	26 „
3. Deficiency, ...	3 „
Total, . . .	100

The deep flexor tendons of the Marsupials are formed of the single tendon belonging to the *fl. hall. long.* and *fl. dig. long.* muscles conjoined. I have found the following results to represent their relative strengths:—

Giant Kangaroo (*long flexors of toes*).

1. First and second toes,	. . .	7 per cent.
2. Third toe,	. . . . .	75 ,,
3. Fourth toe,	. . . . .	18 ,,
		100

Fl. hall. longus, *Fl. dig. longus*,	conjoined,	100 .	Weight. 2.61 oz. av.
Friction, &c.,	. . . .	0 .	
		100	

Wallaby Kangaroo (*long flexors*).

1. First and second toes,	. . .	7 per cent.
2. Third toe,	. . . . .	65 ,,
3. Fourth toe,	. . . . .	27 ,,
4. Deficiency,	. . . . .	1 ,,
		100

Fl. hall. long., *Fl. dig. long.*,	conjoined,	100	Weight. 1.24 oz. av.

In the Giant Kangaroo the deep tendons have exactly the same cross section before reaching the heel, as when distributed to the toes ; and in the Wallaby Kangaroo the difference of cross sections representing friction is only $\frac{1}{8\frac{1}{2}}$ or $1\frac{1}{4}$ per cent.

In the Phalanger, the *fl. dig. long.* muscle is fourteen times the weight of the *fl. hall. long.* muscle, and is distributed, sometimes to all five toes, and sometimes to the outer four only. The *fl. hall. long.* is inserted into the back of the scaphoid bone.

Phalanger (*long flexors of toes*).

Flexor dig. long., . . . 100

1. Second toe,	5·9	per cent.
2. Third toe,	5·9	,,
3. Fourth toe,	29·4	,,
4. Fifth toe,	29·4	,,
5. Deficiency,	29·4	,,
Total,	100.0	

(*l.*) *The Birds.*—The arrangement of the long flexor tendons is very similar in all birds, and the tendons are so long, that the question of the amount of force lost by friction in their passage round the heel possesses great interest.

The *fl. dig. long.* and *fl. hall. long.* muscles are distinct, and their tendons generally unite before reaching the heel.

1. *Ostrich* (*long flexor tendons of toes*).

1. Outer toe,	7.1	per cent.
2. Inner toe,	92·9	,,
	100.0	

		Weight.
1. *Flexor dig. long.*,	49·7 per cent.	5.90 oz. av.
2. *Flexor hall. long.*,	25·7 ,,	2.08 ,,
3. *Friction*,	24.6 ,,	
Total,	100.0	

In the Ostrich, as in the other Struthionidæ, the tendons of the *fl. hall. longus* and *fl. dig. longus* unite into a common tendon, at a point half way down the cannon bone.

2. *Rhea* (*long flexor tendons of toes*).

1. First toe,	22.1	per cent.
2. Second toe,	45.1	,,
3. Third toe,	32.8	,,
Total,	100.0	

		Weight.
1. *Flexor hall. long.*,	17.3 per cent.	0.75 oz. av.
2. *Flexor dig. long.*,	30.3 ,,	2.09 ,,
3. *Friction, &c.*,	52.4	
Total,	100.0	

3. *Jabiru.*—In this bird, which is celebrated for its power of standing on one leg for a long time, the tendons of the *fl. hall. long.* and *fl. dig. long.* do not unite until they reach the sole of the foot; the *fl. hall. long.* flexes the four toes, and the *fl. dig. long.* flexes the second, third, and fourth toes only.

Jabiru (*long flexors of toes*).

1. First toe,	17	per cent.
2. Second toe,	23	,,
3. Third toe,	37	,,
4. Fourth toe,	23	,,
Total,	100	

		Weight.
1. *Flexor hall. long.*,	12.3 per cent.	0.07 oz. av.
2. *Flexor dig. long.*,	30.9 ,,	0.18 ,,
3. *Friction, &c.*,	56.8 ,,	
Total,	100.0	

4. *Weka Weka Rail.*—In this bird, the two long flexor muscles have their tendons united into a single partially ossified tendon, which is about an inch in length, and subdivides into the four long flexors of the toes.

Weka Rail (*long flexors of toes*).

1. First toe, ...	7·7 per cent.
2. Second toe, ...	29·5 ,,
3. Third toe, ...	33·3 ,,
4. Fourth toe, ...	29·5 ,,
Total, . . .	100.0

		Weight.
Flexor hall. long., . .	33·3 per cent. .	0.06 oz. av.
Flexor dig. long., . .	19.2 ,, .	0.03 ,,
Friction, &c., . .	47·5 ,, .	
Total, . .	100.0	

5. *Silver Pheasant.*—In the pheasant the *fl. hall. long.* is distributed, by means of an ossified tendon, to the first toe, and by a tendinous cross band to the ossified tendon of the second, third, and fourth toes, which is pulled by the *fl. dig. longus;* and, with the exception of this cross band, the two tendons are quite distinct.

Silver Pheasant (*long flexors of toes*).

1. First toe, ...	15·7 per cent.
2. Second toe, ...	26.3 ,,
3. Third toe, ...	36.8 ,,
4. Fourth toe, ...	21.2 ,,
Total, . . .	100.0

		Weight.
Flexor dig. long., . .	26.3 per cent. .	0.06 oz. av.
Flexor hall. long., . .	26.3 „ .	0.02 „
Friction, &c., . . .	47.4	
Total, . . .	100.0	

6. *Black Swan.*—In this bird the tendons of the two long flexors unite into a common ossified tendon about three-fourths of an inch in length, which distributes tendons to the second, third, and fourth toes.

Black Swan (long flexors of toes).

1. First toe, ...	0 per cent.
2. Second toe, ...	23 „
3. Third toe, ...	42 „
4. Fourth toe, ...	35 „
Total, . . .	100

		Weight.
Flexor dig. long., . .	40 per cent .	0.06 oz. av.
Flexor hall. long., . .	24 „	0.07 „
Friction, &c., . . .	36 „	
Total, . . .	100	

Collecting together the preceding observations on the flexor tendons of birds, we find that the tendons distributed to the toes exceed in strength the tendons of the long flexors above the heel by the following quantities :—

TABLE XXI.—*Friction of Long Flexor Tendons of Birds.*

	Friction, &c.
1. Ostrich, . . .	24.6 per cent.
2. Rhea, ...	52.4 ,,
3. Jabiru, ...	56.8 ,,
4. Weka Rail, . . .	47.5 ,,
5. Silver Pheasant, .	47.4 ,,
6. Black Swan, . .	36.0 ,,

(*m.*) *Reptiles.*—The only reptile that I have had an opportunity of examining carefully is the Alligator of the Mississippi, from the dissection of which I have obtained the following results :—

Alligator Lucius (*long flexors of toes*).

1. First toe, ...	41.0 per cent.
2. Second toe, ...	29.5 ,,
3. Third toe, ...	18.0 ,,
4. Deficiency, ...	11.5 ,,
Total, . . .	100.0

		Weight.
Flexor hall. long., . .	29.5 per cent.	0.17 oz. av.
Flexor dig. long., . .	70.5 ,,	0.54 ,,
Total, . . .	100.0	

If we arrange the animals already described, according to the nature and amount of friction experienced by the tendons of the long flexors of their toes, we shall obtain the following results :—

TABLE XXII.—*Comparative View of Friction of the Long Flexors of Toes.*

$P > P'$; or Cross Section of Tendons of Toes greater than Cross Section of Tendons of Muscles.	Friction.
1. Pyrenean Mastiff (*Canis familiaris*), ...	65.4 per cent.
2. Lion (*Leo barbarus*), ...	59.0 ,,
3. Fox (*Vulpes vulgaris*), ...	57.6 ,,
4. Jabiru (*Mycteria*), ...	56.8 ,,
5. Rhea (*Rhea Americana*), ...	52.4 ,,
6. Jackall (*Canis aureus*), ...	49.2 ,,
7. Jaguar (*Leopardus onca*), ...	49.2 ,,
8. Weka Weka (New Zealand Rail), ...	47.5 ,,
9. Silver Pheasant (*Gallophasis nycthemerus*), .	47.4 ,,
10. Tiger (*Tigris regalis*), ...	46.0 ,,
11. Leopard (*Leopardus varius*), ...	45.5 ,,
12. Armadillo (*Dasypus sexcinctus*), ...	44.4 ,,
13. Sloth (*Bradypus tridactylus*), ...	42.5 ,,
14. Black Swan (*Cygnus atratus*), ...	36.0 ,,
15. Hare (*Lepus timidus*), ...	36.0 ,,
16. Wolf (*Canis lupus*), ...	34.0 ,,
17. Dingo (*Canis Dingo*), ...	33.8 ,,
18. Japanese Bear (*Ursus Japonicus*), ...	31.7 ,,
19. Virginian Bear (*Ursus Americanus*), ...	23.9 ,,
20. Llama* (*Llama yamma*), ...	25.9 ,,
21. Hedgehog (*Erinaceus Europæus*), ...	25.0 ,,
22. Ostrich (*Struthio camelus*), ...	24.6 ,,
23. Otter (*Lutra vulgaris*), ...	19.8 ,,
24. Man (mean of five subjects), ...	16.2 ,,
25. Spider Monkey† (*Ateles ater*), ...	12.3 ,,
26. Goat (*Hircus ægagrus*), ...	9.5 ,,
27. Rhinoceros (*Rhinoceros unicornis*), ...	9.0 ,,
28. Negro Monkey (*Lagothrix Humboldtii*), . . .	8.0 ,,
29. Ox (*Bos Indicus*), ...	6.8 ,,
30. Macaque (*Macacus nemestrinus*), ...	2.0 ,,
31. Boomer Kangaroo (*Macropus major*), . . .	0.0 ,,

$P > P'$; or Cross Section of Tendons of Toes less than Cross Section of Tendons of Muscles.	Friction.
31. Boomer Kangaroo (*Macropus major*), . . .	0.0 per cent.
32. Wallaby Kangaroo (*Halmaturus Ualabatus*), .	1.0 ,,
33. Alligator (*Alligator Mississippiensis*), . . .	11.5 ,,
34. Porcupine (*Hystrix cristata*), ...	20.0 ,,
35. Phalanger (*Phalangista vulpina*), ...	29.2 ,,

Notes on the foregoing Table.

* I found the following results from the dissection of a fine specimen of the common Llama:—

Llama (*long flexors of foot*).

1. First toe,	49.5 per cent.
2. Second toe,	50.5 „
Total,	100.0

1. *Flexor hall. long.*,	39.6 per cent.
2. *Flexor dig. long.*,	34.5 „
3. *Friction, &c.*,	25.9 „
Total,	100.0

In this animal the cross section of the tendons of the toes of the hind foot exceeds that of the tendons of the fore foot in the proportion of 444 to 411.

† The following are the details of the observations on the Spider Monkey:—

1. First toe,	17.5 per cent.
2. Second toe,	19.3 ,
3. Third toe,	24.7 „
4. Fourh toe,	22.7 „
5. Fifth toe,	15.8 „
Total,	100.0

Flexor hall. long.,	47.4 per cent.	0.24 oz. av.
Flexor dig. long.,	40.3 „	0.15 „
Friction, &c.,	12.3 „	
Total,	100.0	

The relative strengths of the tendons of the toes are very similar to those of the Negro Monkey (p. 95), and very unlike those of the Macaque (p. 94). The *fl. hall. long.* flexes the first, second, third, and fourth toes; while the *fl. dig. long.* flexes the first, second, third, fourth, and fifth. This arrangement resembles that of the flexor tendons of the Capuchin and Negro monkeys, in this respect, that the entire flexion of the fifth toe devolves on the *fl. dig. longus.*

The preceding analysis of the comparative strengths of the flexor tendons of the *fl. hall. long.* and *fl. dig. long.*, and of the toes, proves, that of thirty-five animals examined, the tendons of the toes exceed the tendons of the muscles in thirty instances, and fall short of them in four instances only. This is a result that is, *à priori*, probable; for the use of the foot is not, like that of the hand, to grasp an object by means of the flexion of the digits produced by the muscles of the forearm; but, the toes being applied to the ground as a fulcrum, the muscles of the calf acting on the flexor tendons of the toes (which are fixed), resist their own forcible extension by the weight of the body, by means of the tendons passing round the heel, and so act as a spring to prevent the foot reaching the ground with shock. In other words, in the foot, the force is applied from below, upwards, to the muscles through the intervention of the tendons; whereas, in a grasping organ, such as the hand, the force is applied from above, downwards, from the muscles to the obstacle, or resistance to be overcome.

The difference of strengths, or cross sections in all cases represents the friction overcome by the tendons; while the direction in which the force passes is always from the greater towards the lesser cross section of the tendons.

B. *Comparative Anatomy of the Flexor Tendons of the Hand.*

In the following observations on the comparative anatomy and mechanism of the flexor tendons of the fore foot, or hand, I shall confine myself to the deep flexor tendons of the fingers, as this method will furnish an instructive comparison with the corresponding long flexor tendons of the toes, already discussed. There is one remarkable difference to be noted between the long flexors of the toes and the deep flexors of the fingers; viz., that although both become simplified, as we descend in the scale of animals, into a common tendon giving off similar slips to each toe or finger; yet in the leg, the flexor muscles

do not undergo the same fusion as in the forearm, or as their tendons undergo. Thus, we always find the *fl. hall. long.* to be a muscle quite distinct from the *fl. dig. long.*, even when they act by a united tendon; and, on the contrary, in the forearm when the tendons of the *fl. poll. long.* and *fl. dig. prof.* become united, we generally observe these muscles to become united also. This difference in the muscles of the forearm and leg, I believe to be due to a mechanical necessity. In the propulsion of the body forwards by means of the hind foot, it is necessary to press the inner toe (*hallux*) outwards and backwards against the ground; a motion that is effected by the separate action of the *fl. hall. long.* muscle, and materially aided by the action of the *peronæus longus* muscle, which lifts upwards and inwards the outer side of the near end of the foot. Such an action of the fore foot is seldom necessary or desirable, and hence the differentiation of the *fl. poll. long.*, when it exists, is made to increase the grasping or hand-like action of the thumb, and not for the purpose of progression; and is, therefore, found to be less constant in its occurrence in various animals than the *flexor hallucis longus.*

(*a*). *Man.*—The differentiation of the thumb from the other fingers is carried farther in man than in any other animal, because in man the appropriation of the fore foot to the use of the brain is carried farther. The *fl. poll. long.* is quite distinct from the *fl. dig. prof.*, and in the latter muscle, the tendon of the index finger is usually distinct from the tendons of the third, fourth, and fifth fingers, far above the wrist. It is a curious fact that in the apes, that in other respects most nearly resemble man, the differentiation of the action of the thumb is most imperfect. Thus, in the Chimpanzee (*Troglodytes Aubryi*), Professor Gratiolet found that the thumb is bent by an oblique slip of the tendon of the muscle that bends the other fingers; it is, therefore, influenced by the common movements of flexion, and consequently not

free. This type, according to Gratiolet, is realized in the Gorilla as well as in the Chimpanzee : the small tendon which in these animals flexes the thumb is reduced to a tendinous thread, which exerts no action, for its origin is lost in the synovial folds of the tendons which bend the other fingers, and it abuts on no muscle. The thumb, therefore, in these apes is wonderfully enfeebled, and in none of them is there a trace of the large independent muscle (*fl. poll. long.*) which gives movement to the human thumb ; and, far from becoming more strongly developed, the thumb so characteristic of the human hand seems in the most elevated apes (*Orang*) to incline to a complete annihilation. These apes have, therefore, nothing in the organization of their hand which indicates a passage into the human form.

I have never had an opportunity of dissecting the hand of a Gorilla or of an Orang, but I have dissected the hand of the Chimpanzee (*Troglodytes niger*). In this animal I found the flexor tendon of the thumb to be formed by the union of two small thread-like tendons ; one of which, of silken lustre, was derived from the portion of the *fl. dig. sublimis* muscle corresponding to the index finger, and the other, equally slender, but wanting the silken lustre, proceeded from the tendon of the *fl. sublimis* of the little finger.

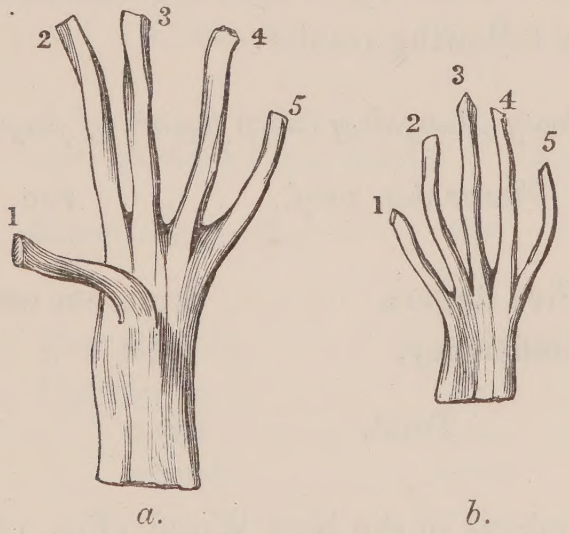
The peculiarities of the human *fl. poll. longus* have always been insisted upon by anatomists as essentially characteristic of Man, as distinguished from the Quadrumans ; but exceptions of the most startling kind are occasionally met with. In 1864, a male subject was brought into the dissecting-room of Trinity College, one of the hands of which was dissected by Mr. Finny, *Med. Schol.*, who made the following note at the time :—

"Abnormal *flexor pollicis longus*. The fleshy origin of this muscle from the bone of the forearm was entirely wanting, and the tendon of the *fl. poll. long.* was attached opposite to the

base of the third metacarpal bone, to the tendons of the *fl. dig. profundus*, on their superficial surface."

This remarkable arrangement of the flexor tendons of the hand is identical with that found in the Macaques, and other monkeys of the Old World, shown in Fig. 17(*a*). Whether this man was a Macaque passing upwards into a man, or a man passing downwards into a Macaque, must be decided by the reader, according as his sympathies are with Lamarck or Buffon.

(*b.*) *The Quadrumans.*—The peculiarities of the thumb in the higher apes have been already alluded to; in the lower monkeys a remarkable difference is found in the deep flexors of the fingers, between the monkeys of the Old and New Continents. This difference is shown in Fig. 17, in



a. *b.*

Fig. 17.

which (*a*) represents the distribution of the flexor tendons in the hand of the monkeys of the Old World, and (*b*) their distribution in the hand of the monkeys of the New World. If we take the Macaque (*Macacus Nemestrinus*) (Fig. 17, *a*), as the type of the Old World monkeys, we find the following arrangement. There is no distinct *fl. poll. long.* muscle, but a tendon branches off to the thumb, starting from the central

portion of the common tendon of the *fl. dig. prof.*, as shown in the drawing.

If we call the cross section of the common tendon 100, we find the cross sections of the thumb and four fingers, as follows :—

1. *Macaque* (*deep flexors of fingers*).

Flexor dig. profundus, . . .	100
1. Thumb, ...	19.3 per cent.
2. Other fingers, . . .	40.0 ,,
3. Deficiency, ...	40.7 ,,
Total, . . .	100.0

In the hand of the Sooty Mangabey (*Cercocebus fuliginosus*) I obtained the following results :—

2. *Sooty Mangabay* (*deep flexors of fingers*).

Flexor dig. prof., . . .	100
1. Five fingers, . . .	50.8 per cent.
2. Deficiency, . . .	49.2 ,,
Total, . . .	100.0

In the monkeys of the New World (Fig. 17 *b.*), a more degraded type of thumb is found to exist; for the flexor tendon of the thumb proceeds, not from the central and upper portion of the common flexor tendon of the fingers, but from the side of that tendon, as shown in the drawing; an arrangement which reduces the thumb, as in animals lower than monkeys, to a simple finger. In some of these monkeys also the thumb itself becomes rudimentary, and is not furnished with any flexor tendon whatever. The *Ateles*, or

Spider monkey, owes its scientific name to this circumstance.

The following observations show the relative cross sections of the flexor tendons of the hand in several of the New World monkeys.

3. *Spider Monkey* (*deep flexors of fingers*).

Flexor dig. prof., . .	100
1. Second finger, . .	18.4 per cent.
2. Third finger, . . .	20.4 „
3. Fourth finger, . . .	20.4 „
4. Fifth finger, . . .	14.3 „
5. Deficiency, ...	26.5 „
Total, .	100.0

4. *Negro Monkey* (*deep flexors of fingers*).

Flexor dig. prof., . .	100
1. Five fingers, . . .	72.6 per cent
2. Deficiency, ...	27.4 „
Total, . . .	100.0

5. *Capuchin Monkey* (*deep flexors of fingers*).

Flexor dig. prof., . .	100
1. Five fingers, . . .	64.7 per cent.
2. Deficiency, ...	35.3 „
Total, . . .	100.0

(*c*). *The Carnivores.* I have examined the deep flexor tendons of the following Carnivores:—

1. The Tiger.
2. The Wolf.
3. The Pyrenean Mastiff.
4. The Bengal Fox.
5. The Virginian and Japanese Bears.

1. *Tiger* (*deep flexors of fingers*).*

Flexor dig. prof., . .	100	
1. First finger, ...	23.4	per cent.
2. Second finger, . . .	15.5	,,
3 Third finger, . . .	12.7	,,
4. Fourth finger, . .	12.5	,,
5. Fifth finger, . . .	11.8	,,
6. Pad refractor, .	1.4	,,
7. Deficiency, ...	22.7	,,
Total, . . .	100.0	

The Tiger has no *fl. poll. long.* distinct from the *fl. dig. long.* muscle, and the thumb is therefore flexed by the same muscular effort as the other fingers; the strength of the tendon that flexes the thumb, as compared with that of the tendons of the other fingers, is remarkable, and shows the great importance of this digit in the grasping action of the animal's paw.

There are some circumstances in the history of this fine tiger, which throw light upon the details of muscular action, and which are, in themselves, sufficiently interesting to justify a slight digression. The tiger and tigress, brother and

* These measurements were taken from the *left* paw.

sister, arrived in the Dublin Zoological Gardens on the 22nd March, 1860, being then about two years old; they were a donation to the Gardens from Major-General Montgomery (Bombay Army), who had shot their mother and carried home the two cubs, and afterwards kept them in his bungalow for nearly one year. While under General Montgomery's care they had become playmates to his children, and it is a remarkable fact, that when visited by two of these children in the Dublin Gardens, after nearly two years' absence, both animals recognised their former friends, and expressed their satisfaction by loud purring and by licking the children's hands gently with the tips of their tongues, being well aware that the rough surface of the tongue would cause pain. The claw of the fourth finger of the right paw of the tiger grew into the finger pad of the foot early in 1866, and was removed by me in May of that year; afterwards the claws of the second, third, and fifth fingers grew into the finger pads, and were cut by me in January, 1867. On both occasions the tiger showed great anger at the pain that was caused by the operation, but shortly afterwards insisted on licking my hands, expressing his desire to do so by constant purring and by rolling himself upon his back, in the manner of a kitten at play.

It is worthy of remark that it required the united strength of eight men, to hold the tiger during these operations, although I have seen a similar operation performed easily upon a large African lion, with the help of five men.

This tiger died on the 20th February, 1869, of malignant putrid fever, after forty-eight hours' illness, and on dissecting his muscles, I paid particular attention to the relative developement of the deep flexors of the right and left paws. The only claw that had again grown into the pads of the foot was that of the third finger of the right foot.

Bengal Tiger (deep flexor muscles).

	Weight.
Right paw (overgrowth of claws), .	10.87 oz. av.
Left paw (uninjured), ...	13.30 ,,

In all the other muscles of the right and left forearm, there was a slight preponderance of the muscles of the left (uninjured) side over those of the right side, but it was not so marked in any as in the deep flexors of the fingers, and in the long palmar muscle, which were directly involved in the injury occasioned by the overgrowth of the claws. The absolute and relative weights of the muscles of the two forearms are shown in the following table :—

TABLE XXIII.—*Comparison of the Weights of the Muscles of the two Fore Paws of a Bengal Tiger, in which the Claws of the Right Foot had grown into the Finger Pads.*

Name of Muscle.	Left Paw.	Right Paw.	Ratio.
	Weight. oz. av.	Weight oz. av.	
1. Pron. rad. teres, . . .	3.51	3.35	96 per cent.
2. Fl. carp. rad., ...	2.12	2.06	97 ,,
3. Palm. long., ...	4.72	3.07	65 ,,
4. Fl. carp. uln., ...	5.57	5.45	98 ,,
5. Fl. dig. subl., ...	0.44	0.37	84 ,,
6. Fl. dig. prof., ... }	13.30	10.87	81 ,,
7. Fl. poll. long., ... }			
8. Pron. quadr., ...	1.99	1.78	89 ,,
9. Sup. rad. long., . . .	3.50	3.13	89 ,,
10. Ext. carp. rad. longr., .	4.20	3.90	93 ,,
11. Ext. carp. rad. brevr., .	3.30	2.87	87 ,,
12. Sup. rad. brev., . . .	1.11	1.08	98 ,,
13. Ex. dig. long., ...	2.60	2.35	90 ,,
14. Auricularis, ...	1.40	1.04	74 ,,
15. Ext. carp. uln., . . .	2.90	2.65	92 ,,
16. Ext. oss. met. poll., . .	2.80	2.28	82 ,,
17. Indicator, ...	0.60	0.58	97 ,,

It appears from the foregoing Table that the muscles that

suffered most from the injury produced by the ulceration of the foot pads, were—

Palmaris longus, . .	65 per cent.
Flexor dig. longus, . .	81 ,,
Ext. oss. met. poll., . .	82 ,,

2. In the European Wolf, I obtained the following results :—

Wolf (*deep flexors of fingers*).

Flexor dig. prof., . .	100
1. First finger, . . .	7.0 per cent.
2. Second finger, . . .	19.1 ,,
3. Third finger, . . .	12.8 ,,
4. Fourth finger, . . .	12.7 ,,
5. Fifth finger, . . .	17.0 ,,
6. Deficiency, ...	31.4 ,,
	100.0

3. In the Pyrenean Mastiff, the following results were found :—

Pyrenean Mastiff (*deep flexors of fingers*).

Flexor dig. prof., . .	100
1. First finger, ...	8 per cent.
2. Second finger, . . .	19 ,,
3. Third finger, ...	24 ,,
4. Fourth finger, . . .	24 ,,
5. Fifth finger, ...	18 ,,
6. Deficiency, ...	7 ,,
Total, . . .	100

The *flex. poll. long.* muscle is not distinct from the *fl. dig. prof.* in either the Wolf or Dog, while in the Fox it constitutes a small but distinct muscle, one-tenth of the weight of the deep flexor, having a tendon of its own, that joins the common tendon of the deep flexor at the wrist. This arrangement of tendons in the paw of the Fox is shown in Fig. 18, and constitutes an additional distinction, separating the Fox from the Dog and Wolf.

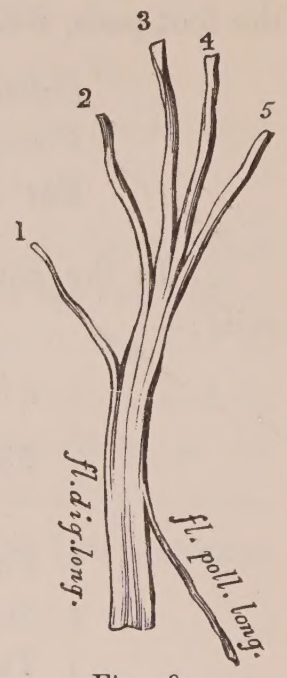


Fig. 18.

4. The Fox examined by me was a Bengal Fox, which gave the following measurements:—

Bengal Fox (*deep flexors of fingers*).

		Weight.
Flexor dig. prof., . .	97.5 per cent.	. 0.15 oz. av.
Flexor poll. long., . .	2.5 ,,	. 0.015 ,,
Total, . . .	100.0	

1. First finger, ...	2.5 per cent.
2. Second finger, . . .	20.7 ,,
3. Third finger, . . .	19.5 ,,
4. Fourth finger, . . .	18.3 ,,
5. Fifth finger, . . .	18.3 ,,
6. Deficiency, ...	20.7 ,,
Total, . .	100.0

The tendon of the *fl. poll. long.* acts upon the common deep flexor tendon in the direction of the prolongation of the

tendon of the thumb, and must exert, by its oblique pull, a distinct action upon that digit.*

5. In the Virginian and Japanese Bears I found the following results :—

Virginian Bear (deep flexors of fingers).

Flexor dig. prof., ...	100
1. First finger, ...	9.1 per cent.
2. Second finger, ...	15.9 ,,
3. Third finger, ...	16.9 ,,
4. Fourth finger, ...	15.3 ,,
6. Fifth finger, ...	7.8 ,,
7. Deficiency,.	35.0 ,,
Total, . . .	100.0

Japanese Bear (deep flexors of fingers).

Flexor dig. prof., . .	100
1. Five fingers, ...	69.4 per cent.
2. Deficiency, ...	30.6 ,,
Total, . .	100.0

(*d*). *The Ungulates.*—The Ungulates examined by me, in reference to the relative strengths of the tendons of the deep flexors of the fingers above and below the wrist, were the Goat and the Llama. I examined the wrist tendons of the Rhinoceros also, and found them to resemble those of the foot already described, both as to arrangement and relative cross sections.

* It has been suggested to me by Dr. Templeton, that the *fl. poll. long.* enables the Fox to climb, an action of which Dogs and Wolves are incapable.

1. In the Bombay Goat I found the following results:—

Flexor dig. profundus, .	100

1. First finger, . . .	49.2	per cent.
2. Second finger, . . .	50.8	,,
Total, . . .	100.0	

2. In the Llama, the following cross sections were obtained:—

Flexor dig. prof., .	85.6	per cent.
Friction, &c., . .	14.4	,,
Total, . .	100.0	

1. First finger, . . .	49.1	per cent.
2. Second finger, . .	50.9	,,
Total, . . .	100.0	

In the hands of the animals previously described, the cross section of the deep tendons of the fingers was less than that of the common tendon above the wrist; which is the essential characteristic of a hand as distinguished from a foot. In these Ungulates, however, we find that in the Goat, the cross sections become equal, and that in the Llama, the cross section of the finger flexors becomes greater than that of the common tendon; thus reducing the fore-foot of this animal to the condition of a true foot, and degrading it from the dignity of a paw, or hand.

(*e*). *The Rodents.*—The only rodent, of which I have

examined the flexor tendons of the hand, is the Porcupine, which gave me the following results :—

Porcupine (deep flexors of fingers).

Flexor dig. prof., . . .	100

1. Second finger, ...	7.7	per cent.
2. Third finger, ...	7.6	,,
3. Fourth finger, ...	7.1	,,
4. Fifth finger, ...	6.6	,,
5. Deficiency, ...	71.0	,,
Total, . . .	100.0	

In the Porcupine, no slip from the deep flexor tendon is given off to the thumb, and there is no *fl. poll. long.* muscle. It will be remembered (p. 110), that the foot of the porcupine resembles a hand, in having the cross section of the flexor tendons greater above the heel than the cross sections of the tendons of the toes ; and it is remarkable, that in the hand of the same animal, the difference between the similar tendons is greater (71 per cent.) than in any other hand examined by me.

(*f*). *The Marsupials.*—In the hand of the Wallaby Kangaroo, I found the following cross sections :—

Five fingers, ...	100

1. *Flexor dig. prof.*, . . .	98.3	per cent.
2. *Friction, &c.*, ...	1.7	,,
Total, . . .	100.0	

If we now collect into one Table, the observations made upon the flexors of the hand, we shall find the difference between these and the flexors of the foot to be very remarkable (*vide* Table XXII.)

TABLE XXIV.—*Comparative View of the Friction of the Deep Flexors of Fingers.*

(A.) $P > P'$; or Cross Section of Tendons of Fingers greater than Cross Section of Tendons of Muscles.	Friction.
1. Llama (*Llama yamma*), ...	14.4 per cent.
2. Wallaby Kangaroo (*Halmaturus Ualabatus*), .	1.7 ,,
3. Goat (*Hircus ægagrus*), ...	0.0 ,,

(B,) $P < P'$; or Cross Section of Tendons of Fingers less than Cross Section of Tendons of Muscles.	Friction.
3. Goat (*Hircus ægagrus*), ...	0.0 per cent.
4. Mastiff (*Canis familiaris*), ...	7.0 ,,
5. Fox (*Vulpes vulgaris*), ...	20.7 ,,
6. Tiger (*Tigris regalis*), ...	22.7 ,,
7. Spider Monkey (*Ateles ater*), ...	26.5 ,,
8. Negro Monkey (*Lagothrix Humboldtii*), . .	27.4 ,,
9. Japanese Bear (*Ursus Japonicus*), ...	30.6 ,,
10. Wolf (*Canis lupus*), ...	31.4 ,,
11. Virginian Bear (*Ursus Americanus*), . . .	35.0 ,,
12. Capuchin Monkey (*Cebus Capucinus*), . . .	35.3 ,,
13. Macaque (*Macacus Nemestrinus*), ...	40.7 ,,
14. Sooty Mangabey (*Cercocebus fuliginosus*), . .	49.2 ,,
15. Porcupine (*Hystrix cristata*), ...	71.0 ,,

The preceding Table demonstrates the fact, that in the hand (or fore foot) of most animals, the cross section of the tendon of the deep flexor, at the wrist, is greater than the united cross sections of its branches subdivided to the various fingers. In fact, the only animal, out of fifteen examined, in which the reverse condition takes place, is the Llama, whose fore foot, in its structure and mode of action, may be regarded as being more truly a foot than a hand.

The following considerations will serve to show more clearly, the important mechanical differences involved in the Tables of cross sections of the flexors of the hand and foot.

The mechanism of a true hand, or grasping organ, may be thus illustrated.

Let OA, AB, and BC, represent three bars turning upon hinges at A and B, and acted upon by a muscle Om, attached

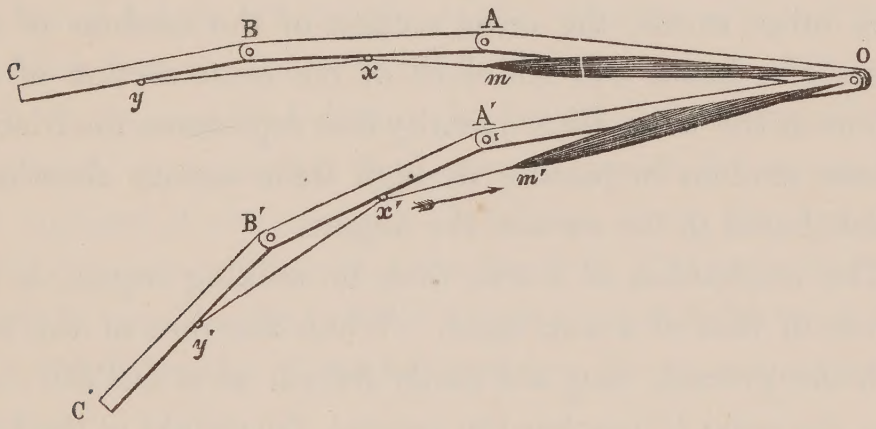


Fig. 19.

to a tendon mxy, this tendon being inserted at y, and passing through a loop or pulley at x. When the muscle Om contracts, the system of bars will take the position $OA'B'C'$, which is more flexed than the position $OABC$, or in other words, the angles at A' and B' are smaller than the angles at A and B. In this movement of flexure, the muscle Om is shortened, while the tendon mxy remains equal in length to the tendon $m'x'y'$; but since the angle at B' is smaller than the angle at B, the base of the triangle, $y'x'$, is shorter than the base yx, and therefore the tendon is drawn through the loop x' in the direction of the arrow, towards the point O; hence the friction of the loop x', which is opposite to the direction of the motion, assists the part of the tendon $y'x'$, which is situated below the loop. If therefore the force of the muscle be denoted by P', and the friction of the loop by F; we shall

have for the strength of the tendon $y'x'$, the equation (*vide* p. 83).

$$P' = F + P. \qquad (15)$$

The tendon $x'm'$ above the loop must be made sufficiently strong to resist the force of the muscle P'; while the tendon $y'x'$ below the loop need only have a strength sufficient to resist the force

$$P = P' - F;$$

or, in other words, the cross section of the tendons of the fingers of a hand, will fall short of the cross section of the tendons at the wrist, by a quantity that represents the friction of these tendons in passing through their various sheaths to be distributed to the ends of the fingers.

The mechanism of a true foot, or walking organ, is the reverse of that of a true hand. When the toes of the foot reach the ground, they are partly flexed, as at $OA'B'C'$, and when the point C' touches the ground, the weight of the body unbends the toes, changing them from the position $OA'B'C'$ to the position $OABC$. The muscle Om' resists the unbending of the toes, and is forcibly extended to the length Om, acting thus as a sort of spring, and preventing the sudden shock that would otherwise be given to the body each time that the foot reaches the ground. Hence it is evident, for the reasons already given, that the tendon $y'x'm'$ (which is inextensible), must be drawn through the loop x, in a direction opposite to that of the arrow, and that the friction of the loop acts towards O, helping the portion of the tendon xm situated above the loop; hence in this case we have

$$P' + F = P; \qquad (15)$$

from which it appears that the cross section of the tendons of the toes must exceed the cross section of the flexor tendons at the heel, by an amount (F) representing the friction of the sheaths.

7. *On the Mechanical Work done by the Human Heart.*—The heart constitutes so peculiar and so important a muscle, that it is worthy of a distinct consideration. It contracts constantly by means of a self-acting nerve supply, and works day and night while life lasts, without ever experiencing a sense of fatigue, or the necessity for rest. Let us examine what is the mechanical equivalent of the work done by an average human heart during the course of a single day and night:—

Let Q denote the weight of the quantity of blood that is expelled from the left ventricle into the aorta at each stroke of the heart, and let h denote the height of a column of blood that would measure the fluid pressure inside the ventricle at the moment of contraction. It follows, from the hydraulic laws of spouting fluids, that the blood Q, if thrown vertically upwards, would ascend to the height h, so that the work done in a single stroke of the left ventricle is represented by the product

$$Q \times h = \text{Work done.}$$

We have now to estimate the values of the quantities Q and h:—

The estimates given by various anatomists of the capacity of the left ventricle vary from 2 to 5 ounces (water), the former estimate being that of the empty ventricle, which admits of considerable distention, especially during life. It is extremely difficult, if not impossible, to say what is the amount of distention experienced by the left ventricle before it contracts upon its contents, and expels them. From some experiments that I have made, it seems to me probable that the distention does not amount to more than fifty per cent., and that we shall not be far from the truth if we take Q to be 3 ounces of blood, neglecting the difference of specific gravity between blood and water.

Professor Donders* assumes the capacity of the left ventricle to be 188 grammes, which are equivalent to 6.631 oz. av., a result which seems to me more like the capacity of both ventricles, than that of one only.

In order to find the quantity h, I shall employ the experiments of the Rev. Dr. Hales.† Having opened a large artery, such as the crural or carotid, in several animals, Dr. Hales inserted a brass or glass tube into the opening, and observed the height at which the blood stood before the animal had lost any large quantity of blood. The following are some of his results :—

No.	Artery.	Height in Tube.
1. Mare, ...	Left Crural, . .	8.25 feet.
2. Gelding, ...	,, ,, .	9.66 ,,
3. Mare, ...	Left Carotid, . .	9.50 ,,
4. Wether, ...	Carotid, . . .	6.46 ,,
5. Fallow Doe, . .	Left Crural, . .	4.17 ,,
6. Dog, ...	Crural, . . .	6.66 ,,

An important fact was ascertained by Dr. Hales during the course of these experiments, viz., that when some of the blood was allowed to flow away, the force of the heart's contraction diminished, no doubt in consequence of the diminished resistance offered by the diminished circulation; the heart, as it were, instinctively measuring the work to be done, and expending upon it exactly the force required. The height of blood in the tube, which measured the force of the heart's contraction, became less and less as the blood was allowed to flow off, until it reached a minimum height, when the animal expired. This minimum, carefully measured in the case of the horses, was found to be—

* Physiologie des Menschen (Leipzig, 1859), p. 109.

† Statical Essays (London, 1769), vol. ii.

No.	Maximum height before loss of blood.	Minimum Height.	Total Blood lost.
1. Mare, . .	8.25 feet.	2.33 feet.	924 cub. in.
2. Gelding,. .	9.66 ,,	2.83 ,,	895 ,,
3. Mare, . .	9.50 ,,	2.42 ,,	833 ,,
Mean, . .	**9.14** feet.	**2.53** feet.	**884** cub. in.

It appears from the foregoing table that the minimum contraction of the horse's heart, corresponding to the resistance offered by a circulation almost drained of its blood, amounts to the pressure of a column of blood 2.53 feet in height. Dr. Hales has also recorded the important fact, that the crural or carotid artery of the horse, when first cut and allowed to spout freely, does not throw the blood much higher than 2 feet. It would appear from this observation that the cutting and free exposure of a cut artery to the air relieves the heart at once of the resistance offered by the capillary circulation, for otherwise it is difficult to understand why the blood should not spout upwards through 9 feet instead of somewhat over 2 feet.

Experiments on the pressure of the blood inside the human heart, similar to those of Dr. Hales on animals, have not yet been made; but it is estimated at 7.5 feet by Dr. Hales from considerations founded on the comparative bulks of the soft parts of man and of the horse supplied by the arterial circulation.

The following observation and calculation of the velocity of blood spouting from a large artery in man, lead, as I believe, to a more correct estimate, although indirect, of the pressure of blood inside the left ventricle.

On the 18th of March, 1863, a large fibro-cellular tumour was removed by Mr. M. H. Colles, in the operating theatre of the Meath Hospital, from the left groin of a middle-aged,

large-sized man; in the course of the operation the external epigastric artery, which appeared enlarged to feed the tumour, was divided, and before it could be ligatured, strong jets of blood from it were thrown in various directions about the room. I noticed, as the poor fellow moved about on the operating table, that the jets of blood fell short, or enjoyed a long range, according to the angle of elevation of the orifice of the bleeding artery, and that there was a certain maximum range on the floor of the theatre, which was not exceeded.

I saw immediately that I had before my eyes the solution of the problem that had puzzled me; as by measuring the co-ordinates of the maximum range, I could calculate the velocity with which the blood left the artery.

In the annexed figure, which is drawn to scale, let *A* represent the orifice of the artery, and *B* the position of the far-

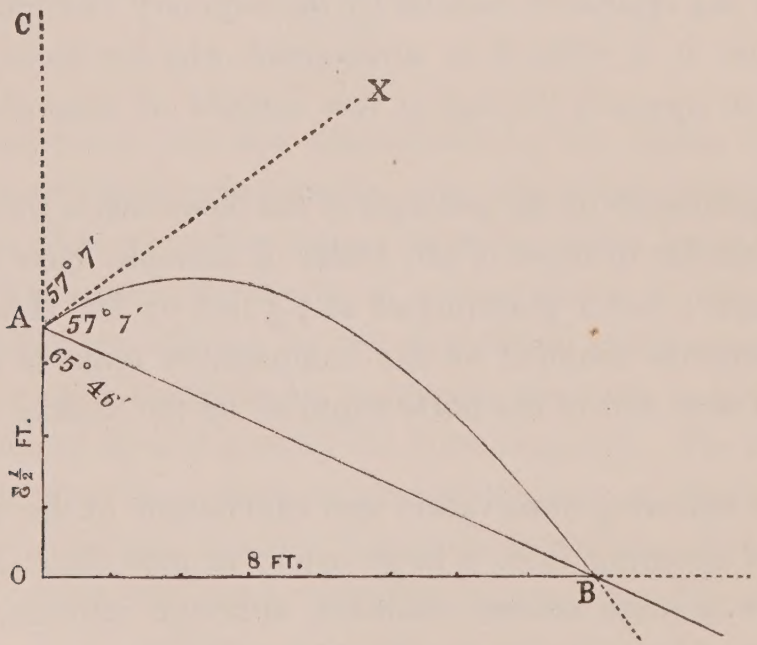


Fig. 20.

thest squirt of blood on the floor; *OA* was found to be 3 ft. 6 in., and *OB* was 8 ft. It is well known that *AX*, the direc-

tion of projection of the maximum range on the plane AB, bisects the angle CAB, and that

$$h = \frac{R \cos^2 i}{4 \cos^2 e}; \qquad (16)$$

where

h = the unknown height due to the velocity of projection;
R = the maximum range AB = 8.773 ft.
i = the angle made by AB with the horizon = 24° 14′;
e = the angle of elevation, made by AX with the horizon = 32° 53′.

Substituting the preceding values in equation (16), I find

$$h = 2.586 \text{ ft.};$$

and since the velocity of projection, v, is given by the equation,

$$v = \sqrt{2gh}; \qquad (17)$$

it follows that

$$v = 12.905 \text{ ft.}$$

The result here found shows that the heart of man, when experiencing the minimum of resistance, such as that offered by the circulation when a large artery is divided, or the vessels nearly empty of blood, contracts with a force measured by 2.58 ft. of a vertical column of blood. The heart of the horse, under similar circumstances, contracts with a force measured by a column 2.53 ft. in height.

The following considerations lead us to the conclusion that it is probable that the maximum hydrostatical force of man's heart is nearly the same as that of the horse. The experiments of Dr. Hales, fully corroborated by the recent observations of Poiseuille, show that the hydrostatical pressure of the blood is the same in all the arteries of the same animal, that have a sensible diameter; from which fact, it is plain that the resistance to the circulation of the blood takes place in the capillary arteries and veins. If therefore we knew the relative

resistances offered in man and in the horse, to the capillary circulation, we could estimate the relative forces with which the blood is propelled in the circulation of these animals.

The experiments of Poiseuille on the discharge of liquids through capillary tubes, prove that the resistance offered by such tubes is directly proportional to the lengths of the tubes, and inversely proportional to the squares of their cross sections.

The quantity of liquid discharged by a capillary tube in a given time is inversely proportional to the resistance, and may be expressed in general by the following formula—

$$Q = A \times \frac{hd^4}{l}. \qquad (18)$$

In this expression, Q denotes the quantity of liquid discharged in a given time; A is a constant; h denotes the charge, or hydrostatical pressure of the fluid; and d and l are the diameter and length of the capillary tube.

There is reason to believe that in animals similar in bulk, the arrangement and structure of the capillaries are such, that the ratio of the squares of their cross sections to the total lengths of the capillaries, is practically constant; as may be shown by a comparison of the sheep and the dog;—the data concerning which animals I take from Dr. Hales' observations and experiments. The left ventricle of the heart of the sheep contains 1.85 cubic inches, and its pulse beats 65 times in a minute; the quantity of blood passing through the capillaries in a given time being proportional to the product of these two quantities. The hæmastatical pressure of the blood in the large arteries was found by Dr. Hales to be 6.46 ft. of blood.

Equation (18) gives us, for the coefficient depending on the capillary resistance,

$$A \times \frac{d^4}{l} = \frac{Q}{h} = \frac{1.85 \times 65}{6.46} = 18.6.$$

The average of the capacities of the left ventricles of the hearts of six dogs examined by Dr. Hales is 0.954 cub. in.; and the average statical pressure in the arteries of sixteen dogs experimented on by him is 4.75 ft. of blood; and the average pulse of the dog is found to be 97 beats in a minute. Hence equation (18) will give us, for the coefficient depending on the capillary resistance in the dog,

$$A \times \frac{d^4}{l} = \frac{Q}{h} = \frac{0.954 \times 97}{4.71} = 19.6.$$

These results, depending solely on the resistance offered by the capillary vessels to the flow of blood, show by their near approach to equality, that in animals, like the dog and sheep, we may safely assume the resistance offered by the structure of the capillaries to be nearly the same.

The sheep and dog differ from each other considerably in size of heart, rate of pulse, and hæmastatical pressure of blood; yet, notwithstanding these differences, the *capillary coefficient* depending on them all comes out to be nearly the same in both animals. The capillary coefficient, found by experiments made on the horse, comes out to be double that of the dog and sheep, showing that the resistance to the circulation in the horse is only half that of the smaller animals.

The left ventricle of the horse contains 10 cub. in., and the rate of pulse is 36 beats in a minute, while the average hæmastatical pressure of the blood in the arteries is 9.14 ft. of blood; hence, we obtain for the capillary coefficient in the horse,

$$A \times \frac{d^4}{l} = \frac{Q}{h} = \frac{10 \times 36}{9.14} = 39.3.$$

This quantity is double that found for the dog, and therefore the capillary resistance is one-half.

Dr. Hales did not make any observations on the hæma-

statical force of the arterial blood in the ox, but there is reason to believe that it is rather greater than in the horse, and nearer to 10 ft. than 9 ft. If we are at liberty to assume it to be 9.66 ft., which is the greatest height measured in the horse, we can calculate the capillary coefficient as follows:—In the ox, the capacity of the left ventricle was found by Dr. Hales to be 12.5 cub. in., and the pulse beats 38 times in a minute. Hence the capillary coefficient in the ox is—

$$A \times \frac{d^4}{l} = \frac{Q}{h} = \frac{12{\cdot}5 \times 38}{9.66} = 39.8.$$

This coefficient is nearly the same as that of the horse, an agreement which serves to show that the capillary resistance depends more on the bulk of the animal than on any other condition.

If we suppose the coefficient of capillary resistance to be the same in man as in the horse, we can calculate the hæmastatical pressure of the blood in the human arteries, as follows:—The human heart has a capacity in its left ventricle, when in action, of 3 ounces, or 5.2 cub. in.; and beats 75 times in a minute. Solving equation (18) for h, we have

$$h = \frac{Q}{A \times \frac{d^4}{l}}.$$

Substituting for Q the product of the capacity of the ventricle and rate of pulse, and for the capillary resistance, its value found for the horse we obtain

$$h = \frac{5{\cdot}2 \times 75}{39{\cdot}3} = 9.923 \text{ feet of blood.}$$

This is the value that I shall assume for the arterial pressure in man. This estimate of the blood pressure approaches to that of Donders,* which is 10.527 ft. (3.21 metres).

* Physiologie, p. 109.

We are now in a condition to calculate in foot tons, the daily work done by the left ventricle of the human heart, which lifts at each stroke 3 ounces through a height of 9.923 ft. Its value is the following:—

Daily work of left ventricle

$$= \frac{3 \times 9.923 \times 75 \times 60 \times 24}{16 \times 2240} = 89.706 \text{ ft. tons.}$$

I find, on taking an average of the estimates of Bouillaud, Laennec, Cruveilhier, Sömmering, and Andral, that the right ventricle has a thickness, which is to that of the left in the proportion of 5 to 13; and as the cavities are of equal dimensions, the thicknesses are proportional to their muscular powers; in order, therefore, to find the total work done by both ventricles of the heart, we must add to the preceding result $\frac{5}{13}$ths of its amount, from which we obtain finally,

Daily work of left ventricle, ...	89.706 ft. tons.
Daily work of right ventricle, ...	34.502 „
Total daily work of both ventricles,	124.208 ft. tons.

The total work done by the heart is somewhat greater than this estimate, for the auricles of the heart do some work, in propelling the blood into the ventricles; of which work no account has been taken in the preceding calculation.

The average weight of the human heart is found from the following Table:—

TABLE XXV.—*Table of Average Weight of Human Heart.*

	Ounces.
1. Meckel, ...	10.0
2. Cruveilhier, ...	7.5
3. Bouillaud, ...	8.375
4. Lobstein, ...	9.5
5. Boyd [æt. 30–40], . .	10.41
6. Boyd [æt. 40–50], . .	10.56
Mean =	**9·39** oz.

From the weight of the heart and the work done by it in one day, we can calculate the work done by the heart per ounce, as follows:—

Work done by the human heart in foot lbs. per oz. per minute,

$$= \frac{124.208 \times 2240}{9.39 \times 24 \times 60} = 20.576.$$

On comparing this result with the work done by other muscles, given in Tables VII. and VIII. (p. 62), we observe that the work done by the heart in a given time (20.576) exceeds considerably the work done by the muscles in a boat race (15.17). In a boat race, the labour can be maintained for a few minutes only; while the heart labours at a greater rate throughout the whole of life.

There is another mode of exhibiting the wonderful energy of the heart which is worth stating, more especially as it affords a means of comparing my own result with that of Professor Helmholtz, the data of whose calculation I am not acquainted with. Let us suppose that the heart expends its entire force in lifting its own weight vertically, then the height through which it could lift itself in one hour is thus found, by reducing the daily work done in ft. tons to the hourly work in foot ounces, and dividing by the weight of the heart in ounces.

Height through which the human heart could raise its own weight in one hour,

$$= \frac{124.208 \times 2240 \times 16}{24 \times 9.39} = 19754 \text{ ft.}$$

Professor Helmholtz* states as the result of his own calculations, that the heart could raise its own weight through 20250 ft. in one hour. It has been already stated (p. 52), that an active climber can ascend 9000 ft. in nine hours,

* "Medical Times and Gazette," May 14, 1864, p. 529.

which is only at the rate of 1000 ft. per hour, or one-twentieth part of the energy of the heart.

When the railway was constructed from Trieste to Vienna, a prize was offered for the locomotive Alp-engine, that could lift its own weight through the greatest height in one hour. The prize locomotive was the "Bavaria," which lifted herself through 2700 ft. in one hour; the greatest feat as yet accomplished on steep gradients. This result, remarkable as it is, is only one-eighth part of the energy of the human heart.

Professor Donders* gives an estimate of the force of the heart, which exceeds considerably the result found by Professor Helmholtz, and by myself. He assumes the contents of the left ventricle to be 0.188 kil., or 188 cubic centimeters, or 11.473 cubic inches, a result intermediate between the capacities of the hearts of the ox and horse, measured by Dr. Hales. Measured in ounces, this capacity is 6.631 ounces, or more than twice the capacity assumed by me. The blood pressure is assumed by Donders at 3.21 meters, or 10.527 ft.; and the force of the right ventricle to be one-third of that of the left ventricle. The other elements of his calculation are the same as my own. From these data, the daily labouring force of the heart comes out equal to

87,400 kilogrammeters = 281.03 ft. tons.

This result, as might be anticipated from the assumed size of the ventricle, is more than double that found by myself and Helmholtz, from independent data.

It may be useful to the reader to state here the results of observers, subsequent to Dr. Hales, as to the pressure of the blood in the large arteries of several animals. Dr. Hales observed the maximum pressure in the arteries, as measured by

* Physiologie, p. 109.

a vertical column of blood, which was noticed to rise and fall. Subsequent observers have shown that in the anterior extremity, the blood pressure depends on the respiration, being greatest during expiration, and least during inspiration; and that this inequality is less sensible in the arteries distant from the heart. In order to compare the observations, it is desirable to reduce Dr. Hales' column of blood to millimeters of mercury. The weight of a cubic inch of blood is 267.7 grs., while that of a cubic inch of water is 252.5 grs. The specific gravity of mercury is 13.596; and 9.14 feet (Dr. Hales' result for the horse) are equal to 2785 millimeters. Hence we find

Maximum mean pressure of blood in the horse, according to Hales,

$$= \frac{2785 \times 267.7}{13.596 \times 252.5} = 217.17 \text{ mm. of mercury.}$$

Poiseuille* directed his attention especially to the determination of the mean pressure of the blood, independent of expiration or inspiration; but has left in his memoir (p. 295), the data for calculating, in the case of the horse, the maximum pressure corresponding to that observed by Dr. Hales. From the experiment there recorded, on the carotid artery of a horse, I have calculated the maximum mean pressure of the blood during expiration, and find

Maximum mean pressure in the carotid artery of the horse, according to Poiseuille,

$$= 229.75 \text{ mm. of mercury.}$$

The minimum mean pressure, during inspiration, in the same artery, may be calculated from the data given, at 63.61 mm. of mercury. The mean of all the maxima and minima being 146.68 mm. of mercury. The maximum pressure found by Poiseuille is a remarkable confirmation of the accuracy of the experiments made by Dr. Hales.

* "Journal de Physiologie par Magendie," tom. viii., p. 272.

Poiseuille (p. 301) gives the following as the mean of all the pressures found in the horse and dog :—

No. 1.	Horse.	Dog.
1,	146.68 mm.	148.88 mm.
2,	147.00 „	147.36 „
3,	157.25 „	141.45 „
4,	154.33 „	157.39 „
5,	182.05 „	145.75 „
6,	———	166.60 „
7,	———	179.04 „
8,	———	141.40 „
9,	———	171.14 „
Mean,	**157·46** mm.	**155·44** mm.

Spengler* repeated the experiments of Poiseuille, and obtained, among others, the following results from the arteries of the horse. The mean maximum pressures during expiration were,

1. First horse, . . . 229.31 mm. of mercury.
2. Second horse, . . 200.67 „
3. Third horse, . . . 171.34 „
4. Fourth horse, . . . 170.00 „
5. Fifth horse, . . . 220.00 „
6. Sixth horse, . . . 230.00 „

Mean maximum, . **203·55** mm. of mercury.

This result agrees well with those of Hales and Poiseuille.

* "Archiv für Anatomie," &c., von Müller (1844), p. 49.

The mean of all the pressures, including maxima and minima, were found by Spengler (pp. 58, 9), to be,

No.	Horse.	Dog.	Goat.
1,	170 mm.	185 mm.	140 mm.
2,	175 ,,	135 ,,	—
3,	160 ,,	150 ,,	—
4,	165 ,,	165 ,,	—
Mean,	**167·5** mm.	**158·75** mm.	**140** mm.

The rate at which the blood circulates through the body may be estimated from the size of the left ventricle, as compared with the total quantity of blood in the body. The total quantity of blood in the body has been found by weighing, before and after death, criminals executed by decapitation. From such observations made by Lehmann, Weber, and Bischoff, the total quantity was found to range from 9 lbs. to 10 bs. If we assume the latter quantity, and take the capacity of the left ventricle at 3 ounces, it is plain that a quantity of blood equal to that contained in all the vessels, will have passed through the heart at the end of 53 strokes, which, if the pulse be assumed at 75 to the minute, will correspond to 42 seconds. This represents the time occupied by the entire circulation of the blood. The time occupied by absorption, circulation, and secretion, combined, may be set down at less than 4 minutes, as appears from the following interesting experiment, devised by Professor Macnamara, of the Royal College of Surgeons in Ireland, at which I had an opportunity of assisting. An old soldier, a patient of the Meath Hospital, suffering from hydrocele, was trained by us to urinate at the word of command, at intervals of one minute, into a succession of test tubes. When his training had been completed by

a few days' instruction, the sac of the hydrocele was emptied and injected with two drachms of tincture of iodine. The patient then urinated at intervals of one minute into the test tubes previously prepared, commencing at one minute after the injection. The fourth test tube responded faintly to the starch test for iodine; and the fifth and subsequent tubes showed its presence very distinctly. From this experiment it is evident that the entire process, *viz.*, absorption by the tunica vaginalis; transmission by the lymphatics and capillary veins to the heart; retransmission by the renal arteries to the kidneys; secretion of urine, and its transmission to the bladder by the ureters—occupied less than four minutes; or about five times the entire time required for the circulation of the blood.

8. **On the Muscular Forces employed in Parturition.**—In the first stage of natural labour, the involuntary muscles of the uterus contract upon the fluid contents of this organ, and possess sufficient force to dilate the mouth of the womb, and generally to rupture the membranes. I shall endeavour to show, from the principles of muscular action already laid down, that the uterine muscles are sufficient, and not much more than sufficient to complete the first stage of labour, and that they do not possess an amount of force adequate to rupture, in any case, the uterine wall itself. In the second stage of labour, the irritation of the fœtal head upon the wall of the vagina provokes the reflex action of the voluntary abdominal muscles, which aid powerfully the uterine muscles to complete the second stage, by expelling the fœtus. The amount of available additional force given out by the abdominal muscles admits of calculation, and will be found much greater than the force produced by the involuntary contractions of the womb itself.

The mechanical problem to be solved for both cases is one

of much interest and some difficulty, as it is the celebrated problem of the equilibrium of a flexible membrane subjected to the action of given forces. It has been solved by Lagrange (*Mecanique Analytique*, p. 147), in all its generality. In the most general case of the problem, the following beautiful theorem can be demonstrated:—Let T denote the tensile strain acting in the tangential plane of the membrane, applied to rupture a band of the membrane one inch broad; let P denote the pressure resulting from all the forces in action, perpendicular to the surface of the membrane, and acting on a surface of one square inch; and let ρ_1 and ρ_2 denote the two radii of principal curvature of the membrane, at the point considered. Then we have the following equation:—

$$P = T \times \left(\frac{1}{\rho_1} + \frac{1}{\rho_2} \right) \tag{19}$$

If the surface, or a portion of it, become spherical, the two principal curvatures become equal, and equation (19) becomes

$$P = \frac{2T}{\rho} \tag{20}$$

In the case of the uterus and its membranes, the force P arises from hydrostatical pressure only, and is therefore easily measured, and the supposition of spherical curvature is approximately admissible.

The natural position of the gravid uterus is shown in Fig. 21, in which OP is the axis of the uterus, and AB a vertical line drawn through G, the centre of gravity of the fœtus; this vertical line must pass through F, the centre of floatation of the fœtus, or centre of gravity of the liquor amnii displaced by the fœtus.

In the first stage of labour, the contraction of the muscular walls of the uterus compresses its liquid contents, and the

membranes are pushed through the mouth of the womb—bulging out into the segment of a sphere, and are in most

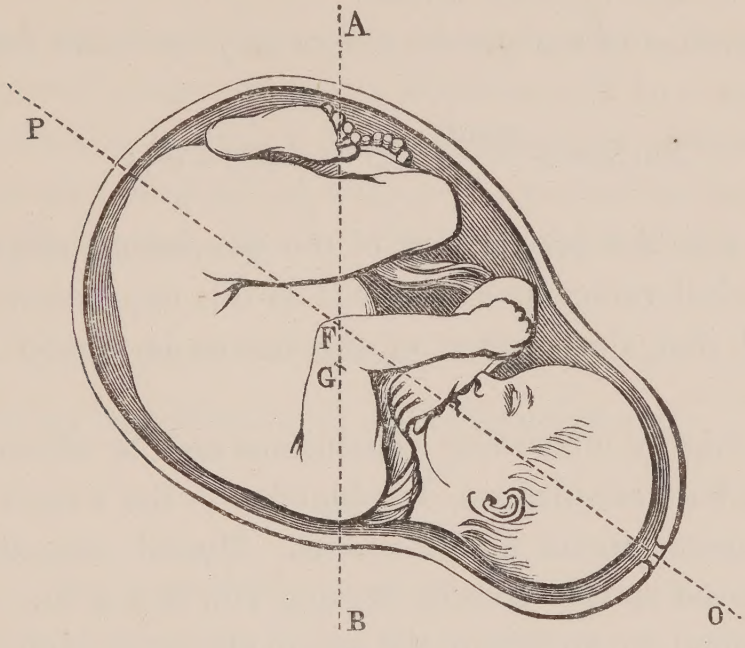


Fig. 21.

cases ruptured by the pressure of the liquor amnii contained in them.

It is evident from the form of the gravid uterus, shown in Fig. 21, that its curvature is greatest near its mouth, and equation (20) shows that for a given hydrostatical pressure, the tensile strain is proportional to the radius of curvature; hence this strain will be greatest at the fundus of the uterus. If we assume the shape of the uterus to be that of a prolate ellipsoid, whose longer diameter is 12 inches, and shorter diameter 8 inches; its mean curvature will be that of a sphere whose diameter is 9.158 inches.

The volume of the gravid uterus is found from the expression—

$$\text{Volume} = \frac{4}{3}\pi\, ab^2; \qquad (21)$$

in which a and b are the semiaxes, and π is the ratio of the

circumference of a circle to its diameter; substituting for a and b their numerical values, we find the contents of the uterus to be 402.13 cubic inches.

The surface of the gravid uterus may be found from the equation—

$$\text{Surface} = \frac{2\pi ab}{e}\left(\sin^{-1}e + \sqrt{1-e^2}\right); \qquad (22)$$

in which e is the eccentricity of the generating ellipse. If the numerical values be substituted in this expression, it will be found that the surface of the uterus is 270.66 square inches.*

Some highly interesting conclusions may be drawn from the preceding calculations, combined with the weight of the total muscular tissue of the uterus. Heschl estimates the weight of the uterine muscles at from 1 lb to 1.5 lb. Montgomery found the muscles of the gravid uterus to weigh 1.5 lb.; and Levret estimates them at 51 cubic inches, which, with a specific gravity of 1.052 (*vide* p. 84), I find to be equivalent to 1.93 lb. Taking the mean of these estimates, we have—

Weight of Muscular Fibres of Gravid Womb.

Heschl, ...	1.25 lb.
Montgomery, . . .	1.50 ,,
Levret, ...	1.93 ,,
Mean, . .	**1.56** lb.

If we now suppose this quantity of muscle to be spread over the entire surface of the uterus, we find,

* Levret estimates the contents of the gravid uterus at 408 cubic inches, and its surface at 339 square inches.

Poppel estimates the contents at 300 cubic inches, and the surface at 210 square inches.

Mean thickness of muscular wall of uterus,

$$= \frac{1.56 \times 7000 \times 1000}{252.5 \times 270.66 \times 1052} = 0.1519 \text{ inch.}$$

If we suppose a ribbon, one inch in width, to be formed from the wall of the uterus, its thickness will be 0.1519 inch; and as each square inch of cross section of muscular fibre (p. 69) is capable of lifting 102.55 lbs., we find for the greatest tensile force producible by the contraction of the uterine muscles.

Tensile strain of uterine wall per inch = $102.55 \times 0.1519 = 15.577$ lbs.

Substituting this value of T, in the equation (20)

$$P = \frac{2T}{\rho}$$

and for ρ, its mean value 9.158 in., we obtain the maximum hydrostatical pressure inside the gravid uterus, that can be produced by the contraction of its muscular fibres.

Maximum hydrostatical pressure produced by uterine contraction

$$= \frac{2 \times 15.577}{9.158} = 3.402 \text{ lbs.}$$

One hundred experiments were made by Duncan and Tait, upon the hydrostatical pressure necessary to rupture the membranes which contain the liquor amnii, which are recorded in Dr. Duncan's book* (pp. 306–311). The greatest pressure observed was 3.10 lbs., and the least was 0.26 lb.; and I find that the mean rupturing pressure of all their experiments was **1·2048** lb.

Combining this experimental result with the calculation already given, of the amount of pressure producible by the muscular tissue of the womb, we may conclude that the uterine muscles are capable of rupturing the membranes in every

* "Researches in Obstetrics." Edinburgh: 1868.

case, and possess, in general, nearly three times the amount of force requisite for this purpose.

The weight of muscle in the virgin uterus is about one and a half ounces, and this muscle is increased to one and a half pounds during pregnancy; that is, it is increased about sixteen fold. If ever there was a muscular system produced to effect a specific object, the uterine muscle may be regarded as such; and if the principle of least action in nature be valid, we should expect such a muscle to be developed only to the extent required, and not beyond it. Considered from this point of view, the preceding calculation must be regarded as interesting; for it shows that the uterine muscle possesses a force of 3.4 lbs., intended to overcome a maximum resistance of 3.1 lbs. It would be a waste of power to endow the uterus with more force than I have shown it to possess, for it is not necessary that the uterus should complete the second stage of labour, as the abdominal muscles are available for this purpose; so that by using them, and not giving the uterus more force than is absolutely necessary for the first stage of labour, an admirable economy of muscular power is effected.

Viewed in connexion with the doctrine of Final Causes, the uterine muscle presents a subject well worthy of contemplation. It is plainly necessary that the first stage in the expulsion of the fœtus should not be entrusted to a voluntary muscle, and hence an involuntary muscle is gradually provided, which takes the initiative, and commences the process of parturition, the completion of which is then accomplished by the aid of voluntary muscles, to the employment of which, at this stage, no moral objection can be raised. It is also necessary (if the Contriver be all-wise, or if the principle of Least Action in nature be true), that the involuntary muscle, so produced, should not possess more or less force than is requisite for its purpose. The uterine muscle does not grow, to meet a grow-

ing resistance (as happens frequently in other cases), and its precise degree of strength cannot be produced by a tentative process; for, in healthy gestation, the uterine muscle never tries its force against the membranes it is called upon to rupture, until the actual period of parturition has arrived. We witness, therefore, the instructive spectacle of a growing force intended to overcome, at a definite future period, a definite resistance, against which it never measures its strength until the actual moment of conflict has arrived.

The mind must be singularly constituted, which fails to see in such an arrangement the clear evidence of a Divine Contriver, adapting means to an end, foreseen by Himself.

The extreme force of uterine contraction produces a pressure of 3.402 lbs. per square inch, which is equivalent to a pressure of 54.106 lbs. acting upon a circle of four and a half inches in diameter, which is assumed as the average area of the pelvic canal.

The maximum force used to expel the fœtus, by both uterine and abdominal muscles combined, is estimated by Joulin,* from forceps experiments made on the dead body, at 110.23 lbs., a result which is regarded by Dr. Duncan† as too large. Dr. Duncan himself considers 80 lbs. as the maximum force ever employed in difficult cases. This would correspond with an hydrostatical pressure inside the uterus of 5.03 lbs. per square inch, which is greater than the uterine muscles, unaided, are capable of producing.

Dr. Duncan's experiments on the strength of the uterine wall itself show that an hydrostatical pressure of from 17 to 25 lbs. per square inch is necessary to its rupture; a result which conclusively demonstrates, when taken in conjunction with

* "Traité complet des Accouchements," p. 447.

† "Researches in Obstetrics," p. 323.

the preceding calculations, that the uterus can never destroy itself by its own contraction. It would, indeed be, *à priori*, ridiculous to suppose that Nature would produce an involuntary muscle capable of destroying itself, and so causing the death of its owner, by its own blind force.

In concluding my remarks on the uterine muscles, I may observe that the arrangement of these muscles on various parts of the organ affords a confirmation of the accuracy of Lagrange's principle as applied to them. It is well known that these muscles attain their greatest thickness at the fundus of the womb, where they are fully double the thickness of the muscular layer at the neck of the womb. Now, so long as the contents of the womb are fluid, or semi-fluid, a constant pressure is exerted perpendicularly to all portions of its walls, requiring by Lagrange's theorem (20), a tensile muscular force varying as the diameter of curvature at each point. From observations on the curvature of the gravid uterus, it may be inferred that the diameters of curvature at the fundus and neck of the womb are nearly in the proportion of 29 to 17. When the womb is emptied of its contents, and the uniform hydrostatical pressure inside is removed, the uterine walls contract with degrees of force proportional to the diameters of curvature they possessed in their former condition; the fundus contracting with twice the force of the parts near the neck—an action eminently calculated to promote the safe and speedy reduction of the organ to its diminished bulk, without risk of inversion.

It is worth while to consider, for a moment, the explanation offered by the Lamarckian Theory of Natural Selection, of the relation between the strength of the uterine muscles and the resistance of the membranes they are required to rupture. According to this theory, an animal that possessed such a proportion as I have shown to exist between the force

and resistance to be overcome, must be the descendant from ancestors who attained this proportion accidentally, and transmitted it to their posterity, who thus acquired an advantage over their neighbours in the competition for life. Thousands and tens of thousands of those who have preceded us must have perished in the race, from not having had the precise balance between uterine force and resistance, which we happily possess; and we may regard ourselves as the survivors of a contest in which the slain greatly outnumber the victors. According to this School of Philosophy, an economy of two per cent. of wax will make a bee; and a saving of a quarter ounce of uterine tissue may, possibly, constitute the sole difference between a Celtic and a Teutonic woman.

Happy is the man whose mother possessed the hereditary tendency to form no more uterine muscle than was absolutely necessary to push him into the world! Happy is the bee who can form her cell with a saving of two grains of wax in the hundred, for she will distance in the race of life her competitors who have not inherited this precious art!

In the second stage of labour, the voluntary action of the abdominal muscles is called into play, to aid the expulsive efforts of the uterine muscles. I have attempted to calculate the force available from the contraction of these muscles as follows:—

The abdominal muscles are four in number, viz. *rectus abdominis*, *obliquus externus*, *obliquus internus*, and *transversalis*. The last three muscles form curved sheets, acting upon the corresponding muscles of the opposite side by means of tendinous *aponeuroses* which meet in the linea alba, and form the sheath of the vertical *rectus abdominis* muscle. From the arrangement of all four, it is plain that the tensile force of muscular contraction in the curved wall of the belly from the xiphoid cartilage to the symphysis pubis, is to be measured

by the sum of the united forces of all the muscular sheets. If we knew the force of each muscle, and the principal curvatures of the belly in the middle line, we could calculate, by Lagrange's theorem, the hydrostatical pressure inside the abdominal cavity, and available to expel fæces, urine, or a fœtus.

In order to ascertain the force of the muscles, I measured carefully their average thicknesses in three subjects, of whom one was a young woman who had borne children, and the others were men of ordinary size and appearance. The results obtained were the following :—

TABLE XXVI.—*Thickness of Abdominal Muscles.*

	No. 1.—Male.	No. 2.—Female.	No. 3.—Male.
Rectus abdominis, . .	0.275 in.	0 29 in.	0.34 in.
Obliquus externus, . .	0.200 ,,	0.25 ,,	0.19 ,,
Obliquus internus, . .	0.235 ,,	0.17 ,,	0.24 ,,
Transversalis, . . .	0.127 ,,	0.15 ,,	0.14 ,,
Totals, . .	0.837 in.	0.86 in.	0.91 in.

The average total thickness of the muscular walls is 0.869 inch, which is nearly identical with the measurements obtained from the female subject. It has been already explained (p. 46), that we must add 50 per cent. to the weight of muscles in the dead subject, in order to bring them to the living weights. This correction gives us 1.3053 inches, for the mean thickness of the muscles causing tension in the central line of the belly, where the forces of all the muscles come into play together. Multiplying this thickness by 102.55 lbs., or coefficient of muscular contraction, we find,

$$T = 1.3035 \times 102.55 = 133.67 \text{ lbs.}$$

This is the tensile strain producible by the contraction of the

abdominal muscles along the curved central line of the belly.

There remain now to be ascertained the principal curvatures of the abdominal surface, and to use the equation, (19)

$$P = T\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right),$$

so as to determine P, the hydrostatical pressure per square inch inside the cavity of the belly, and available, either in whole or in part, for the expulsion of the fœtus during the second stage of labour.

In order to ascertain the curvature of the belly, I made experiments upon three young men, placed lying upon their backs upon the floor, and made them depress and raise the abdominal wall as much as possible. The result was as follows:—Taking a straight line from the upper part of the symphysis pubis to the xiphoid cartilage as the fixed line of comparison, it was found possible to depress the navel one inch below this fixed line, and to raise it two inches above it. When the belly was distended to the utmost, by the action of the abdominal muscles, I measured the longitudinal and transverse curvatures, by measuring the sagittas corresponding to a given length of tangent, with the following results:—

No.	Diameter of Longitudinal Curvature.	Diameter of Transverse Curvature.
J. G. H., . .	22.93 inches.	12.30 inches.
H. O., . . .	22.73 ,,	12.80 ,,
S. H., . . .	22.52 ,,	12.80 ,,
Mean, . .	**22·727** inches.	**12·633** inches.

The curvature of the distended belly, at the navel, is found to be, from the foregoing measurements,

$$\frac{1}{\rho_1} + \frac{1}{\rho_2} = \frac{1}{11.3635} + \frac{1}{6.3166} = \frac{1}{4.0596}.$$

Multiplying this curvature into the tension of the abdominal muscles at the navel, already found, viz. 133.67 lbs. per inch, we obtain, finally,

$$P = \frac{133.67}{4.0596} = 32.926 \text{ lbs. per sq. inch.}$$

This amount of expulsive force, per square inch, is available, although not usually employed to assist the uterus in completing the second stage of labour. If we suppose it applied to a surface of a circle $4\frac{1}{2}$ inches in diameter, the usual width of the pelvic canal, we find that it is equivalent to 523.65 lbs. pressure.

Adding together the combined forces of the voluntary and involuntary muscles, we find,

Involuntary muscles =	54.106 lbs.
Voluntary muscles, =	523.65 ,,
Total, . . .	**577·75** lbs. av.

Thus, we see that, on an emergency, somewhat more than a quarter of a ton pressure can be brought to bear upon a refractory child that refuses to come into the world in the usual manner.*

In order to determine, by actual experiment, the expulsive

* The preceding result will, no doubt, remind the curious and well-informed reader, of the statement made by Mr. Shandy, on the authority of *Lithopædus Senonensis de partu difficili*, that the force of the woman's efforts, in strong labour pains, is equal, upon an average, to the weight of 470 lbs. avoirdupois, acting perpendicularly upon the vertex of the head of the child.

force of the abdominal muscles, I placed two men, of 48 and 21 years of age, respectively, lying on a table upon their backs, and put a disk measuring 1.87 in. diameter just over the navel; weights were placed upon this disk, and gradually increased until the extreme limit of weight that could be lifted with safety was reached; this limit was found to be, in both cases, 113 lbs. As the circle whose diameter is 1.87 in. has an area of 2.937 square inches, the pressure perpendicular to the abdominal wall produced by the action of the abdominal muscles is,

$$P = \frac{113}{2.937} = 38.47 \text{ lbs. per sq. inch.}$$

This force is somewhat greater than that calculated from the curvature of the belly, and the coefficient of muscular contraction, already determined; viz. 32.93 lbs. per square inch. It should be remembered, however, that in my experiments on the abdominal muscles, I pushed the trial of strength almost to the verge of producing slight *peritonitis*, as both the subjects of my experiment complained of sharp shooting pains in the abdominal walls for 24 hours after the experiments were made.

The foregoing theory of the muscular forces employed in parturition leads us naturally to be cautious in the use of chloroform, and other anæsthetics, which produce their effects, first upon the voluntary, and afterwards upon the involuntary muscles. When chloroform is used, in parturition, beyond the stage at which it produces simple drunkenness, and indifference to pain, it is positively injurious; for it destroys the action of the voluntary abdominal muscles, which constitute the chief part of the force employed in difficult labours. Hence, the muscular tissue of the uterus is left to overcome an obstacle beyond its strength, and without the aid that nature intended to be given by the abdominal muscles. The

result is, that labour ceases until consciousness returns, and enables the ill-treated woman to avail herself of the apparatus of voluntary abdominal muscles provided by nature for her use.

In these remarks I confine myself altogether to the physical aspects of the use of chloroform in labour, and forbear to express an opinion as to the moral effects likely to be produced by the exhibition of an intoxicating agent, administered merely for the luxurious purpose of deadening the pain of the exercise of a purely natural physiological function.

9. **On the Classification of Muscles and their Mode of Action.**—It has been already shown that muscles consist of linear fibres, which may be variously arranged; and as lines form surfaces, and combinations of surfaces form solids, it is evident that a natural classification of muscles must be essentially geometrical, and that the mechanical action of muscles must depend upon their geometrical configuration.

The following classification of muscles is proposed as a simple and natural arrangement:—

I. Muscular Fibres lying in the same Plane.

1. *Fibres parallel and rectilinear.*

(*a*) Direct Prismatic.
(*b*) Rhomboidal.
(*c*) Penniform.

2. *Fibres intersecting and rectilinear.*

(*a*) Triangular.
(*b*) Deltoidal.
(*c*) Quadrilateral.

3. *Fibres curvilinear and parallel.*

(*a*) Sphincters.

II. Muscular Fibres forming Curved Surfaces.

1. *Fibres rectilinear.*

(*a*) Skew Surfaces.

2. *Fibres curvilinear.*

(*a*) Ellipsoidal Surfaces.

Fibres in same plane, parallel to each other, and rectilinear.—This natural group of muscles has been described by Borelli,* who has named their several species, *Direct Prismatic*, *Rhomboidal*, and *Penniform*.

Direct Prismatic Muscles.—If we suppose two bars, AA and BB, joined by a number of parallel elastic strings, to be drawn asunder, as represented in Fig. 22, the elasticity of the strings will draw back the bars into their original position, by the same kind of action as the contractile fibres of a *Direct Prismatic* muscle. If f denote the contractile force of each fibre, and n their number, the resultant force of the muscle will be

$$F = nf, \qquad (23)$$

and will act parallel to the fibres, in the direction ab. If the parallel sheets of muscular fibres be united, so as to form a solid muscle, the re-

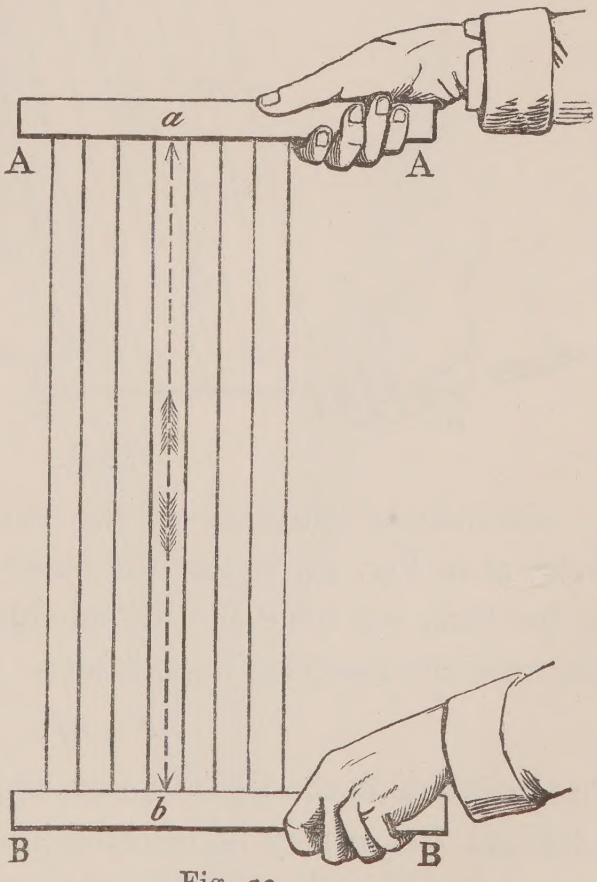


Fig. 22.

* De Motu Animalium, vol. i., p. 8.

sultant will pass through the centres of gravity of the areas marked on the bones by the origin and insertion of the fibres.

Good examples of the *Direct Prismatic* muscle may be found in the *sartorius* and *omo-atlantic* muscles of various animals.

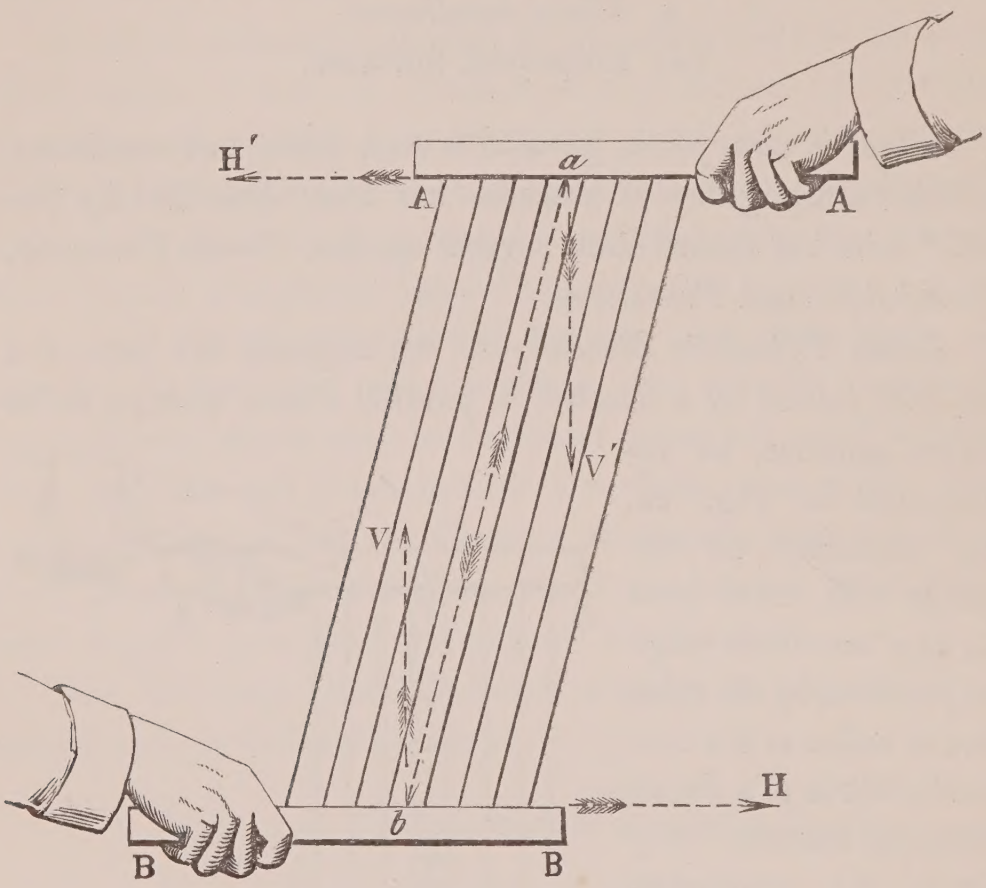


Fig. 23.

Rhomboidal Muscles.—If the two bars, AA and BB, be held, as in Fig. 23, so that the fibres joining them are oblique to the bars, we have the Rhomboidal muscle of Borelli. In this case the resultant force is also

$$F = nf;$$

but as it acts in the right line ab, which is oblique to the bars AA and BB, it may be resolved into two forces at each bar, viz., bH, bV, and aH', aV'; of these forces, bH and aH' act parallel to the bars, and tend to bring them into the position of the prismatic muscle; the other forces bV and aV' tend to

draw the bars together. If ϕ denote the angle abH, the horizontal and vertical forces exerted by the muscle will be

$$H = F \cos \phi$$
$$V = F \sin \phi \qquad (24)$$

The *intercostal* and *rhomboid* muscles afford good examples of the *Rhomboidal* type of muscle.

Penniform Muscles. — The structure of penniform muscles is illustrated by Fig. 24, in which two bars, AA, BB, parallel to each other, are connected by two sets of parallel fibres, with a third bar, CC, parallel to the other bars. It is a combination of two rhomboidal muscles, which give a vertical resultant in the direction CV.

If F denote, as before, the resultant force of each rhomboidal muscle,

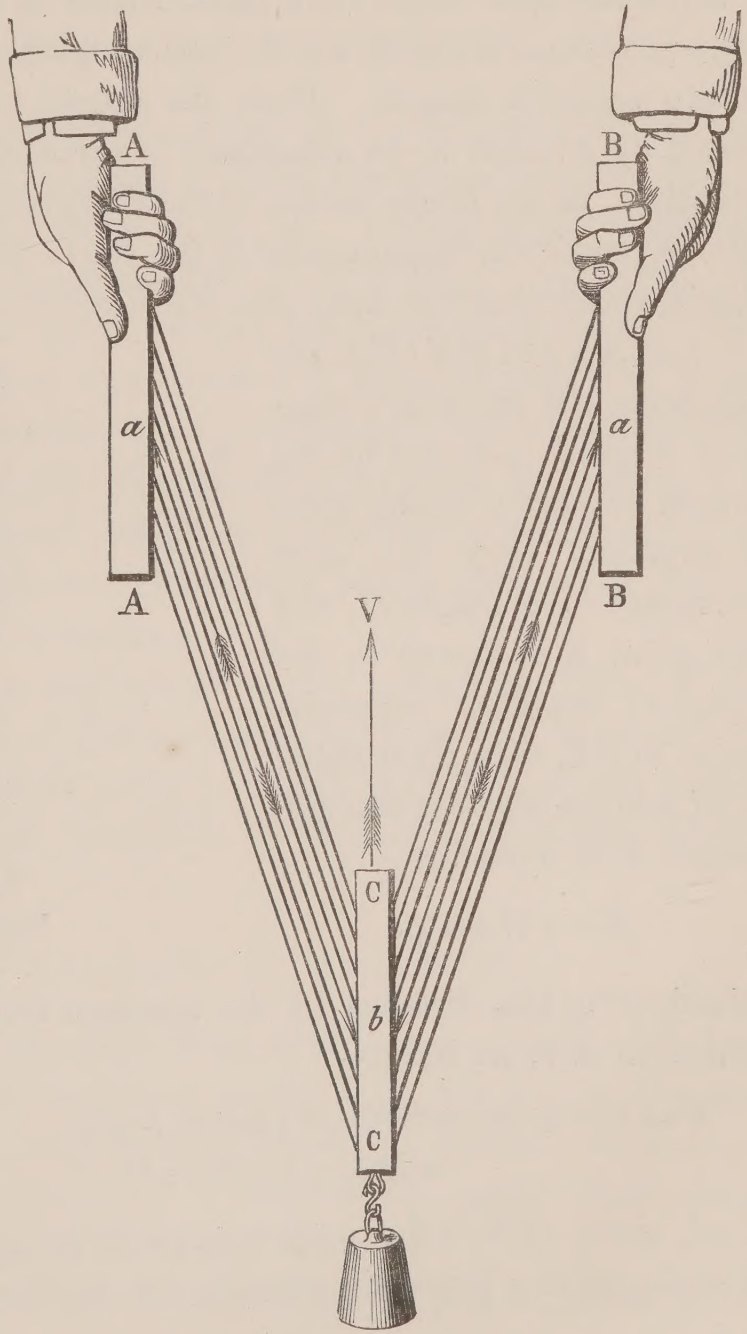


Fig. 24.

and ϕ denote the angle abV, made by its fibres with the parallel bars, the resultant force of the penniform muscle will be

$$R = 2F \cos \phi. \qquad (25)$$

Many muscles, when dissected, are found to contain parallel sheets of fascia, resembling the bars AA, BB, and the muscular fibres take origin from these sheets of fascia, forming true penniform muscles, which, taken together, make up the entire complex muscle. Thus, the *deltoid* muscle is considered by Albinus to be composed of seven distinct pennate muscles. In the *biceps humeri* of the Ungulates I have found, also, a penniform arrangement of the fibres with reference to fascial sheets that run along the whole course of the muscle.

Let AX and BX (Fig. 25) represent two fibres of a pennate muscle, and let l be the length of each; during their contraction, they are each shortened by a length δl, and the point X is moved to the point x; draw xp perpendicular to BX, then Xp is equal to δl, and the distance through which X is moved is

$$Xx = \delta l \sec \phi.$$

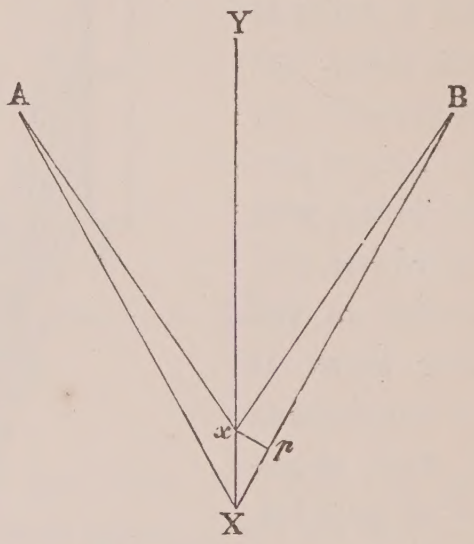


Fig. 25.

Multiplying this distance by the resultant force (25), in the direction XY, we have—

Work done by the contraction of a pennate muscle

$$= 2F \cos \phi \sec \phi \delta l = 2F\delta l. \qquad (26)$$

And, since $2F\delta l$ is the work *inherent* in the muscular fibres, if arranged in a prismatic manner, we conclude that there is

neither gain nor loss of work due to the penniform arrangement.

In the Nylghau, the muscular fibres of the *biceps humeri* are 1.84 inches in length, and make an angle of 50° with the central line. These penniform fibres will therefore act like the prismatic fibres of an ordinary *biceps* muscle, whose length is

$$l = 1.84 \times \sec 50° = 2.85 \text{ inches.}$$

The actual length of the *biceps* in the Nylghau is 6 inches, and yet the total amount of its contraction is only that due to a muscle of 2.85 inches in length. The intention of this arrangement of penniform fibres is to produce a powerful pull through a small space. In the *brachiæus* muscle of the same animal, the fibres run parallel together through the entire length of the muscle, which is prismatic, and wound spirally round the humerus. The length of the fibres is 8 inches, and from their parallel, and not penniform arrangement, this muscle is capable of a contraction nearly three times greater than that of the penniform *biceps*.

In the Ass, the length of the penniform fibres of the *biceps humeri* is 1.06 inches, and they form with the tendinous axis an angle of about 24°. Hence the action of this penniform biceps will be similar to that of a prismatic muscle whose length is

$$l = 1.06 \times \sec 24° = 1.15 \text{ inches.}$$

The actual length of the *biceps* muscle in the Ass is 6.6 inches. The fibres of the *brachiæus* muscle in the Ass are parallel, and run along its entire length, which is 9 inches. Hence the *brachiæus* is capable of a contraction nearly eight times greater than that of the *biceps* muscle.

Let it be required to find a prismatic muscle, whose action

shall be exactly the same as that of a given penniform muscle. In order that the action of the two muscles shall be, in all respects, the same, it is necessary that they shall contract through the same space and with the same force, the product of the space and force being the work done by the muscle in a single contraction. If l denote the length of the penniform fibres ab (Fig. 24), and ϕ the angle abV; then the contraction due to the penniform muscle will be equal to that of a prismatic muscle whose length is

$$L = l \sec \phi. \tag{27}$$

Let $aAa\,bBb$ (Fig. 26) represent a penniform muscle, whose length AB is λ, the angle ABC being ϕ. If perpendiculars AC and AD be drawn from A to the fibres Bb and Bb produced, it is plain that the lines AC and AD will represent the total number of fibres employed in the muscle; hence, if f be the force of each fibre, the total force employed at each side will be

$$F = nf = f \times AD = f\lambda \sin \phi;$$

but

$$R = 2F \cos \phi.$$

Hence

$$R = 2f\lambda \sin \phi \cos \phi = f\lambda \sin 2\phi \tag{28}$$

This is equivalent to the force of a prismatic muscle, whose cross section is $\lambda \sin 2\phi$, or equal to the line CD.

For,

$$\lambda = AB.$$

$$\lambda \sin \phi = AB \sin \phi = AD.$$

$$\lambda \sin \phi \cos \phi = AD \cos \phi = DE.$$

$$2\lambda \sin \phi \cos \phi = 2DE = CD.$$

the actual width of the penniform muscle is

$$w = 2l \sin \phi.$$

Hence the width of the equivalent prismatic muscle may be found: let this width be called W, and we have

$$\frac{W}{w} = \frac{CD}{bb} = \frac{2\lambda \sin\phi \cos\phi}{2l \sin\phi} = \frac{\lambda \cos\phi}{l};$$

and, finally,

$$W = \frac{w\lambda \cos\phi}{l}. \quad (29)$$

These formulæ, applied to the *biceps* muscle of the Nylghau and Ass, give the following equivalent prismatic muscles.

In the Nylghau, the *biceps* muscle has the following measurements:

$l = 1.84$ in., $\phi = 50°$, $\lambda = 6$ in.

Hence,

$$L = l \sec\phi = 2.85 \text{ in.}$$

$$W = w \frac{\lambda \cos\phi}{l} = 2.09\, w.$$

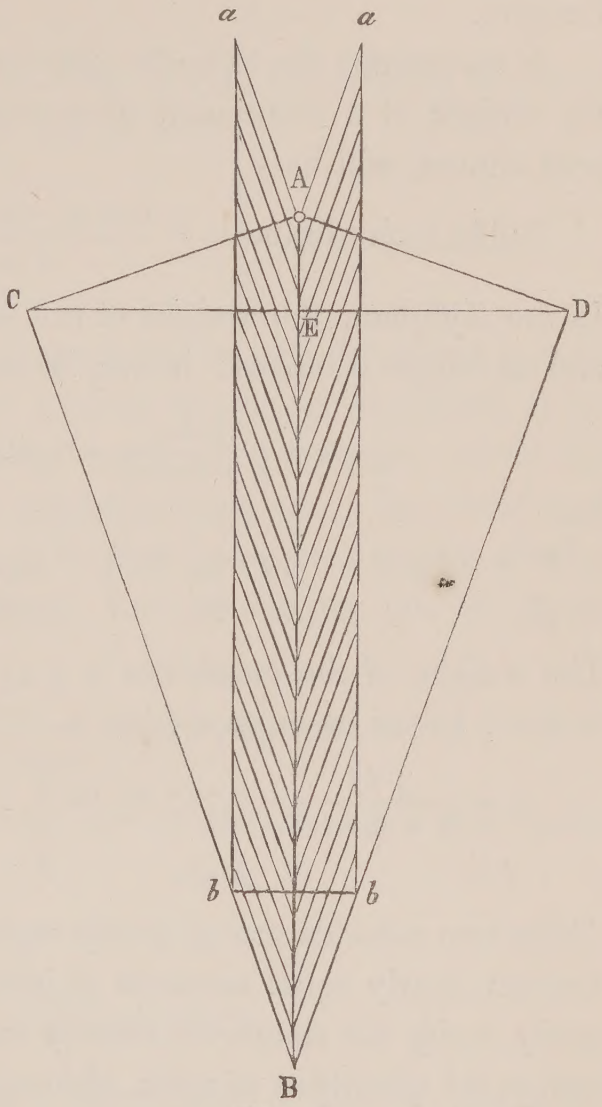


Fig. 26.

In the Ass we have,

$$l = 1.06 \text{ in.}, \quad \phi = 24°, \quad \lambda = 6.6 \text{ in.}$$

Hence,

$$L = l \sec\phi = 1.15 \text{ in.}$$

$$W = w \frac{\lambda \cos\phi}{l} = 5.688\, w.$$

It is interesting to observe that the short thick prismatic muscles, to which the penniform *biceps* of the Nylghau and of

the Ass are equivalent, are totally different from the long thin *brachiæus* muscles, associated with them in the flexion of the forearm.

If we assume the specific gravity of muscle to be 1.050, the weight of a cubic inch of muscle, expressed in avoirdupois ounces, will be—

$$\text{Cubic inch of muscle} = \frac{1.050 \times 252.5 \times 16}{7000} = 0.606 \text{ oz.}$$

In the Nylghau, the weight of the *biceps* muscle is 3.44 oz., and its length 6 inches; hence, its cross section, w, is

$$w = \frac{3.44}{6 \times 0.606} = 0.946 \text{ sq. in.}$$

but

$$\left.\begin{aligned} W &= 2.09\, w = 1.977 \text{ sq. in.} \\ L &= 2.85 \text{ in.} \end{aligned}\right\} \text{ Equivalent Prismatic } \textit{biceps} \text{ of Nylghau.}$$

The weight of the *brachiæus* is 3.43 oz., and its length is 8 inches; hence its cross section is—

$$\left.\begin{aligned} w &= \frac{3.43}{8 \times 0.606} = 0.707 \text{ sq. in.} \\ l &= 8 \text{ in.} \end{aligned}\right\} \textit{Brachiæus} \text{ of Nylghau.}$$

These two muscles are of nearly equal weight, and therefore contain nearly equal amounts of inherent work, and we can easily verify the statement already made (26), that they give out equal quantities of work, although one has its fibres prismatic and the other penniform.

Work done by *biceps* (penniform)

$$= 1.977 \times 2.85 = 5.63.$$

Work done by *brachiæus* (prismatic)

$$= 0.707 \times 8 = 5.65.$$

In the Ass, the weight of the *Biceps* is 6.83 oz., and its length is 6.6 inches; hence its cross section, w, is

$$w = \frac{6.83}{6.6 \times 0.606} = 1.71 \text{ sq. in.}$$

Hence,

$$\left.\begin{aligned} W &= 5.688\, w = 9.726 \text{ sq. in.} \\ L &= \qquad 1.15 \text{ in.} \end{aligned}\right\} \text{ Equivalent prismatic } \textit{biceps} \text{ of Ass.}$$

The weight of the *brachiæus* is 3.23 oz., and its length is 9 inches; hence its cross section is—

$$\left.\begin{aligned} w &= \frac{3.23}{9 \times 0.606} = 0.592 \text{ sq. in.} \\ l &= \qquad 9 \text{ in.} \end{aligned}\right\} \textit{Brachiæus} \text{ of Ass.}$$

In a fine Sambur Stag examined by me, the *biceps humeri* muscle was found to be pennate in the arrangement of its fibres; their length being 2.56 in., the angle made by them with the fascia of insertion being 34°. The weight of the muscle was 5.91 oz. In this case, the length of the *biceps* muscle was 10 inches, from which we find its mean cross section

$$w = \frac{5.91}{10 \times 0.606} = 0.975 \text{ sq. in.}$$

From this we find, as before, the prismatic muscle equivalent to the pennate biceps—

$$L = l \sec \phi = 3.09 \text{ in.}$$

$$W = w \frac{\lambda \cos \phi}{l} = 3.16 \text{ sq. in.}$$

The *brachiæus* muscle in the Sambur is 4.95 oz. in weight and 9.5 inches in length, and it is a prismatic muscle; its cross section is,

$$w = \frac{4.95}{9.5 \times 0.606} = 0.860 \text{ sq. in.}$$

It is interesting to contrast together the two muscles which flex the forearm in the three animals described, in order to see, at a glance, the very different uses which these muscles are intended for.

TABLE XXVII.—*Ungulate Biceps and Brachiæus.*

	Biceps.		Brachiæus.		Ratio of Force of Biceps to Brachiæus.	Ratio of Contraction of Brachiæus to Biceps.
	W.	L.	*w.*	*l.*		
	Sq. In.	In.	Sq. In.	In.		
Nylghau, .	1.98	2.85	0.71	8.0	2.80	2.81
Ass, . . .	9.72	1.15	0.59	9.0	16.47	7.83
Sambur, .	3.16	3.09	0.86	9.5	3.67	3.08

This Table shows clearly the very different actions of the *biceps* and *brachiæus* muscles in the Ungulates. The *biceps* muscle contracts with a force much greater than that of the *brachiæus*, while the space through which it acts is much less than that of the *brachiæus*. The two muscles differ but little in external appearance, but by means of the penniform arrangement of the fibres of the *biceps*, a rope-like muscle is enabled to do the duty of a flat short muscle, which it would be impossible to place directly upon the bones of the arm.

The rapid lift of the fore foot, which is so characteristic of the swift Ungulates, and by none more beautifully shown than by the Deer tribe, is mainly due to the powerful action of the *biceps* through a short space; while the slower and longer flexions of the forearm are due to the distinctive action of the *brachiæus* muscle.

In a Brahmin Cow examined by me, the arrangement of the fibres of the *biceps* was penniform, though I omitted to measure their length, and the angle made by them with the central fascial sheet of their origin. The measurements of the *biceps* and *brachiæus* muscles in this animal were as follows. The *biceps* weighed 3.88 oz., and had a total length of 6 inches. The *brachiæus* weighed 2.66 oz., and its fibres, which were arranged as in a prismatic muscle, measured

8.15 inches in length. These measurements correspond with the following mean cross sections:—

Biceps, ...	1.067 sq. in.
Brachiæus, ...	0.539 „

The angular velocity with which the forearm turns on the elbow joint is not proportional merely to the force of the muscle, but to the product of that force into the distance from the point of insertion to the axis of the elbow joint. I measured these distances carefully, for the purpose of obtaining a comparison between the angular velocities produced by the *biceps* and *brachiæus* muscles respectively. The following Table contains the measurements and results; and I have added corresponding measurements taken from the arm of a very fine Lioness.

In the Lioness the *biceps* muscle weighed 6.70 oz., and had a length of 8 inches; and the *brachiæus* muscle weighed 2.60 oz., and had a length of 8.75 inches.

TABLE XXVIII.—*Angular Velocity of Forearm produced by Biceps and Brachiæus Muscles.*

	Biceps.		Brachiæus.		Ratio of Angular Velocities produced by *Biceps* and *Brachiæus*.
	Cross Section.	Distance of Insertion from Axis of Joint.	Cross Section.	Distance of Insertion from Axis of Joint.	
	Sq. In.	In.	Sq. In.	In.	
Nylghau, .	1.98	1.02	0.71	2.26	1.26
Ass, . . .	9.72	1.24	0.59	2.00	10.21
Sambur, .	3.16	1.72	0.86	2.44	2.59
Lioness, .	1.38	2.29	0.49	2.16	3.00

The total angular movement of the forearm produced by each muscle will be proportional to the length of the muscle

divided by the distance of its point of insertion from the axis of the joint; the ratios of the angular motions caused by the two muscles is shown in the following Table:—

TABLE XXIX.—*Angular Motion produced by the Biceps and Brachiæus Muscles.*

	Biceps.		Brachiæus.		Ratio of Angular Movement, produced by Biceps and Brachiæus.
	Length.	Distance of Insertion.	Length.	Distance of Insertion.	
	In.	In.	In.	In.	
Nylghau, .	2.85	1.02	8.0	2.26	0.789
Ass, . . .	1.15	1.24	9.0	2.00	0.206
Sambur, .	3.09	1.72	9.5	2.44	0.461
Lioness, .	8.00	2.29	8.75	2.16	0.862

Bringing together all the preceding results, we obtain the following Table:—

TABLE XXX.—*Comparison of the Action of the Biceps and Brachiæus Muscles.*

	Ratio of Forces.	Ratio of Velocities.	Ratio of Angular Motions.
Nylghau, . . .	2.80	1.26	0.789
Ass, ...	16.47	10.21	0.206
Sambur, . . .	3.67	2.59	0.461
Lioness, . . .	2.82	3.00	0.862
Man,*	1.50	1.82	1.159

The Ass greatly exceeds the other animals in the relative

* The data from which the comparison of the *Biceps* and *Brachiæus* muscles in Man is made are given in pp. 63, 64.

force of the *biceps*, and the velocity of motion produced by it, as compared with the *brachiæus ;* and the angle through which it bends the forearm is only one-fifth the angle of flexion due to the *brachiæus;* while in the Lioness these angles are nearly equal, and in Man the angular motion caused by the *biceps* is greater than that produced by the *brachiæus.*

These peculiarities are common to the Ass and Horse, in which latter animal the "gig action" of the fore foot, so much admired, is altogether due to the same cause as in the Ass, viz., the rapid flexion of the elbow joint caused by the pennate arrangement of the fibres of the *biceps* muscle, producing a powerful contraction through a short space.

The *mylohyoid* muscle, which forms the floor of the mouth (*diaphragma oris*), is another good example of the penniform arrangement of muscular fibres. This muscle, as it exists in Man, is shown in Fig. 27, where *H* denotes the hyoid bone,

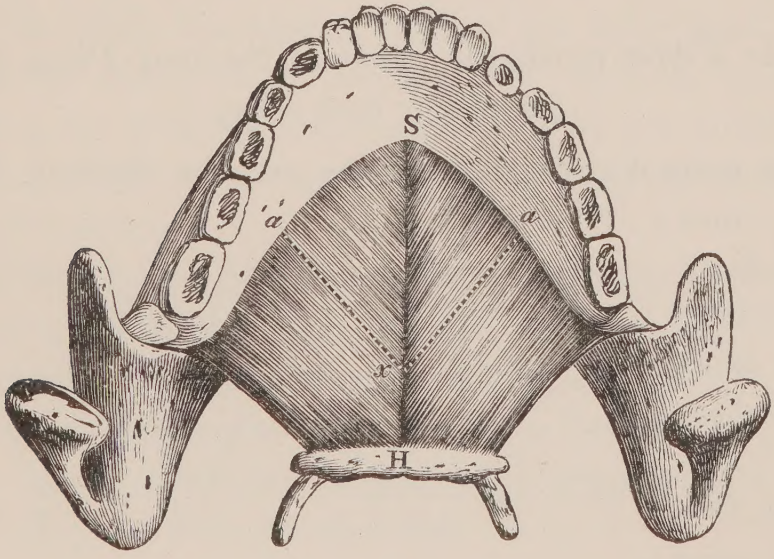


Fig. 27.

and *S* the symphysis. The posterior fibres of the muscle are attached directly to the tongue bone, while the anterior fibres

meet in a central *raphe*, *HS*, which is the direction of the resultant motion produced by the simultaneous contraction of all the penniform fibres. The lengths of the fibres measured from behind forwards increase from 0.95 in. to 1.50 in., and then decrease gradually down to zero. The angle ϕ, made by them with the central raphe, is 45°, when the top of the tongue bone is brought into the same plane as the floor of the mouth. If l denote the length of any fibre, and δl the amount of its contraction, it follows that any point x, lying in the central raphe, will be drawn upwards and forwards through a space represented by $\delta l\sqrt{2}$, where l is the length of the line xa, which is proportional to the distance of x from the symphysis of the jaw; hence the point S will have no motion at all, and the velocity will increase from S to the centre of H, where the fibres are of the greatest length.

Another example of penniform muscle may be seen in the *accelerator urinæ*, in which the fibres form angles of 23° with the central raphe.

Muscular Fibres rectilinear, lying in the same Plane, and not parallel to each other.

This natural group of muscles may be divided, like the former, into three distinct types, which may be named, respectively, *Triangular*, *Deltoidal*, and *Quadrilateral* muscles.

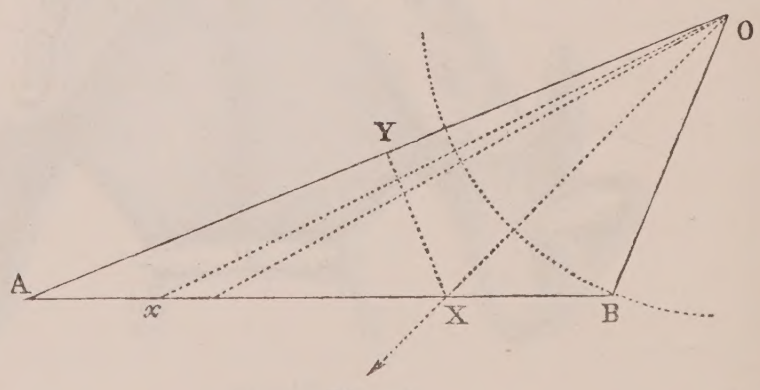


Fig. 28.

Triangular Muscles.—In these muscles, the origin is a

point, and the insertion of the fibres is a right line Let O (Fig. 28) denote the origin, and AB the line of insertion. The muscular fibres diverge from the point O, lying between the lines OA and OB, and each fibre contracts with the same force, whatever be its length. Draw the line OX bisecting the angle AOB. If the muscular fibres be taken in pairs, making equal angles at opposite sides of OX, the resultant of each pair of fibres will lie along the right line OX, which will therefore be the direction of the resultant of the whole muscle; and since the bisector of the angle AOB divides the base AB into segments AX, XB, proportional to the sides AO and OB, it is easy in all cases to find the point X, in the line of insertion, through which the whole force of the contracting muscle is directed.

One of the best examples of triangular muscle, with which I am acquainted, is to be found in the *biceps femoris* of the large *Felidæ*. In a large Lion examined by me, the *biceps* weighed 27.75 oz. It forms a massive triangular sheet, having for its origin the highest point of the *tuber ischii*, and is inserted by strong fascia, continuously, from the middle of the femur, covering the *vastus externus*, down the entire length of the outer side of the leg, as far as the *tendo Achillis*, round which it is fastened.

The total length of the fibres from the origin to the lowest point of insertion was 22.25 inches, and the length of the fibres passing to the highest point of insertion was 8.75 inches. The base of the triangle, when the leg is extended, was 17.25 inches. If we divide this length in the proportion of OA to OB, or 22.25 to 8.75, we shall find

$$AX = 12.38 \text{ inches.}$$
$$BX = 4.87 \quad ,,$$

This point corresponds exactly with the top of the fibula, so

that the effect of the whole triangular muscle is the same as that of a single prismatic muscle passing from the tuber ischii at the top of the fibula.

Another remarkable example of triangular muscle is found in the anterior fin muscle of the Angel Shark; this muscle consists of a number of distinct muscular bundles, each belonging to a single ray of the fin, and forming altogether a triangular muscle, whose sides measured 5 in. and $2\frac{1}{2}$ in. respectively, the base measuring in a straight line 7 inches; the muscle itself is not terminated by this base, but has a curved base, as shown in the Fig. 29. In this case, as in the

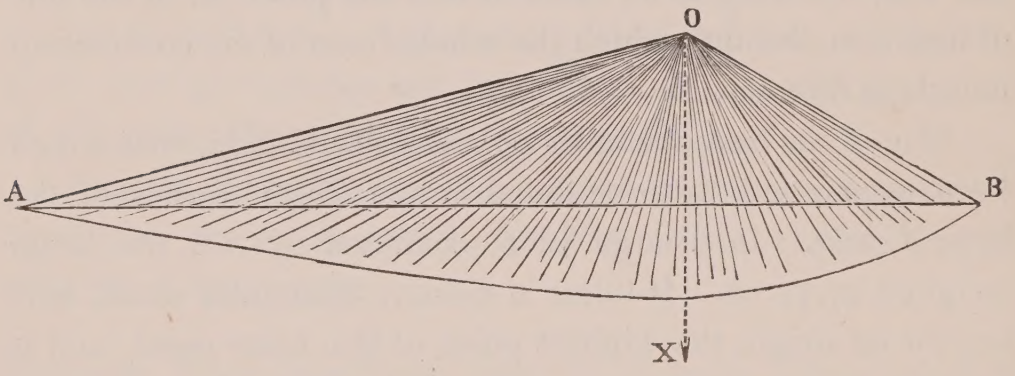


Fig. 29.

last, the bisector OX, of the angle AOB, will be the direction of the resultant of the radiating fibres, whatever be the shape of the curve that forms the base of insertion of the muscular fibres. In the Angel Shark I found 20 distinct bundles lying between OB and OX, and 30 bundles lying between OA and OX; but the first bundles were 50 per cent. thicker than the second, so that the muscular force was distributed equally through equal angles round the point O.

Let us suppose the radiating fibres of a triangular muscle to be so arranged that equal angles will contain equal numbers of muscular fibres; and let it be required to calculate the work done by the contraction of such a muscle, and

compare it with the inherent or potential work of the muscular fibres contracting separately.

We may imagine a circle described round the point O, with a radius assumed equal to unity, and such that the fibres passing to their insertion completely fill the circumference of the arc of this circle. If θ denote (Fig. 28) the angle XOx, made by any muscular fibre with the bisector OX, and f denote the force of contraction of the muscular fibres applied perpendicularly to a unit length; then $fd\theta$ will be the force acting in the line Ox, and the resultant of all these forces, estimated in the direction OX, will be

$$R = f \int_{-\theta}^{+\theta} \cos\theta d\theta = 2f \sin\theta.$$

When the whole muscle AOB contracts, each fibre is shortened so as to allow the base AB to be drawn towards O, in the direction OX, remaining parallel to itself. If b denote the bisector OX, and δb be the amount of contraction, the total work done by the triangular muscle will be—

$$\text{Work of triangular Muscle} = R\delta b = 2f\delta b \sin\theta = 2f\delta q, \quad (30)$$

where q denotes the perpendicular (XY) let fall from the point X upon the side of the triangle.

If we assume, that the cross section of a triangular muscle is everywhere the same, this cross section may be conveniently used as the unit to which f is applied, and we obtain the following important theorem:—

The Work done by the contraction of any triangular muscle is equal to the Work done by a prismatic muscle having the same cross section and a length equal to twice the perpendicular dropped on the side, from the foot of the bisector of the vertical angle of the triangular muscle.

The total amount of work *inherent* in the muscle may be found as follows: if l denote the length of any fibre Ox, then $ld\theta$ will be proportional to the work of its contraction; and the total inherent work will be—

$$\text{Inherent Work of Triangular Muscle} = f \int_{-\theta}^{+\theta} \delta l d\theta$$

If we call β the angle BXO (Fig. 28), we have

$$l = \frac{b \sin \beta}{\sin (\beta - \theta)}, \text{ and}$$

$$\text{Inherent Work} = f\delta b \sin \beta \int_{-\theta}^{+\theta} \frac{d\theta}{\sin (\beta - \theta)}$$

$$= f\delta b \sin \beta \log \frac{\cot \frac{1}{2}(\beta - \theta)}{\cot \frac{1}{2}(\beta + \theta)} \quad (31)$$

This result admits of a geometrical illustration (Fig. 30). In the triangle AOB draw the bisector of the vertical angle

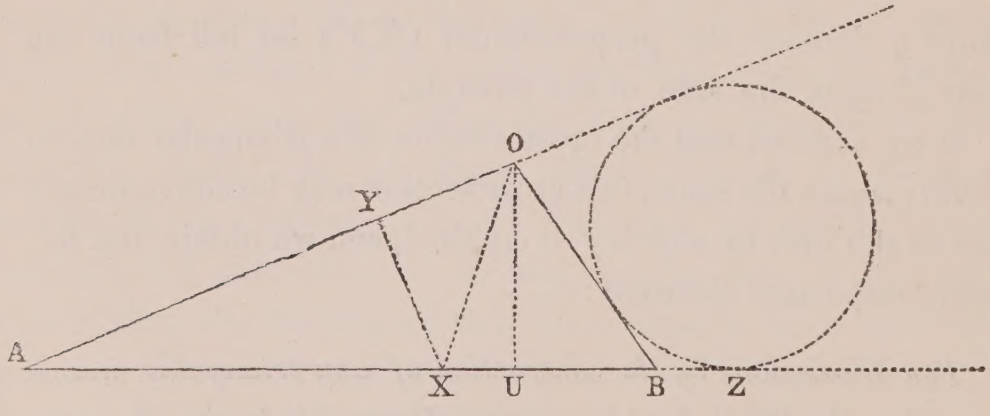


Fig. 30.

OX, and from the point X let fall the perpendicular $XY = q$. Produce the sides AO and AB, and draw the exscribed circle,

touching AB at the point Z, and let fall the perpendicular $OU = p$, from the vertex upon the base of the triangle.

It is easy to see that

$$OAZ = \beta - \theta$$

$$OBZ = \beta + \theta$$

$$\frac{\cot \frac{1}{2}(\beta - \theta)}{\cot \frac{1}{2}(\beta + \theta)} = \frac{AZ}{BZ}$$

$$OU = p = b \sin \beta.$$

Therefore

The work inherent *in a triangular muscle is proportional to the perpendicular let fall from the vertex upon the base, multiplied by the Napierian Logarithm of the ratio of the segments of the base made by the point of contact of the exscribed circle;*

and

The work *done* by the same muscle will be proportional to double the perpendicular let fall upon the side of the triangle from the foot of the bisector of the vertical angle.

In the case of the *biceps femoris* of the Lion, I made the following measurements :—

$$p = 3.28$$

$$AZ = 9.56$$

$$BZ = 2.72$$

$$q = 1.85.$$

From these *data* we find—

$$p \log_e \frac{AZ}{BZ} = 3.28 \times 1.25 = 4.10,$$

and

$$2q = 3.60.$$

Hence, the *inherent* work of the muscle is to the work *done* by it as 41 to 36; or the work done is 88 per cent. of the inherent work, and the *loss* of work amounts to 12 per cent., in consequence of the triangular arrangement of the fibres.

In the preceding investigation, I have assumed that the muscular fibres proceeding from *AB* to *O* are distinct from each other, and pass continuously from origin to insertion without interfering or blending with each other. On this supposition, the fibres are supposed to be arranged, not in one plane, but like the radiating blades of a fan, in such a manner that the cross section of the muscle made by a sphere described with *O* as centre and with any radius, shall be always constant.

In order to ascertain the law of distribution of fibres in a triangular muscle, I made the following measurements in the great pectoral muscle of the wing of the common Buzzard.

The weight of the muscle was 0.705 oz. av., and the length of its extreme fibres was 6 inches. I took the breadth and thickness of the cross section of the muscle at 6, 4, and 2 inches, respectively, from its humeral insertion, and calculated the area of the cross section from these measurements, with the following results:—

Pectoral Muscle of Buzzard.

Distance of Section from Insertion.	Length of Section.	Mean Thickness of Section.	Area of Cross Section.
2 inches.	1.92 inches.	0.22 inches.	0.422 sq. in.
4 ,,	2.26 ,,	0.185 ,,	0.418 ,,
6 ,,	6.00 ,,	0.07 ,,	0.420 ,,

This Table demonstrates the accuracy of the supposition

made as to the distribution and arrangement of the fibres, and I believe that the same arrangement exists in every muscle, however complicated in form; or, in other words, that each muscular fibre passes from origin to insertion, retaining a constant thickness or cross section.

In the common Heron, I obtained results quite conformable with those found from examination of the pectoral muscle of the Buzzard. In Fig. 31 is shown the pectoral muscle of the Heron: *OP* is the insertion on the ridge of the arm bone; *XYZ* is the curve formed by the origin of the muscular fibres, which radiate in a fan-shaped manner from their insertion, *OP*, to their origin, *XYZ*. I measured the length of the line *XYZ*, and took the thickness of the muscle at various points along this line; and also along a curved line of section, *xyz*, made so as to divide all the radiating fibres. The cross sections along *XYZ*, and *xyz* are shown in Fig. 32.

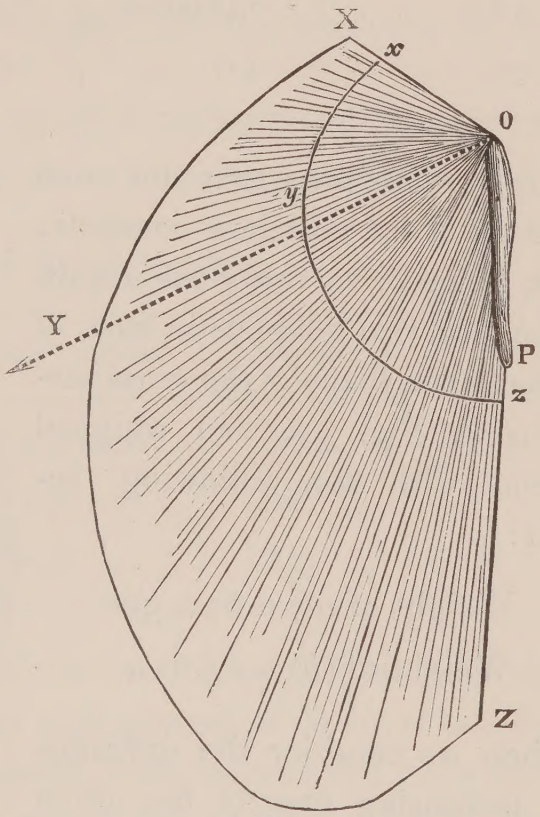


Fig. 31.

The section *XYZ* had a mean thickness of 0.22 in. through two-thirds of its length, and of 0.335 in. through one-third, giving a mean thickness of 0.258 in. The section *xyz* had a mean thickness of 0.56 in. in the middle third, and of 0.50 in. and 0.38 in. in the outer thirds, giving a total

mean of 0.48 in. The length of the line *XYZ* was 7.42 in., and that of *xyz*, 4.27 in.* Hence we find the following :—

Pectoral Muscle of Heron.

Section.	Length.	Thickness.	Area.
XYZ, . . .	7.42 inches.	0.258 inches.	1.914 sq. in.
xyz, . . .	4.27 ,,	0.48 ,,	2.049 ,,

This result shows that the cross section of the muscle is constant; but, in order to verify the result, I cut out two templets in zinc, of exactly the shape of the cross sections in Fig. 32, and weighed them, with the following result:—

Weight of (*a*) = 63.4 grs.

Weight of (*b*) = 65.6 ,,

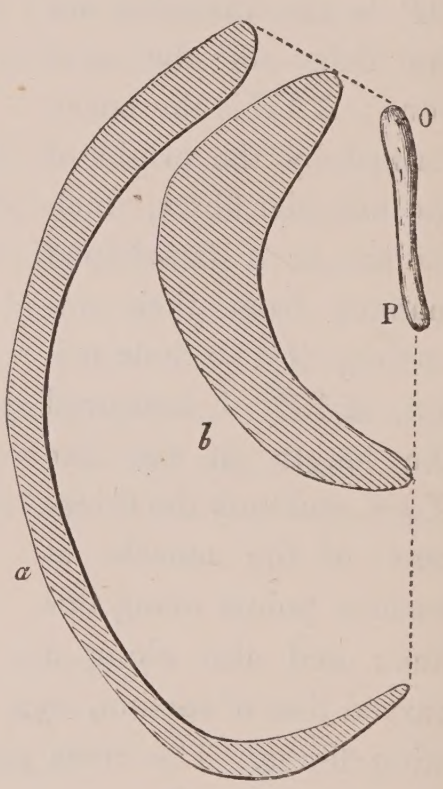


Fig. 32.

When we consider the difficulty of measuring exactly the cross section of a muscle, the foregoing results must be regarded as proving that its cross section is, practically, constant.

Among the advantages that arise from this arrangement of fibres in complex muscles, one of the most obvious is the equal strength which it gives to all parts of the muscle, which would not be secured by any web-like arrangement, allowing

* The total weight of the muscle was 2.68 oz. av.

of the blending of the fibres with each other. Thus, in a triangular muscle formed from a uniform sheet of muscle of equal thickness throughout, the strength of the section near the vertex would be much less than of that near the base, and the muscle might be torn by the application of external violence; whereas, in a muscle whose fibres are arranged in a fan-shaped form, the cross section being the same everywhere, the muscle is of equal strength, and the tendency to tear across is lessened.

The following measurements, made on a male human subject, confirm the results obtained from the pectoral muscle in Birds:—

Pectoral Muscle in Man.

Length of Section.	Mean Thickness.	Cross Section.
2.20 inches.	0.31 inches.	0.682 sq. in.
4.25 ,,	0.16 ,,	0.680 ,,
11.50 ,,	0.06 ,,	0.690 ,,

There is always a loss of Work done in a triangular muscle, which is due to the peculiar arrangement of its fibres in a fan-like form; and, in this respect, muscles with radiating fibres differ remarkably from the penniform muscles, in which, as I have shown, the Work *done* is always equal to the *inherent* Work. This may be proved as follows: let P, P', P'', &c., be any forces, and let p, p', p'', &c., be the displacements (estimated in the directions of the forces) undergone by their points of application; and let R, r, be the resultant of the forces, and the displacement of its point of application, estimated in the direction of the resultant itself;—then we have, by Lagrange's principle of virtual velocities,

$$Rr = Pp + P'p' + P''p'' + \&c. \qquad (32)$$

Let AOB, Fig. 33, be a triangular fan-shaped muscle, and let OY be the bisector of the vertical angle, and its length

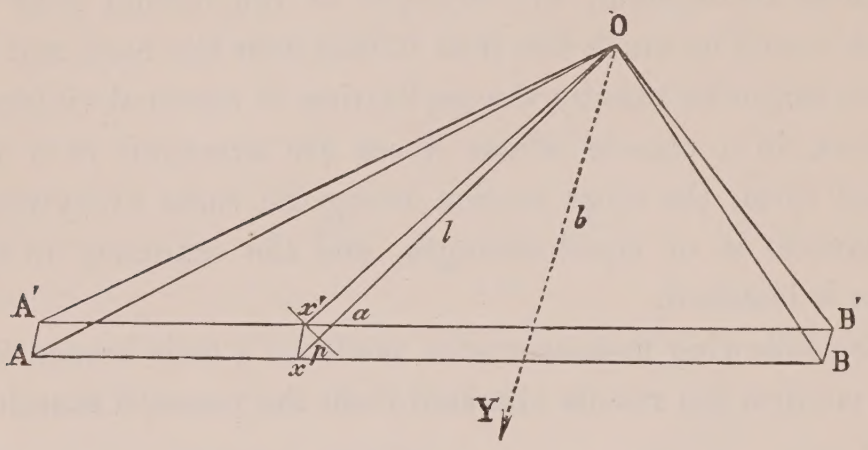


Fig. 33.

equal b. Since the line of insertion, AB, remains constant in length, when the muscle contracts, the line AB will move, parallel to itself in the direction YO, into a position $A'B'$, and the fibre OY will contract to its full amount of inherent work, through a space $AA' = BB' = \delta b$; if we take any other fibre, $Ox = l$, and draw xx' parallel to OY, and with O as centre describe the circle $x'p$, then xp will be the space through which the fibre l contracts; but this space is less than $xa = \delta l$, which bears to l the same ratio that $AA' = \delta b$ bears to b. Hence the fibre l, in contracting, only gives out the work

$$f d\theta \times xp = f \delta b \,.\, \cos\, \theta d\theta,$$

instead of the work

$$f \delta l \, d\theta,$$

which is inherent in it.

If we suppose (Fig. 34) AB and $A'B'$ of indefinite length, and draw xy parallel to OY in such a manner that $Ox = Oy$; it is easy to see that all the fibres lying to the left of xy will be shortened by the transference of AB to $A'B'$, while all the fibres

lying to the right of xy will be lengthened. No Work at all is done by the fibre Ox, or those to the right; and of the

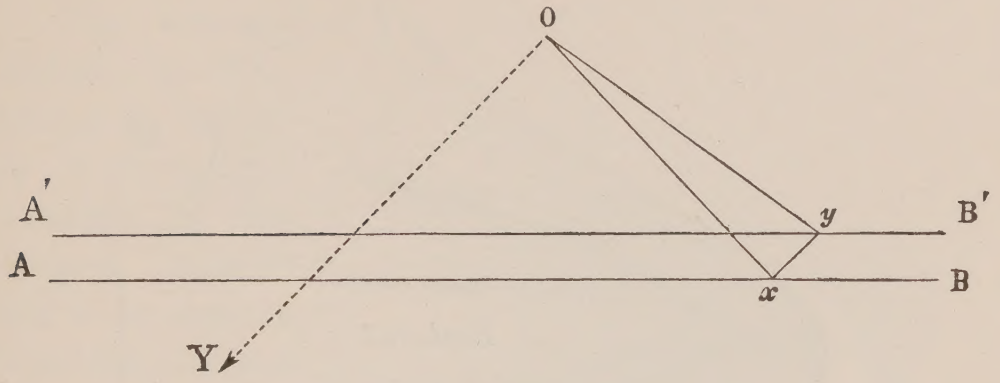


Fig. 34.

fibres to the left of xy, all do some Work; but the Work done by them, with the single exception of the fibre OY, is less than their inherent Work. Taking the sum of all the Work done by each fibre, we have—

$$\text{Work done} = \int_{-\theta}^{+\theta} f\delta b \cos\theta d\theta = 2f\delta b \sin\theta,$$

a result that coincides with the Work done, found directly from the resultant of all the forces (30).

When a fan-shaped muscle, instead of being triangular in shape, has a curved base, as in the pectoral fin of the Angel Shark, Fig. 29, the Work done by the muscle will still be represented by double the perpendicular let fall from the foot of the bisector of the vertical angle upon the side; and the work inherent in the muscle will depend upon the integral of $ld\theta$, taken all round the curve.

If this curve be a portion of a circle whose centre coincides with the vertex of the radiating fan-shaped fibres, the comparison of the inherent Work and Work done becomes very simple. This case is nearly realized in the great pectoral

muscle of man, Fig. 35. Let 2θ be the angle contained between the extreme radiating fibres. Then, if l be the length

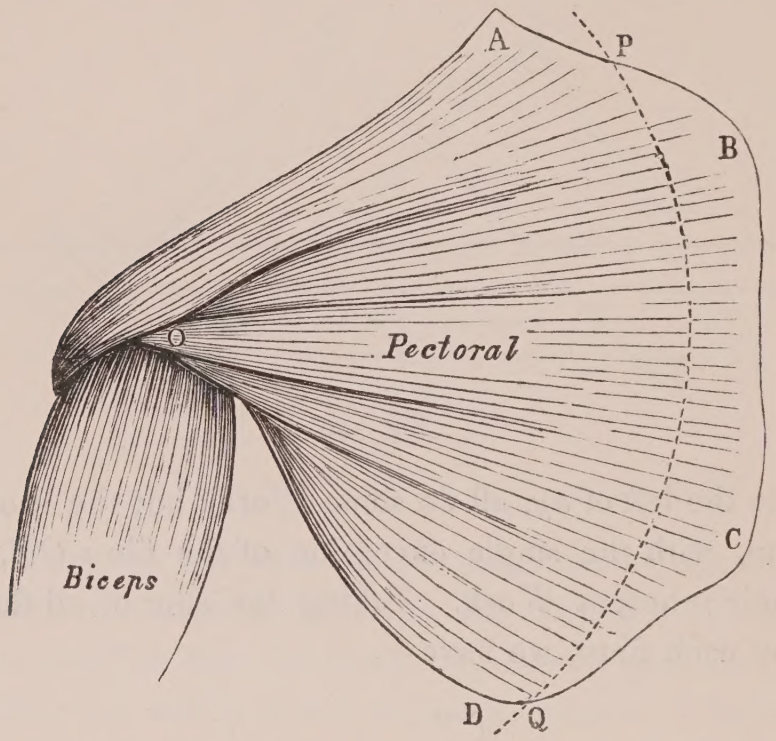


Fig. 35.

of each fibre,

$$\text{Inherent Work} = 2f\delta l \times \theta.$$
$$\text{Work done} = 2f\delta l \sin\theta.$$

and

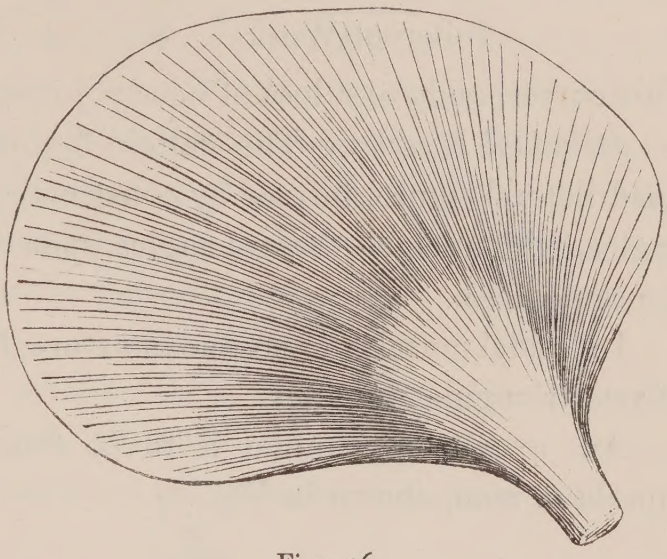
$$\frac{\text{Work done}}{\text{Inherent Work}} = \frac{\sin\theta}{\theta}. \qquad (33)$$

In the great pectoral muscle of man, Fig. 35, a circle PQ, described with a point O on the humerus as centre, coincides appromixately with the origin of the fibres, $ABCD$; and since the angle 2θ, contained between the extreme radiating fibres, is 90°; equation (33) becomes for the pectoral of man,

$$\frac{\text{Work done}}{\text{Inherent Work}} = \frac{\sin\theta}{\theta} = \frac{2\sqrt{2}}{\pi} = 0.9003.$$

Hence there is 10 per cent. of Work lost, in the pectoral muscle, by means of the fan-shaped arrangement of its fibres.

The *temporal* muscle in man, shown in Fig. 36, is a remarkable example of fan-shaped muscle; and there must be a considerable loss of work occasioned by its shape; this loss of work would be very difficult, however, to calculate, in consequence of the elliptical form of the line of origin of the fibres.



Fi g. 36.

The *latissimus dorsi* in man may be considered as a muscle whose fibres take origin from an arc of a circle, whose centre is the point of insertion in the arm bone; the mean angle contained by the extreme fibres is 35°—hence we find, from equation (33),

$$\frac{\text{Work done}}{\text{Inherent Work}} = \frac{\sin\theta}{\theta} = \frac{3007}{3054} = 0.98.$$

This shows a loss of only 2 per cent., caused by the fan-shaped arrangement of the fibres, whereas the pectoral muscle loses 10 per cent., in consequence of the greater angle contained between its extreme fibres.

The radiating fibres of the muscle of the *iris* give us an example of the maximum loss of Work caused by fan-shaped arrangement. We have seen that when the angle between the extreme fibres increases from 35° to 90°, the loss of Work

done increases from 2 to 10 per cent. When the fibres radiate through a semicircle, we have

$$\frac{\text{Work done}}{\text{Inherent Work}} = \frac{\sin\theta}{\theta} = \frac{2}{\pi} = 0.636.$$

This corresponds to a loss of nearly 36 per cent.

Deltoidal Muscles.—The deltoid figure of geometers is a quadrilateral figure, formed by constructing two isosceles triangles, with unequal vertical angles upon the opposite sides of the same base.

Deltoidal muscles are found in nature, formed by the union of two triangular muscles.

An excellent example is to be found in the *trapezius* muscle of man, shown in Fig. 37.

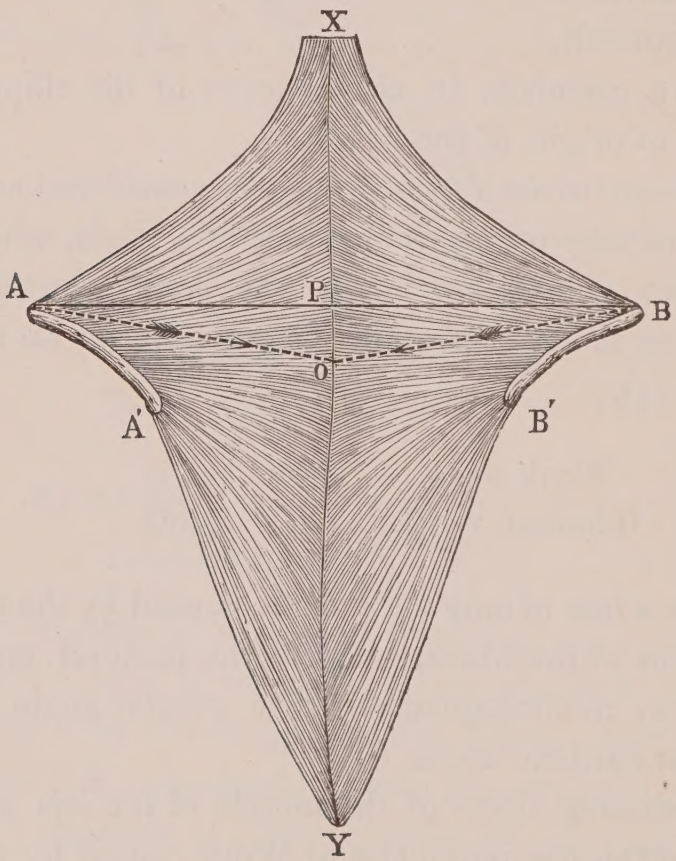


Fig. 37.

The whole figure $XAYB$ is a deltoid, and is formed by the union of the two triangular muscles XAY and XBY; the

spines of the scapulæ are shown at AA' and at BB' ; the line AB joining the vertices of the two triangular muscles is necessarily perpendicular to the spinal column XY; and the bisectors of the vertical angles (AO and BO) are found to fall behind and below the line AB; so that when the whole trapezius is in action, the resultant forces draw the scapulæ in the directions AO and BO, shown by the arrows. This action of the *trapezius* muscle is usefully employed, for expanding the chest, in the kalisthenic exercises used by young ladies.

Using the same notation as in the case of the *biceps femoris* muscle of the lion, I have found the following results from measurements made on the *trapezius* muscle of man :

XY = 16 inches.	$\beta = 83° = AOX.$
XA = 9 „	$\theta = 47°30' = OAX.$
YA = 12 „	p = 7 in.
	q = 5·16 in.

From these data we find, from equations (30) and (31),

$$\frac{\text{Work done}}{\text{Inherent work}} = \frac{2q}{p \log_e \frac{\cot \frac{1}{2}(\beta - \theta)}{\cot \frac{1}{2}(\beta + \theta)}} = \frac{10.32}{7 \log_e \left(\frac{\text{cot. } 17° 45'}{\text{cot. } 65° 15'} \right)} = 0.7703 \qquad 34$$

The *work done* by the *trapezius* muscle is, therefore, only 77 per cent. of its inherent work, and there is a loss of 23 per cent. due to the fan-shaped arrangement of its fibres.

The *pectoral* muscles of many animals, especially the rodents, form, by union of the muscles of the right and left sides, admirable examples of *Deltoidal* muscles.

In fig. 38 I have drawn the pectoral muscles of the common squirrel, O, O being the humeral insertions of the two

muscles, and AB the sternum; OX is the bisector of the angle BOA.

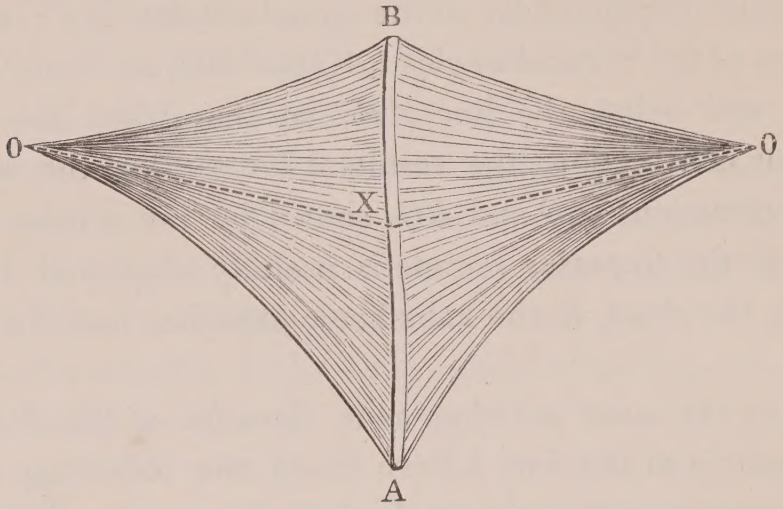


Fig. 38.

Using the same notation as in equation (34), we have

$$\beta = 80°,\ p = 27,$$
$$\theta = 30°,\ q = 10,$$

and, finally,

$$\frac{2q}{p \log_e \dfrac{\cot (25°)}{\cot (55°)}} = 0.66.$$

This shows a loss of 34 per cent., arising from the fan-shaped arrangement of the fibres.

Quadrilateral Muscles.—It frequently happens that the fibres composing a triangular muscle do not run on completely to a narrow insertion, but are inserted into a bone or fascia, lying in the same plane with the line of origin. Such a muscle may be called a quadrilateral muscle, and its origin and insertion, like the base of the triangular muscle, may become curved lines instead of right lines. The typical quadrilateral muscle is shown in fig. 39, in which AB, $A'B'$ are the origin and insertion of the muscular fibres. Produce the extreme fibres AA', BB', to meet at O, and draw $OX'X$

bisecting the angle AOB; and from X, X', let fall XY $X'Y'$, perpendicular to OA.

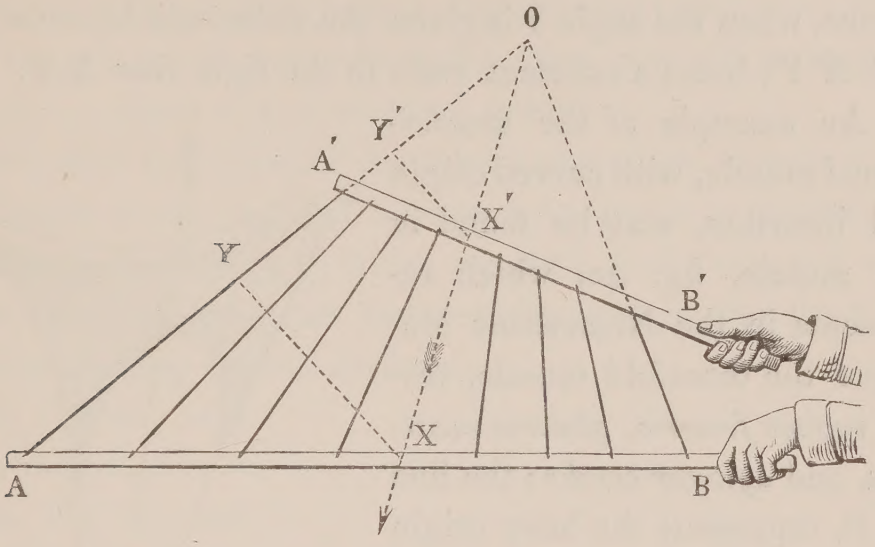


Fig. 39.

The quadrilateral muscle may be regarded as a triangular muscle, AOB, from which a portion $A'OB'$ has been cut off, and the properties of the quadrilateral muscle may be readily deduced from those of triangular muscles.

I have shown (equation 30) that the work done by the triangular muscle AOB, is proportional to the perpendicular XY; but the work done by the part cut off, $A'OB'$, is proportional to the perpendicular $X'Y'$; hence the total work done by a quadrilateral muscle is proportional to the difference of XY and $X'Y'$. Hence, whatever be the shape of two quadrilateral muscles, if the difference between XY and $X'Y'$ be the same in both, the two muscles will do the same work in contracting: also, if the inclination, AOB between the extreme fibres of two quadrilateral muscles, be the same, the work done by each will be proportional, simply, to the length of the right line XX'; for

$$XY = XO.\ \sin\theta$$
$$X'Y' = X'O.\ \sin\theta$$

and, therefore

$$XY - X'Y' = (XO - X'O).\ \sin\theta = XX'.\ \sin\theta$$

Hence, when the angle θ is given, the difference between XY and $X'Y'$, bears a constant ratio to the right line XX'.

An example of the *Quadrilateral* muscle, with curved origin and insertion, may be found in the muscle, fig. 40, which represents in the Nemestrine Macaque the threefold muscle, *tensor vaginæ femoris*, *glutæus maximus*, and *agitator caudæ;* the line *Is*, *Il*, represents the bony origin of this threefold muscle, and the lower curved line represents its insertion into the fascia of the thigh; its fibres converge towards a certain point *O*, the angle between the extreme fibres being 40°; and the resultant force of all the fibres lies in the line *OX*, which bisects the angle between the extreme fibres.

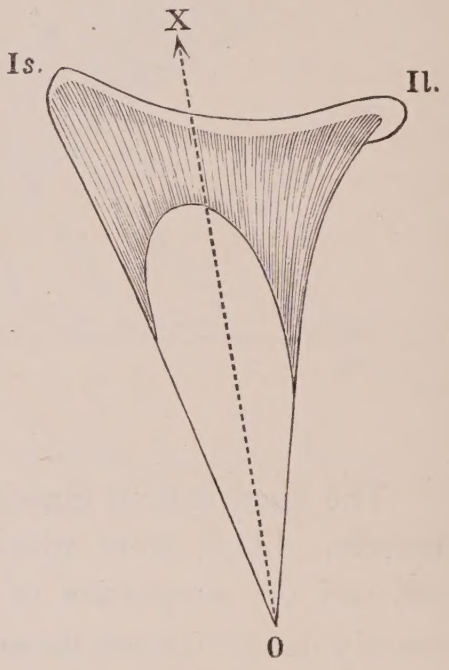


Fig. 40.

Quadrilateral muscles, whose lines of origin and insertion lie, strictly, in the same place, are very rare in nature, and the whole group of muscles, called quadrilateral, tends to merge itself in the more complex form of muscle, presently to be described, in which the fibres, although rectilinear, form, when taken together, ruled or skew surfaces.

Sphincter Muscles, whose Fibres are curvilinear and parallel, lying in the same Plane.

In a sphincter muscle, each fibre forms a closed curve, circular, or elliptical in shape; and the collection of such

fibres forms the entire muscle. Sphincter muscles are usually placed round an orifice, which is closed by the contraction of the fibres. These two systems of fibres counteract each other, and by means of their alternate action the orifice may be opened or closed to any required extent.

We shall first consider the theory of a single circular fibre, but before doing so, it is necessary to prove an elementary theorem in Geometry, of which we shall make use, not only in the theory of sphincter muscles, but also in the theory of muscles forming curved surfaces.

If from any point O, outside a circle, Fig. 41, two tangents be drawn and the chord joining their points of contact, and the diameter passing through O be also drawn:

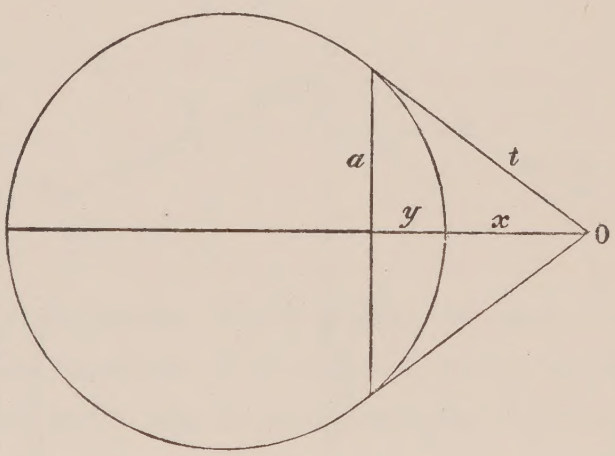


Fig. 41.

Then let

r = radius,
t = tangent,
a = half the chord,
x = intercept between circle and point O,
y = intercept between circle and chord :—

I say that when the point O approaches indefinitely near the circle, the intercepts x and y will become equal to each other.

For

$$t^2 = (2r + x)\, x \text{ (Euc. iii. 36).}$$
$$a^2 = (2r - y)\, y \text{ (Euc. iii. 35).}$$

Subtracting these equations from each other, we have

$$2r\,(x - y) + (x^2 + y^2) = t^2 - a^2.$$

But t and a become equal to each other, and to half the arc contained between the tangents when O indefinitely approaches the circle; hence the foregoing equation becomes

$$2r\,(x - y) + (x^2 + y^2) = 0;$$

and, as x and y are indefinitely small, the term $x^2 + y^2$ disappears, being of the second order, and the equation becomes, finally,

$$2r\,(x - y) = 0$$

or

$$x = y \qquad Q.\ E.\ D.$$

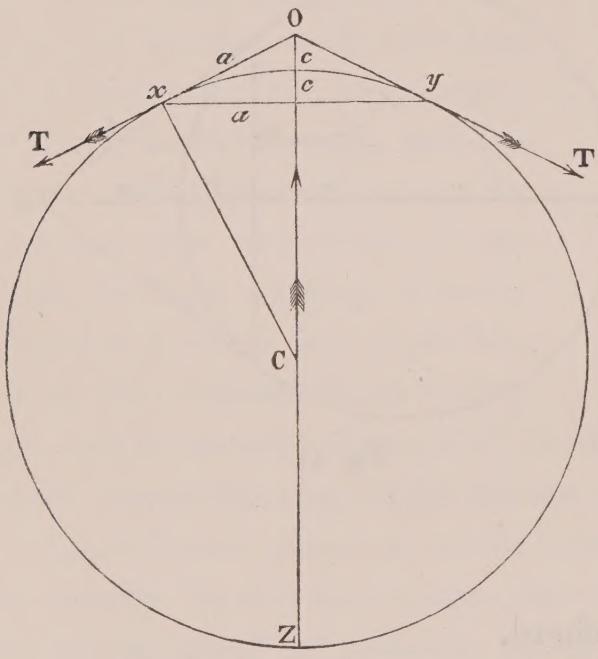


Fig. 42.

Let xyZ, Fig. 42, represent a circular fibre of a sphincter muscle, and let any point O, indefinitely near the fibre, be taken, and from this point imagine two tangents drawn to the circle. When the whole fibre contracts there will be produced a tangential strain at each point of the circle; and it is necessary for equilibrium that forces perpendicular to the circle shall act at each point. We are required to find the relation between the system of tangential and perpendicular forces.

It must be remembered that the sphincter fibres can produce tangential strains only, for a muscle can only contract; and that the perpendicular forces, which equilibrate the tangential strains must be produced by a distinct set of radiating fibres.

At the points of contact x, y, the tangential strains act in the directions, Ox and Oy, and they must be equilibrated by a perpendicular force acting in the direction CO, passing through the centre. Let T represent the tangential strain at each point x and y; the resultant of these two forces is

$$R = 2T \cos TOC.$$

Since the arc xy, is small, we have the intercepts between O and the circle, and between the circle and chord xy, equal to each other; let the tangent, arc, or semichord be called a, and the intercept c; then we have

$$\cos TOC = \frac{2c}{a} = \frac{a}{\rho},$$

and therefore

$$R = 2T. \frac{a}{\rho}$$

If P represent the perpendicular force acting on each unit length of the circle, the resultant R just found must equilibrate the force P applied to the whole arc intercepted between x and y; and hence we have

$$R = 2Pa.$$

Equating these two values of R, we obtain,

$$2Pa = 2T \frac{a}{\rho}$$

and, finally,

$$P = \frac{T}{\rho} \qquad (35)$$

Hence, if the tangential strain be given, the perpendicular force will vary inversely as the radius of the circle, or directly as its curvature; and if the perpendicular force be given, the tangential strain will vary directly as the radius of the circle.

The first of these cases is one of common occurrence, as the sphincter is often called upon to contract upon a resisting body; in this case, the cross section of the sphincter ring measures the total amount of the tangential contraction possible, and the body filling the orifice will be compressed by a force which increases as the aperture of the orifice diminishes.

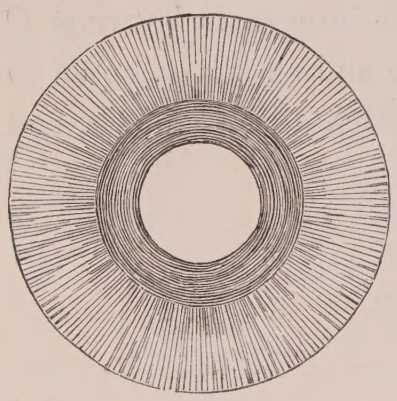


Fig. 43.

The best example of sphincter and radiating fibres to be found in nature occurs in the muscles of the *iris*, which close and open the aperture of the pupil of the eye. These are shown in Fig. 43.

The *sphincter* muscle forms a flat narrow band on the posterior surface of the inner rim of the *iris* close to the pupil, and is about $\frac{1}{40}$th of an inch wide.

The radiating fibres, which are antagonistic to the sphincter fibres, commence at the ciliary, or outer margin of the iris, and are directed inwards, in bundles, towards the pupil; at the pupil itself, they blend with the sphincter, some of the radiating fibres reaching as far as its inner margin.

Under the influence of these opposing muscles, the diameter of the pupil ranges from $\frac{1}{3}$rd to $\frac{1}{20}$th of an inch. At any given diameter of the pupil, equilibrium exists between the forces T and P, developed by the sphincter and radiating fibres; the ratio between these forces, however, does not remain constant, but ranges from 3 to 20, according to the diameter of the pupil.

The *sphincter* muscle of the iris derives its motor supply from the third pair of nerves, while the *dilator* muscle is supplied from the sympathetic; thus, the opposing forces of these antagonist muscles are instructed to act from distinct nerve centres.

The relations between the fibres of these muscles (as revealed by the microscope) are shown in Fig. 44, which represents a portion of these fibres, in the iris of the Albino-rabbit, (Kölliker) magnified 350 times.

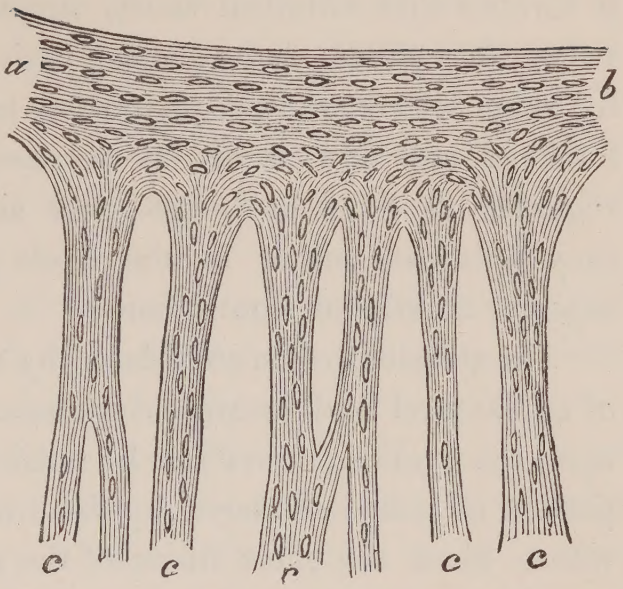


Fig. 44.

In this figure *ab* represents the sphincter, and *cccc* the radiating fibres.

The arrangement of these fibres shows in a clear manner how admirably they are constructed to react upon each other in the manner already described.

The sphincter muscle of the eye forms, also, an excellent example of this class of muscles.

This muscle *OO* (*orbicularis palpebrarum*) is shown in Fig. 45;

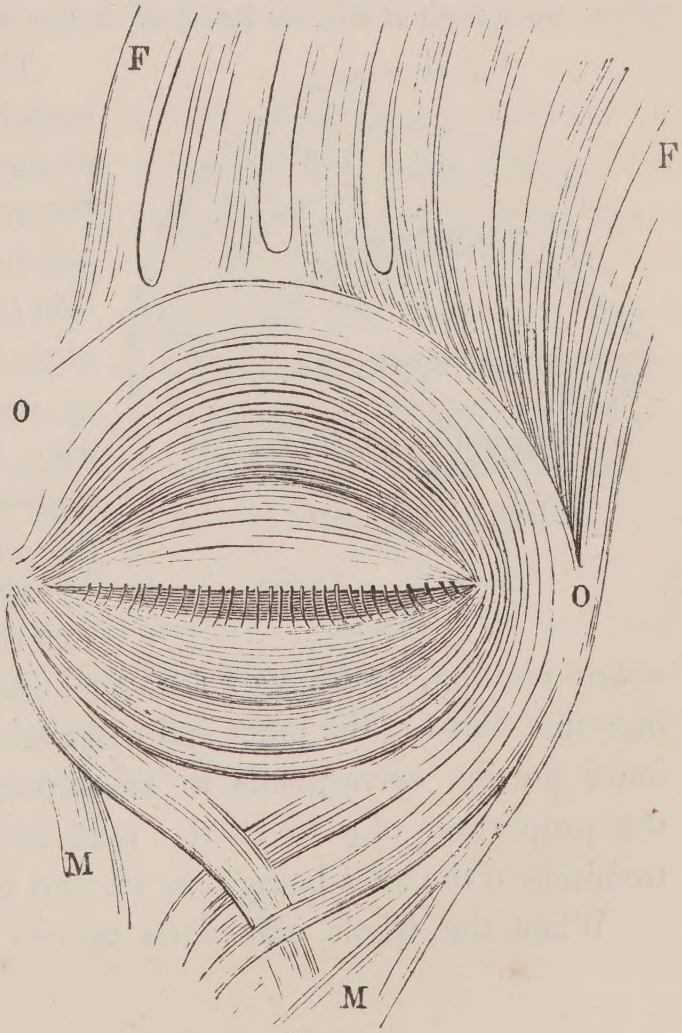


Fig. 45.

it forms a thin elliptical sheet, surrounding the fissure between the eyelids, and covering their surface, and spreading for some distance outwards on the temple, upwards on the forehead, and downwards on the cheek. The fibres may be regarded as fixed at *I*, the inner angle of the eye, where they terminate either in the *tendo palpebrarum* or in the superior maxilla or frontal bone.

The muscle is often considered by anatomists as composed of an external and internal ring, according as the fibres lie upon the eyelids (*ciliaris*) or lie outside them. The external portion is bounded above by the frontal muscle, *FF*, with whose fibres the outer fibres of the sphincter are blended; and the lower portion of the external sphincter sends off cross slips, by which it blends itself with the malar muscles, *MM*.

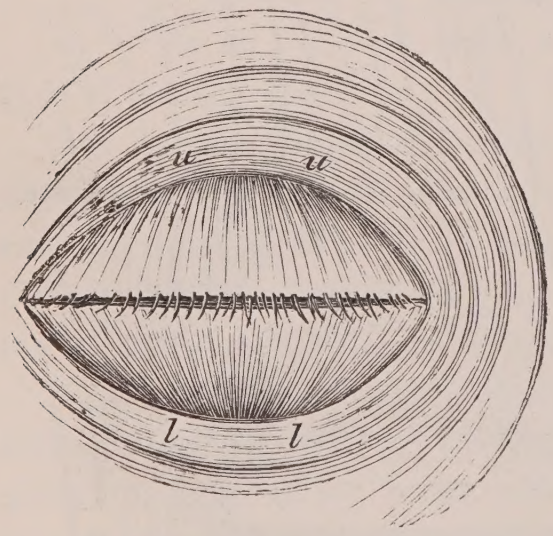


Fig. 46.

The frontal and malar muscles are the antagonists of the external portion of the sphincter; but the antagonists of the inner portion (*ciliaris*) are the radiating fibres shown in Fig. 46, named *levator palpebrarum*, *uu*, *levator palpebræ superioris*, *ll*, a portion of the *orbicularis* muscle.

The outer fibres of the external sphincter are nearly circular, and have a radius of curvature equal to 0.89 in.; the fibres of the inner sphincter are elliptical, and their outer portion corresponds to an ellipse, whose axes are in the proportion of 9 to 5—the radii of curvature at the extremities of the axes being 0.25 in. and 0.45 in. respectively.

When the whole *orbicularis* muscle contracts, its outer

fibres, being circular, contract symetrically towards the centre ; but the elliptical fibres, being attached to the *tendo palpebrarum* on the inner side, on contracting draw with them the entire orbicular muscle towards the nose; the force with which this is done varying as the curvature of the fibres at the outer side of the eye.

The motor nerves of the *orbicularis* muscle are derived from the temporal branch of the facial nerve (7th pair). These same nerves furnish the motor supply of the *frontalis* muscle. The other antagonist of the *orbicularis* muscle, viz., *levator palpebræ*, derives its motor supply from the 3rd pair of nerves (*motores oculorum*). There would thus seem to be a more complete antagonism between the *levatores palpebrarum* and the *orbicularis internus* than between the *frontalis* and *orbicularis externus.*

The mouth furnishes us with an extremely complex but most beautiful example of sphincter and radiating muscles, whose varied action produces an infinite variety in the shape and expression of the lips.

The orbicular muscle of the mouth, like that of the eye, is elliptical, and constitutes the only force available for closing the lips. The radiate muscles of the mouth are eighteen in number, nine at each side, placed symetrically in relation to those of the opposite side. The nine muscles at each side of the mouth are arranged in two sheets, an outer and an inner; the outer sheet consists of six muscles, and the inner sheet consists of three muscles, which overlap the muscles of the outer sheet at the angles of the mouth, where the curvature of the orbicular muscle is greatest.

These muscles are—

Outer Layer.

1. *Quadratus labii superioris* (*levator labii superioris*).
2. *Orbicularis malaris.*
3. *Zygomatici* (*major et minor*).
4. *Risorius.*
5. *Subcutaneus colli* (*platysma*).
6. *Triangularis oris* (*depressor anguli oris*).

Inner Layer.

7. *Quadratus labii inferioris* (*quadratus menti*).
8. *Buccinator.*
9. *Caninus* (*levator anguli oris*).

In fig. 47 I have drawn the directions of the foregoing muscles in the relative position which they occupy with respect to the sphincter muscle.

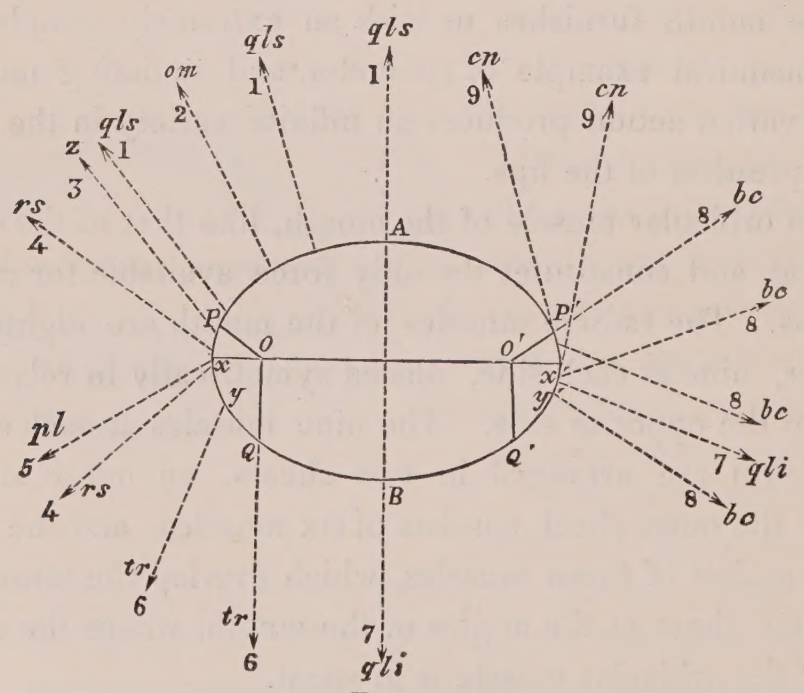


Fig. 47.

The latter muscle is represented by the ellipse, whose foci are O and O'; on the left-hand side of the ellipse are shown the directions of the outer layer of radiating muscles, and

(to avoid confusion) the deep layer of radiating fibres is shown on the right-hand side of the diagram.

1. 1. 1. *Quadratus labii superioris.* The fibres of this sheet of muscle act perpendicularly to the *orbicularis oris*, from the extremity of the minor axis to a line passing through the focus, and making an angle of 40° with that axis. This sheet of muscle acts upon three-fourths of the elliptic quadrant, and is the only radiate muscle that counteracts the *orbicularis* in this part of its extent.

2. *Orbicularis malaris.* This little muscular slip is a portion of the *orbicularis oculi*, on the outer side, and passes into the upper lip, with the fibres of the *quadratus labii superioris.* Its mechanical action is feeble, but identical with that of the fibres of the latter muscle. Its chief use is not mechanical but expressional, for it correlates the action of the sphincter of the eye with that of the lifter of the upper lip; as in the highly natural gesture of winking with one eye, and lifting at the same time the corner of the corresponding lip. It was fully described by *Santorini*, who calls it *perpetuus lacertulus ab imo orbiculari.*

3. The *Zygomatic* muscular slips occupy the position shown in the diagram, and require no further description, as they are well known.

4. 4. *Risorius.* This muscle forms a thin sheet of radiating fibres, extending from 35° above the axis major to 40° below the same; in conjunction with the deeper seated *buccinator*, it is intended to control the action of the *orbicularis* at the extremity of its major axis, where its curvature is greatest, and consequently the perpendicular force also required to be greatest.

5. The *Platysma* of the neck (*subcutaneus colli*) sends a sheet of fibres to the angle of the mouth, which act in a direction 30° below the major axis.

6. 6. The *triangularis oris* (*depressor anguli oris*) forms a sheet, radiating from 68° below the major axis to a line, *OQ*, very nearly at right angles to the same axis.

7. 7. *Quadratus labii inferioris* (*quadratus menti*) forms a radiating sheet whose fibres are perpendicular to those of the *orbicularis*, through four-fifths of the lower elliptic quadrant, from the minor axis to a direction 22° below the major axis, near the corner of the mouth.

To understand exactly the relative positions of the deep and superficial layers, we should imagine the ellipse folded on its minor axis, so as to bring all the radiating lines on the same side of the mouth.

This muscle acts alone on the *orbicularis*, from *B* to *Q*, where the curvature of the sphincter is least, and is assisted by other muscles, as it approaches the angle of the mouth where the curvature is greater. It corresponds precisely in its action and position with the *quadratus labii superioris* on the upper side of the ellipse.

8. 8. 8. 8. *Buccinator.* This important muscle consists of a flat, thin, strong set of fibres, in contact with the mucous membrane, and forming a considerable portion of the wall of the mouth ; its fibres range from 34° above to 34° below the major axis, the upper fibres passing into the upper lip, the lower fibres into the lower lip, and the central fibres decussating, so as to pass, as shown in the figure, into the opposite lip.

This whole arrangement gives a maximum of perpendicular force along the major axis, at the extremity of which the cuvature is greatest. Its upper fibres *OP'* or *OP*, mark the boundary of overlapping of the deep and superficial muscles on the upper side of the mouth. Thus the whole ellipse may be divided into regions as follows :—

PAP' and *QBQ'* counteract *orbicularis* by one sheet of muscles only.
PQ and *P'Q'* counteract *orbicularis* by two sheets of muscle.

9. 9. The *canine* muscle (*levator anguli oris*) acts in a manner different from all the preceding; its fibres are tangential to the ellipse in the neighbourhood of the extremity of the axis major, and therefore aid the orbicularis rather than counteract it. It may be regarded as a force in reserve to aid the *orbicularis* where its curvature is greatest, and where the most powerful apparatus of radiate muscles is placed.

If we examine the outer sheet of muscles, we find that they form a continuous radiating muscle through APQ, with one break between the lower border of the *risorius* (4) and the *triangularis oris* (6), below the angle of the mouth; this break is marked xy on the figure, and is feebly supplied by a few fibres of the *platysma* and *triangularis*. If we now examine the inner sheet of muscles, we find them to form a continuous sheet through $BQ'P'$ (or BQP), with the exception of a space, $x'y'$, through which the *buccinator* and *quadratus labii inferioris* overlap each other, forming a double sheet.

It is interesting to observe that this space, $x'y'$, overlaps the space xy, when the figure is folded on its minor axis, so that the overlapping of the sheets of inner fibres compensates for the deficiency of the outer sheet.

The nineteen muscles of the mouth are all supplied with motor power from the seventh pair, and their constantly varied action produces the wonderful play of the lips, required in articulation; as the curvature of each portion of the *orbicularis* changes, corresponding changes must take place in the action of the radiating fibres that counteract it, the result being an endless variety in the expression of the most expressive feature of the face.

This consideration furnishes an argument against the use of the beard, which not only takes away from its wearer the

power of showing his feelings by the play of his lips, but also confers upon him the dishonest advantage of concealing their expression by means of a barricade of red, black, or grey bristles.

The sphincter muscles already described—viz., of the iris, eye, and mouth, have a corresponding set of antagonist radiating muscles, intended to open the apertures round which the sphincters are placed; but there are other sphincters surrounding apertures, which it is not necessary to open by muscular agency. Such sphincters are the anal and vaginal sphincters, which close their respective apertures, and are not interfered with by any antagonist muscles.

The *anal sphincter* consists of two distinct parts; an internal part, composed of circular fibres of involuntary muscle, and an external part, composed of elliptical fibres, which are under the control of the will. The fibres of the elliptical sphincter take originfrom behind, in the *os coccygis*, and passing round the *anus*, unite in front of it, at the central point, (*tendo perinæi*); from this point again, in the female, the fibres of the *vaginal sphincter* proceed, opening out to surround the vaginal orifice and vestibule, and are finally inserted on the *corpora cavernosa clitoridis*. The two sphincters thus form a muscular sheet, resembling the figure 8, and may act together or separately.

The *levator ani*, which forms the pelvic diaphragm, does not act as an antagonist to the anal sphincter; on the contrary its action aids the sphincter, for those of its fibres which blend with the sphincter fibres are tangential to the curve of the latter (like the fibres of the *canine* muscle in the mouth), and therefore assist instead of opposing the constricting action of the sphincter muscle.

During parturition, the *levator ani* and *sphincter ani* cooperate to close the aperture of the anus, so as to allow the

abdominal muscles to exert their full force in expelling the contents of the uterus and vagina.

Muscular Fibres forming curved Surfaces.

In all the preceding cases, we have supposed the sheet formed by the muscular fibres to form a plane, but this condition is rarely fulfilled with mathematical exactness. The muscular sheets, in general, form surfaces more or less curved, and it becomes necessary to take into consideration the various curvatures of the several surfaces formed by the sheets of the muscular fibres.

Surfaces are divisible into two classes—viz., those whose curvatures at any point are in the same direction, and those whose curvatures at any point have opposite directions. The first class of surfaces may be called convex, or ellipsoidal surfaces; the second class may be called skew surfaces.*

If we imagine a convex surface, like an egg, uterus, or bladder, and at any point taken on the surface draw a line perpendicular to it, and imagine a plane passing through this line to turn round upon it, this plane in each of its positions will intersect the surface in a certain curve. In the convex surface, the curvatures of all these curves at the point in question will be all convex or all concave together, in the same direction.

If we imagine a skew surface, like a saddle, or dice box, and draw a perpendicular to it at any point, the plane revolving round this perpendicular will intersect the surface in certain curves, some of which are convex, and some concave; and the convex curves are separated from the concave curves by two lines intersecting upon the surface at the given point;

* This term is here used in a sense slightly different from its meaning when employed by geometers.

so that the surface possesses a convex curvature in one of the regions formed by those intersecting lines, and possesses a concave curvature in the other region. In the direction of the intersecting lines themselves, the surface has no curvature at all, for these lines divide the convex from the concave portion of the surface.

The lines drawn upon skew surfaces, along which the surface is not curved, may become right lines, and, as the surface itself is the aggregate of all the lines composing it, we may have a skew curved surface composed altogether of rectilinear fibres. This case frequently occurs in animal mechanics, the rectilinear generators being the actual fibres of the muscles.

As the ellipsoidal muscles are more easily understood than the skew muscles (notwithstanding that they are composed altogether of curved and not rectilinear fibres), I shall commence my description of curved muscular surfaces, with that of ellipsoidal muscles, and proceed afterwards to show the properties of skew muscles.

Ellipsoidal Muscles.—If a tangent plane be drawn at any point of a convex or ellipsoidal surface, it will touch the surface in one point only; and if a plane be drawn parallel to the tangent plane and very close to it, so as to intersect the surface, this plane will cut the surface in an ellipse, or circle, according as the principal curvatures at the point in question are unequal or equal. If tangent planes be now drawn to the surface along the elliptic curve of intersection, they will form a tangent cone to the surface, and this cone will be a circular or elliptic cone, according as the curve of intersection is a circle or ellipse.*

* The curve of intersection with a surface made by a plane very near and parallel to a tangent plane is called by geometers the *indicatrix* curve.

In the simplest case of ellipsoidal muscles, that in which the curvature at a given point coincides with that of a certain sphere, let O be the vertex of the tangent cone, Fig. 48, OT, OT being sides of the cone touching the surface at t, t.

Let C be the centre of the circular *indicatrix*, $txyt$, and join OC. This joining line pierces the surface at the point S, at which point, a tangent plane would be parallel to the plane

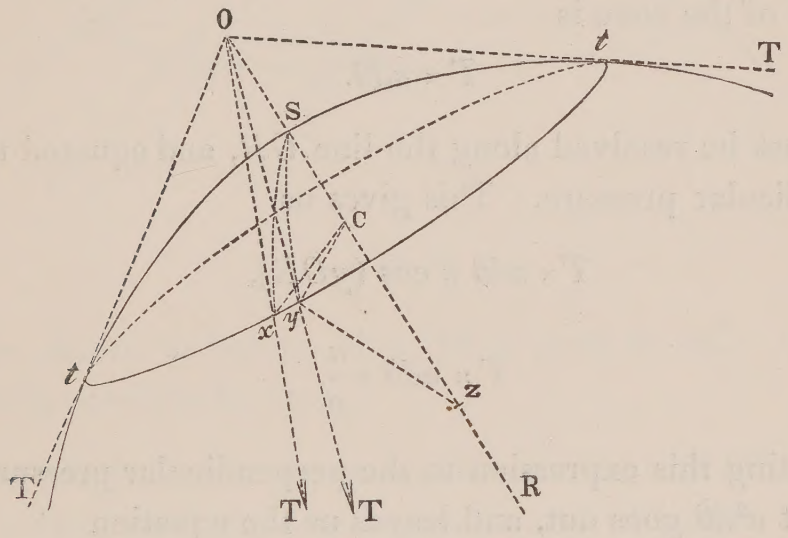


Fig. 48.

of the *indicatrix*; and it can be shown, as in the case of the circle p. 198, that the line OC is bisected at the point S, because O is indefinitely near the surface.

Let the surface now be supposed to be composed of muscular fibres, and to contain a fluid reacting perpendicularly to the surface, it is required to determine the conditions of equilibrium. Let P denote the force acting perpendicularly on the unit surface, and let T denote the tangential strain caused by the muscular contraction, acting along the side of the tangent cone, upon the unit of length of the *indicatrix*. It is necessary and sufficient for equilibrium, that the perpendicular pressure upon any element xyC, of the *indicatrix*, shall equilibrate the corresponding muscular strain acting in the element

of the tangent cone xyO. Let z be the centre of the circle Sx or Sy, and ρ its radius. Since O is very near the surface, Oy and Cy are to be considered equal; let each of them be called a.

The perpendicular pressure acting on the element Cxy is

$$P \times \tfrac{1}{2}a^2 d\theta$$

where $d\theta$ is the angle xCy; and the tangential strain in the element of the cone is

$$T \times a d\theta.$$

This must be resolved along the line OR, and equated to the perpendicular pressure. This gives us

$$T \times a d\theta \times \cos(yOR),$$

or

$$T \times a d\theta \times \frac{a}{\rho}.$$

Equating this expression to the perpendicular pressure, we find that $a^2 d\theta$ goes out, and leaves us the equation

$$P = \frac{2T}{\rho}.$$

This is the well known equation which is used by architects in the problem of the equilibrium of a spherical dome.

When the surface has a spherical curvature at any point, as in the preceding case, the tangential strain T, is the same in every azimuth at each point; but when the curvature becomes ellipsoidal, and the *indicatrix* is an ellipse, the tangential strain T is different in different azimuths, while the perpendicular pressure P remains constant.

Let us now examine the general case.

Let S (Fig. 49) be the point of the ellipsoidal surface in question; and let $txyt$ be the *indicatrix* ellipse formed by a

plane parallel to the tangent plane at S; and let $Otxyt$ be the tangent cone drawn to the surface along the *indicatrix*. It

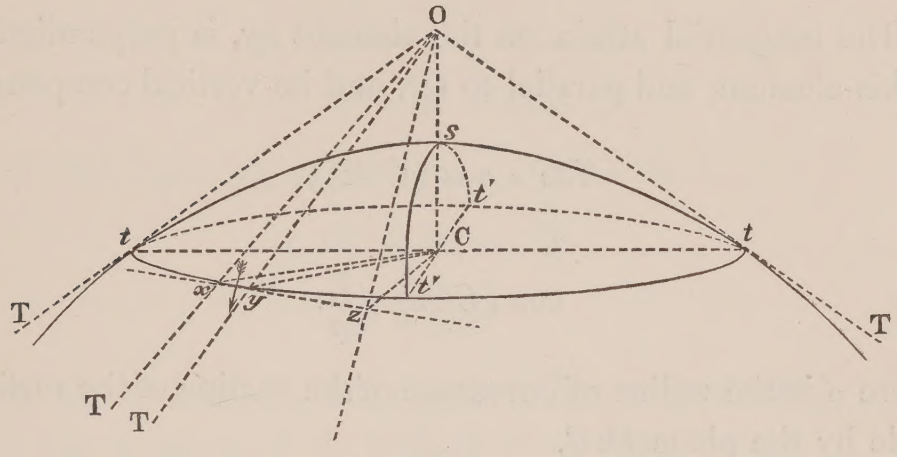


Fig. 49.

can be shown as before, that OS is equal to SC. Let the following notation be used.

P = perpendicular pressure acting on the unit of surface.
T = tangential strain acting on the unit of length.
$a = Ct$ = semiaxis major of *indicatrix*.
$b = Ct'$ = semiaxis minor of *indicatrix*.
$c = CS = SO$.
$r = Cx$, Cy, = radius vector of *indicatrix* at any point.

Let xy be any element of the *indicatrix*, and produce the tangent xyz to meet a perpendicular Oz, let fall upon it from the vertex of the cone; and draw Cz. The lines Oz and Cz are both perpendicular to the tangent to the *indicatrix*, and are to be considered equal to each other, because the point O is very near the surface. Let the element xy be called ds, and let Oz or Cz be called p. It is necessary and sufficient for equilibrium that the pressure P acting upon the elementary triangle Cxy, shall be equilibrated by the vertical component of the tangential strain acting on the element xy.

The perpendicular pressure acting on the elementary triangle Cxy is

$$P \times \tfrac{1}{2}\, pds.$$

The tangential strain on the element xy, is perpendicular to this element, and parallel to Oz, and its vertical component is

$$Tds \times \cos\,(COz);$$

but

$$\cos\,(COz) = \frac{p}{\rho}.$$

where ρ is the radius of curvature of the section of the surface made by the plane OzC.

Hence, finally,

$$P \times \tfrac{1}{2}\, pds = Tds \times \frac{p}{\rho}.$$

or

$$P = \frac{2T}{\rho}. \qquad (36)$$

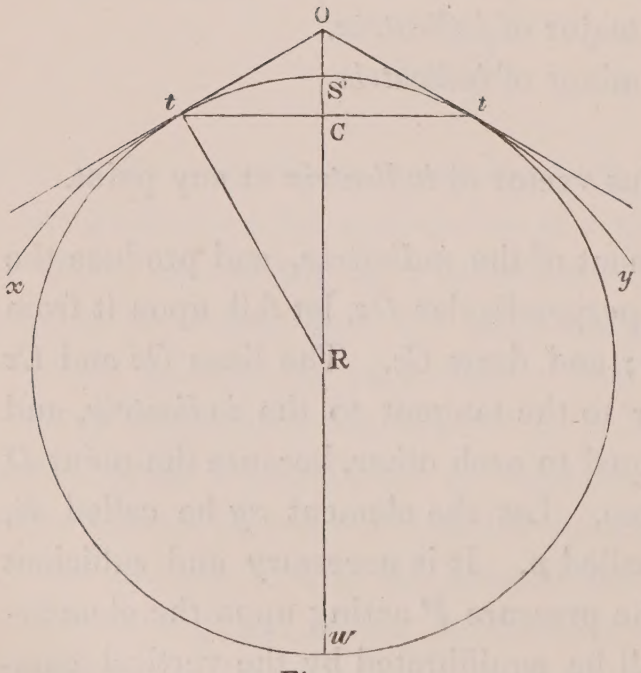


Fig. 50.

In using this equation it must be constantly borne in mind that ρ is the radius of curvature of the section passing through the perpendicular let fall on the tangent to the *indicatrix*, and not that of the section passing through the radius vector.

Let $xtSty$, Fig. 50, represent this

section, and *wz Stw* the circle of curvature at the point *S*, having its centre at *R*; then

$$c = OS = SC$$
$$p = zO = zC$$
$$\rho = Rz = RC.$$

In the right angled triangle *RzO*, we have

$$(zO)^2 = RO \times CO;$$

or

$$p^2 = 2c\rho. \qquad (37)$$

Since $2c$, the height of the vertex of the tangent cone, is the same for all points of the *indicatrix*, we see that

$$\rho \propto p^2,$$

and since (because *P* is constant), by equation (36) *T* varies as ρ, it must vary as p^2. Hence, we have the following elegant theorem.

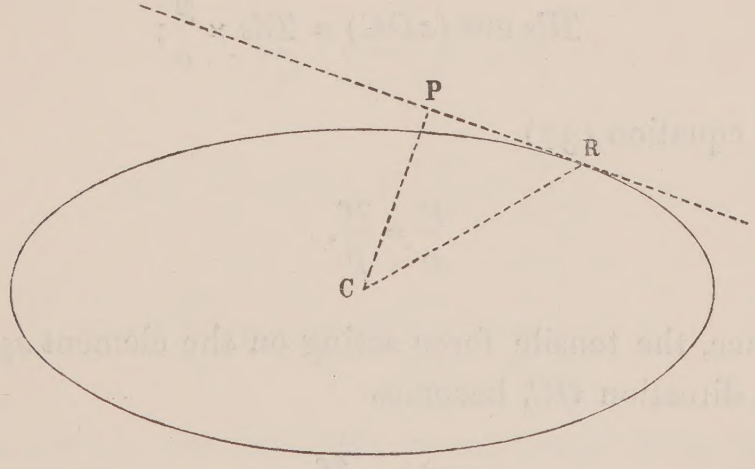


Fig. 51.

Let Fig. 51 represent the *indicatrix* ellipse at any point of a convex surface; let *C* be its centre, and *CR*, *CP*, the radius vector and perpendicular on tangent at any point. The surface forms an elliptic dome, of very small height, standing

upon the indicatrix. It follows from the foregoing, *that the tension of the surface in any azimuth CR, varies directly as the square of CP.*

Hence the greatest and least tensions will be at the extremities of the major and minor axes of the ellipse, and will be proportional to the squares of those axes.

The preceding problem, as is evident, contains the solution of the conditions necessary in architecture to construct an ellipsoidal dome.

It is sometimes convenient in problems in Animal Mechanics to use the mean tension of the muscular wall, without regard to its variation as the azimuth varies; and for this purpose, we must take the mean of all the tensions in every possible azimuth.

The tensile strain at each point of the indicatrix is Tds, acting parallel to the line Oz, Fig. 49, and perpendicular to xy. The component of this tension, in the direction OC, is

$$Tds \cos (zOC) = Tds \times \frac{p}{\rho};$$

but, by equation (37)

$$\frac{p}{\rho} = \frac{2c}{p},$$

and hence, the tensile force acting on the element xy, in the vertical direction OC, becomes

$$Tds \times \frac{2c}{p}$$

We must now regard T as a constant, having the mean value of the tensions taken in every azimuth, and integrate the foregoing expression all round the ellipse.

To do this we must find the value of

$$\int \frac{ds}{p}$$

in an ellipse.

Assume

$$x = a \sin \phi$$

$$y = b \cos \phi,$$

then

$$ds = d\phi \sqrt{a^2 \cos^2 \phi + b^2 \sin^2 \phi}$$

$$p = \frac{ab}{\sqrt{a^2 \cos^2 \phi + b^2 \sin^2 \phi}}.$$

$$\frac{ds}{p} = \frac{(a^2 \cos^2 \phi + b^2 \sin^2 \phi)\, d\phi}{ab}.$$

The integral of this expression taken all round the ellipse is

$$\int \frac{ds}{p} = \pi \frac{a^2 + b^2}{ab};$$

and

$$2cT \int \frac{ds}{p} = \pi ab \times T \left(\frac{2c}{a^2} + \frac{2c}{b^2} \right);$$

or finally

$$2cT \int \frac{ds}{p} = \pi ab \times T \left(\frac{1}{\rho_1} + \frac{1}{\rho_2} \right),$$

where ρ_1 and ρ_2 are the radii of curvature of the surface along the axes of the ellipse.

The vertical component of all the elementary tensions, just found, must be now equated to the sum of all the perpendicular pressures acting on the *indicatrix*, which is

$$P \times \int \tfrac{1}{2} p ds = P \times \pi ab.$$

Hence we obtain

$$P \times \pi ab = \pi ab \times T \left(\frac{1}{\rho_1} + \frac{1}{\rho_2} \right),$$

or

$$P = T \left(\frac{1}{\rho_1} + \frac{1}{\rho_2} \right),$$

which is Lagrange's theorem, already employed at page 152.

The first example of ellipsoidal muscles that I shall discuss is the case of the abdominal muscles, already considered in a general way in treating (p. 151) of the muscular force employed in parturition. The principal curvatures of the abdomen at the navel (p. 161) were shown to correspond to circles of curvature of 22.7 in. and 12.6 in. diameter respectively; and the tensile strain in the walls of the abdomen is produced by the simultaneous action of the eight abdominal sheets of muscle. According to the theory just developed, the tensile strains produced by all the muscles in the vertical and transverse directions at the navel, should be in the proportion of the diameters of curvature in the same directions; and the axes of the *indicatrix* ellipse at the navel should be in the proportion of the square roots of those diameters.

A severe test may thus be applied to the theory, and to the measurements of the forces produced by the abdominal muscles. For the purpose of applying this test I shall make use of the muscles of subject No. 2 (female), p. 160, the measurements of whom corresponded nearly with the mean of all.

The four muscles at each side of the central line have the following mean thickness:—

1. *Rectus abdominis*, ... 0.29 inches.
2. *Obliquus externus*, ... 0.25 ,,
3. *Obliquus internus*, ... 0.17 ,,
4. *Transversalis*, ... 0.15 ,,

Of these muscles, the *rectus abdominis* produces a tensile strain altogether in the vertical direction, and the *transversalis* causes a tension in the horizontal direction only. These two tensions are nearly in the proportion of 2 to 1, which approximates the ratio of the diameters of curvature. If therefore these muscles, alone, were in action, the theory would be fully vindicated. The *oblique* muscles, however, produce, each of them, a vertical and horizontal component strain at the navel, which must be added to the former, to determine the total strain acting in the walls of the abdomen in those directions. Let us now proceed to calculate these components, and test the entire theory.

Let *O*, *E* (Fig. 52), represent the *obliquus externus* muscle, and *SNP* the *linea alba*, *S* denoting the end of the sternum, *N* the navel, and *P* the pubes;—Il. being the crest of the ilium. The line *AB* represents the inner limit of the muscular fibres, where they become inserted into the fascial sheet enclosing the *rectus abdominis*. The fibres of the external oblique muscle are approximately parallel to each other, and constitute a *prismatic* muscle spread in a sheet over a convex surface,—and the prolongations of its fibres to the linea alba make angles of 35° with that line. Each of the

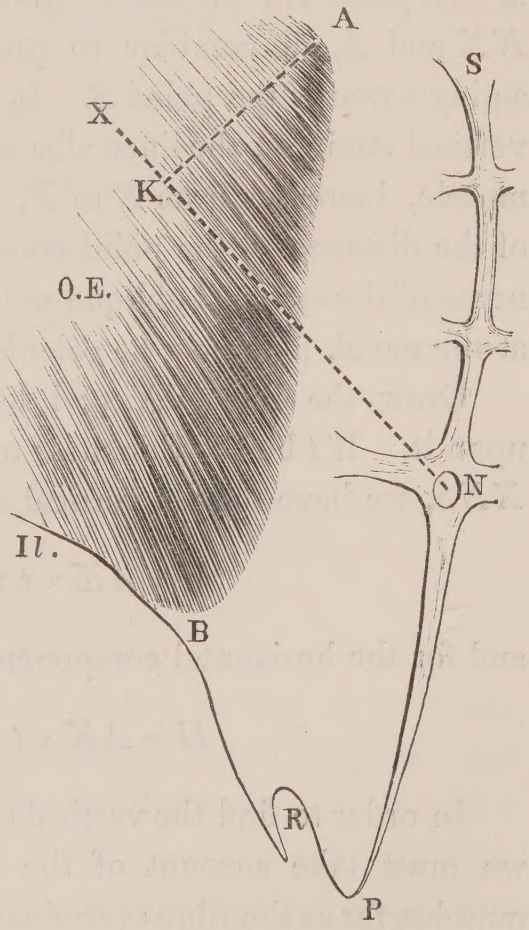


Fig. 52.

muscular fibres, when combined with the corresponding fibre of the other side, produces a vertical strain acting from P towards S, and a horizontal strain which is equilibrated by the equal and opposite strain on the other side of the abdomen;—and there is a remarkable difference between these two tensile strains, for the transverse strain is constant along the entire linea alba, while the vertical strain is greatest at P, and diminishes from P to S. Take, for example, the fibres NX, passing through the navel; the muscular fibres between NX and B, produce along the line PN vertical components acting towards N, and consequently they produce no strain whatever at the point N; on the contrary, the fibres lying between NX and A, all combine to produce a vertical strain at N, acting towards the point S. It is therefore evident that the vertical strain on the linea alba caused by the external oblique muscle, increases from S to P, and varies in the proportion of the distance of the point considered from S. The components of the external oblique muscle, acting upon a linear inch at the navel, may now be calculated.

Draw the line NX, and let fall the perpendicular AK upon it. If t be the thickness of the muscle, and ϕ the angle XNS, we have for the vertical component,

$$V = AK \times t \times 2 \cos \phi ;$$

and for the horizontal component,

$$H = AK \times t \times \sin \phi .$$

In order to find the vertical tensile strain, on a linear inch, we must take account of the width of the *recti abdominis* muscles, for as the fibres terminate in the line AB, at the outer border of the *recti* muscles, their vertical component is to be regarded as distributed along the entire breadth of those muscles.

Hence, if w denote the width of the *recti* muscles, and p the length of AK, the vertical component per linear inch will be

$$V = \frac{p \times t \times 2 \cos \phi}{w} \tag{38}$$

The horizontal strain corresponding to a linear inch of the linea alba will not, of course, be the strain corresponding to a width (AK) of muscle, but is produced by a width equal to $\sin \phi$; substituting this width for AK, we find

$$H = t \times \sin^2 \phi. \tag{39}$$

I find, by careful measurements, the following values to be substituted in the foregoing expressions—

$$AK = 4.25 \text{ in.}$$

$$t = 0.25 \text{ ,,}$$

$$\phi = 35^\circ$$

$$w = 5{,}41 \text{ in.}$$

Hence we obtain, finally,

$$V = \frac{4.25 \times 0.25 \times 2 \cos 35^\circ}{5.41} = 0.321$$

$$H = 0.25 \times \sin^2 35^\circ = 0.082.$$

The *internal oblique* muscle is shown in Fig. (53), where P, N, $Il.$, denote the pubes, navel, and crest of ilium, and AB the insertion of the muscle into the outer border of the *rectus* sheath. The *internal oblique* muscle is not prismatic, but quadrilateral, and its action is more complicated than that of the *external oblique*. The upper fibres give a vertical component which draws the *linea alba*, downwards, towards the

navel, and the lower fibres give a vertical component which draws the linea alba, upwards, towards the navel; hence, neither sets of fibres can produce any tensile strain at the navel itself. The only portion of the muscle that causes a vertical tension at the navel, is the portion contained between the horizontal fibres XY and the line NX, which is the direction of the fibres passing through the navel itself. This portion of the muscle, and the corresponding portion of the similar muscle, at the other side, will behave as a triangular muscle, whose resultant ZX bisects the angle NXY, and is equal to (p. 181)

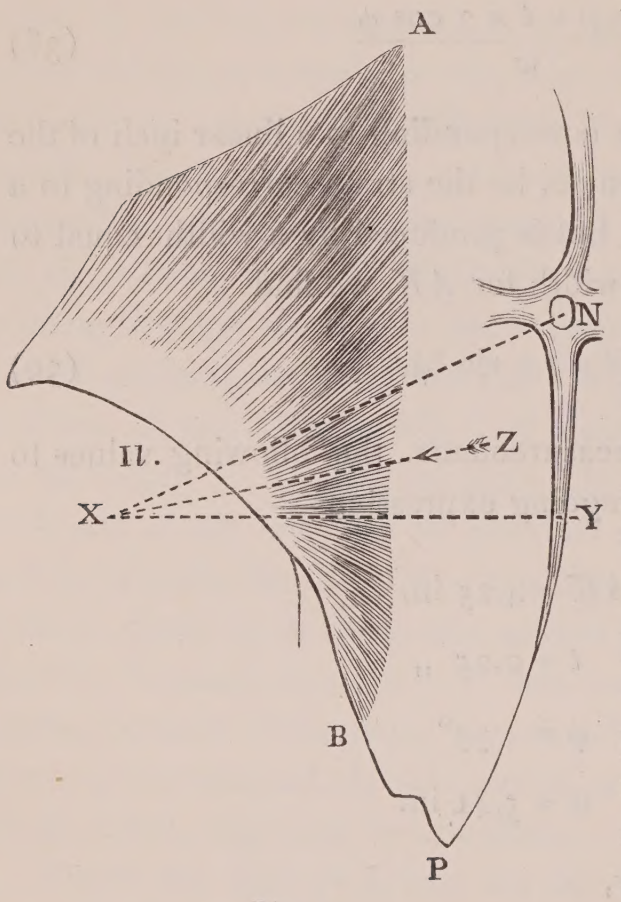


Fig. 53

$$2 f \sin \theta,$$

where θ denotes half the angle NXY, and f denotes the cross section of the muscle. Hence the vertical resultant of the internal oblique muscle at both sides is

$$2 f \sin \theta \times 2 \cos \phi,$$

where ϕ denotes the angle made with the linea alba by the line ZX, and is the complement of θ. This resultant must be divided by the total width of the *recti* muscles, in order to find

the strain per linear inch, as in the case of the external oblique. Hence we find, finally,

$$V = \frac{2f \sin \theta \times 2 \cos \phi}{w} = \frac{2f \text{ versin } 2\theta}{w}. \qquad (40)$$

In this equation, f is equal to the product of the cross section of the muscle, multiplied by its width enclosed between the lines NX and XY.

Cross section of *internal oblique* = 0.17 in.
Width of do.. . . . . = 2.97 ,,
$\theta = 90° - \phi$ . . . . . = 13°
Width of *recti* muscles, . . . = 5.41 in.

Hence we have

$$V = \frac{2 \times 0.17 \times 2.97 \times \text{versin } 26°}{5.41} = 0.019.$$

The transverse strain produced at the navel by the internal oblique muscle, depends on the direction of the fibres in its neighbourhood only, and is found by the same formula as the transverse strain of the external oblique muscle, viz.—

$$H = t \times \sin^2 \phi, \qquad (41)$$

which gives us, for the transverse strain, per linear inch at the navel,

$$H = 0.17 \times \sin^2 64° = 0.137.$$

Adding together the vertical and transverse tensile strains produced by all the muscles at the navel, we find—

Vertical Tension due to Abdominal Muscles.

Rectus abdominis, ...	0.290
Obliquus externus, ...	0.321
Obliquus internus, ...	0.019
Transversalis, ...	0.000
	0.630

Transverse Tension due to Abdominal Muscles.

Rectus abdominis, ...	0.000
Obliquus externus, ...	0.082
Obliquus internus, ...	0.137
Transversalis, ...	0.150
	0.369

From the preceding results, it follows that the transverse strain at the navel is 58.57 per cent. of the longitudinal strain—

$$\frac{\text{Transverse strain}}{\text{Longitudinal strain}} = \frac{369 \times 100}{630} = 58.57 \text{ per cent.}$$

At p. 161, I have given the measurements of the abdominal curvatures of three young men, when the abdomen was distended to its utmost extent. From the mean of all three we obtain, in the case of young men, the result that the diameter of the transverse circle of curvature is 55.58 per cent. of the diameter of the longitudinal circle of curvature—

$$\frac{\text{Transverse diameter}}{\text{Longitudinal diameter}} = \frac{12633 \times 100}{22727} = 55.57 \text{ per cent.}$$

As it was desirable to ascertain the amount of curvature of the abdomen, at the time when its muscles are used in the act of parturition, I obtained the following measurements of five healthy women, taken at the commencement of labour. I instructed the head midwife of Sir Patrick Dun's Hospital Maternity to measure for me, with tape, the following quantities:—

1. Height of woman.
2. Girth transversely round navel.
3. Length of curved arc from the end of the sternum to the top of the symphysis pubis.

In well formed skeletons the distance from the top of the symphysis to the end of the sternum is $\frac{17}{67}$ths of the height of the body, so that from the measurement of the height, I could find the length of the chord of the longitudinal arc, whose length was measured directly.

If θ denote half the arc subtended by this chord, we have the following equations, to determine ρ_1 and ρ_2, the radii of longitudinal and transverse curvatures :

$$\text{chord} = \frac{17}{67} \times \text{height} \qquad (a)$$

$$\frac{\text{arc}}{\text{chord}} = \frac{\theta}{\sin\theta} \qquad (b)$$

$$\frac{\text{chord}}{\sin\theta} = 2\rho_1 \qquad (c)$$

$$\frac{\text{girth}}{\pi} = 2\rho_2 \qquad (d)$$

The measurements made were the following :—

Parturient Women.

	Height.	Longitudinal Arc.	Transverse Girth.
No. 1.	60 in.	16.8 in.	39.3 in.
No. 2.	63.6 ,,	19.0 ,,	31.2 ,,
No. 3.	61.5 ,,	16.8 ,,	44.3 ,,
No. 4.	59.8 ,,	15.7 ,,	46.2 ,,
No. 5.	69.9 ,,	21.8 ,,	39.6 ,,

From these data, the following table was calculated by means of the foregoing equations :—

Parturient Women

No.	Chord.	Semiangle $=\theta$	Diameter of longitudinal Curvature $=2\rho_1$	Diameter of transverse Curvature $=2\rho_2$	Ratio of Curvatures $=100\times\frac{\rho_2}{\rho_1}$
1.	15.22 in.	43°42′	22.03 in.	12.51 in.	56.8
2.	16.13 ,,	55°48′	19.50 ,,	9.93 ,,	50.9
3.	15.60 ,,	37°36′	25.51 ,,	14.10 ,,	55.3
4.	15.17 ,,	26°00′	34.60 ,,	14.71 ,,	57.6
5.	17.73 ,,	62°24′	20.01 ,,	12.63 ,,	63.1
				Mean	**56.74** per cent.

The mean of the ratios in the last column gives us, for parturient women,

$$\frac{\text{Transverse diameter}}{\text{Longitudinal diameter}} = 56.74 \text{ per cent.}$$

This ratio agrees very well with the 55·57 per cent., found from the measurement of the abdominal curvatures in young men, and they both agree with the corresponding proportion of transverse and longitudinal tensile strains (58.57 per cent.), deduced by calculation, from the dissection of a healthy young woman, who had borne several children.

If we compare together the mean curvatures of the several cases, we shall find, as we might expect, a considerable difference depending on height and general configuration of body. But it is worthy of remark, that the mean value of all the curvatures differs very little from the mean maximum curvature of the abdomen, voluntarily distended:—

Mean Curvature of Abdomen in Parturient Women.

No.	Longitudinal Diameter $= 2\rho_1$	Transverse Diameter $= 2\rho_2$	Mean Curvature $= \frac{1}{\rho_1} + \frac{1}{\rho_2}$
1.	22.03 in.	12.51 in.	$\frac{1}{3.987}$
2.	19.50 ,,	9.93 ,,	$\frac{1}{3.289}$
3.	25.51 ,,	14.10 ,,	$\frac{1}{4.540}$
4.	34.60 ,,	14.71 ,,	$\frac{1}{5.161}$
5.	20.01 ,,	12.63 ,,	$\frac{1}{3.872}$

Mean $\frac{1}{4.1738}$

This mean curvature differs little from the mean curvature of the voluntarily distended abdomen of young men—viz.: $\frac{1}{4.0596}$, given in p. 162.

We can calculate, from the vertical and transverse tensions of the abdominal muscles at the navel, given in p. 223–4, and from the curvatures just given, the perpendicular pressure per square inch, caused by the action of those muscles, in each of the preceding cases. For, by equation (36), we have

$$P = \frac{2T}{\rho};$$

which becomes, on adding 50 per cent. to the measurement of the dead muscles,

$$P = \frac{4 \times 0.945}{2\rho_1},$$

for the longitudinal strains; and

$$P = \frac{4 \times 0.553}{2\rho_2},$$

for the transverse strains. Substituting in succession, the values of $2\rho_1$ and $2\rho_2$, we obtain—

Pressure producible by Abdominal Muscles acting at navel, in Parturient Women.

	Deduced from Longitudinal Curvature.	Deduced from Transverse Curvature.
No. 1.	17.59 lbs. ℔ sq. in.	18.13 lbs. ℔ sq. in.
No. 2.	19.88 „ „	22.84 „ „
No. 3.	15.19 „ „	16.09 „ „
No. 4.	11.20 „ „	15.42 „ „
No. 5.	19.37 „ „	17.96 „ „
Mean	**16.646 lbs.**	**18.088 lbs.**

These pressures are produced by the abdominal muscles, acting at the navel, under the conditions of curvature, proper to each case; and are only about half as great as the pressure at navel, obtained by direct experiment (page 163); but they do not represent the maximum pressures producible by the abdominal muscles. In order to show this, let us calculate the tensile strains produced at each point of the linea alba.

The longitudinal and transverse strains, produced by the *recti* and *transversalis* muscles, are constant at each point of the linea alba; but this is not the case with the *oblique* muscles. The vertical and horizontal strains of the *external oblique* are, as shown by equations (38) and (39),

$$V = \frac{2t \cos \phi}{w} \times p$$

$$H = t \sin^2 \phi.$$

In these equations, everything is constant except p, the perpendicular thickness of the portion of the external oblique

above the point of the linea alba under consideration. The value of p, at the navel, is 4.25 in. (p. 221), and it increases as we go downwards towards the pubes, where it becomes 8.11 inches. While, therefore, the horizontal component of the external oblique is everywhere the same, its vertical component reaches a maximum just above the pubes.

This maximum is represented by

$$\frac{8.11 \times 0.25 \times 2 \cos 35^\circ}{5.41} = 0.614$$

The vertical and transverse strains produced by the *internal oblique*, are both variable, as shown by equations (40) and (41)—

$$V = \frac{2f \operatorname{versin} 2\theta}{w}$$

$$H = t \sin^2 \phi.$$

In these equations, f, θ, and ϕ are all variable, and in the following manner:—Let the point be taken on the linea alba (about half way between the navel and the pubes), where the fibres of the muscle become horizontal. At this point there is no vertical strain, and the horizontal strain is a maximum, because ϕ becomes a right angle; the transverse strain at other points varies as the square of sin ϕ, but is always less than at the point named. With respect to the vertical strain at any point, it is thus found: $2\theta = 90^\circ - \phi$ is the angle made with the horizontal line, by the fibres passing towards the point, and f varies, as the base of the triangular muscle formed by these fibres and the horizontal fibres. Now, it is evident that the versine of 2θ and the base of the triangular muscle both increase as we pass along the linea alba, in either direction, above or below the central point between the navel and pubes, where the whole action of the internal oblique is transverse; so that the vertical strain produced by this muscle has two maxima, one near the pubes, and the other near the ster-

num. The measurements for these two points are,

$$\text{At pubes, } 2\theta = 65^\circ\ f = 3.12 \times 0.17,$$
$$\text{At sternum, } 2\theta = 45^\circ\ f = 7.16 \times 0.17.$$

Hence, we find the vertical strain producible at the sternum, by the internal oblique muscle, to be

$$\frac{2 \times 7.16 \times 0.17 \times \text{versin } 45^\circ}{5.41} = 0.135.$$

and the vertical strain, producible at the pubes, by the same muscle,

$$\frac{2 \times 3.12 \times 0.17 \times \text{versin } 65^\circ}{5.41} = 0.113.$$

Let us now add together the vertical strains at the point in the linea alba, just above the pubes, and we shall find,

Rectus abdominis, ...	0.290
Obliquus externus, ...	0.614
Obliquus internus, ...	0.113
Transversalis, ...	0.000
	1.017

We have seen that the vertical tension, producible by the muscles acting at the navel, was represented by 0.630 only; so that in order to calculate the maximum hydrostatical pressure, producible inside the abdomen by the abdominal muscles acting to the greatest advantage, we should increase the numbers given in the table, page 228, in the proportion of 1017 to 630.

If we take the largest number in the table, corresponding to No. 2, we have

$$\text{maximum pressure inside abdomen} = 22.84 \times \frac{1017}{630} = 36.87 \text{ lbs. per sq. in.}$$

This result agrees well with that given, from direct experiment (p. 163), as the maximum expulsive force of the abdominal nuscles—viz. 38.47 lbs.

It is interesting to observe that the tension in the vertical direction produced by the external oblique muscle, varies as the distance of the point of the linea alba under consideration from the sternum, which is precisely the law according to which the pressure of the contents of the abdomen, regarded as semi-fluid, would vary; viz., as the depth of the fluid measured from its surface.

The *Diaphragm*, which forms the upper surface of the abdominal cavity, furnishes an example of a convex ellipsoidal muscle; it is used continually in respiration, and occasionally in the expulsion of the contents of the abdomen. Its force is considerable, although not equal to the maximum effort of the abdominal muscles; and being an involuntary muscle not under our control, when it provokes the abdominal muscles to take on a violent reflex action, as in coughing, its effects are sometimes dangerous, as, for example, in cases of hernia. It is, therefore, important, not merely in a scientific but also in a practical point of view, to know the amount of hydrostatical pressure it is capable of producing upon the contents of the abdomen.

From measurements made by me, I infer that the mean curvatures of the diaphragm, when most convex, correspond with circles of 5.601 inches, and 5.109 inches in diameter at the two sides of the body, and that the mean thickness of its muscular walls is 0.18 in. Adding 50 per cent. to the latter measurement as before, and taking the mean diameter of curvature, we obtain from equation (36)

$$P = \frac{2T}{\rho} = \frac{4T}{2\rho};$$

and from the coefficient of muscular force (p. 69),

$$K = 102.55 \text{ lbs.}$$

$$P = \frac{4 \times 0.27 \times 102.55}{5.355} = 20.68 \text{ lbs. per sq. in.}$$

This result is comparable with the expulsive force of the abdominal muscles acting at the navel, although inferior to their maximum force exerted near the pubes.

Muscular Skew Surfaces.

The last and most complex muscular surfaces to be considered are those corresponding with skew surfaces in geometry. The simplest form of skew muscle occurring in nature is that shown in Fig. 54, where AB and $A'B'$ are two

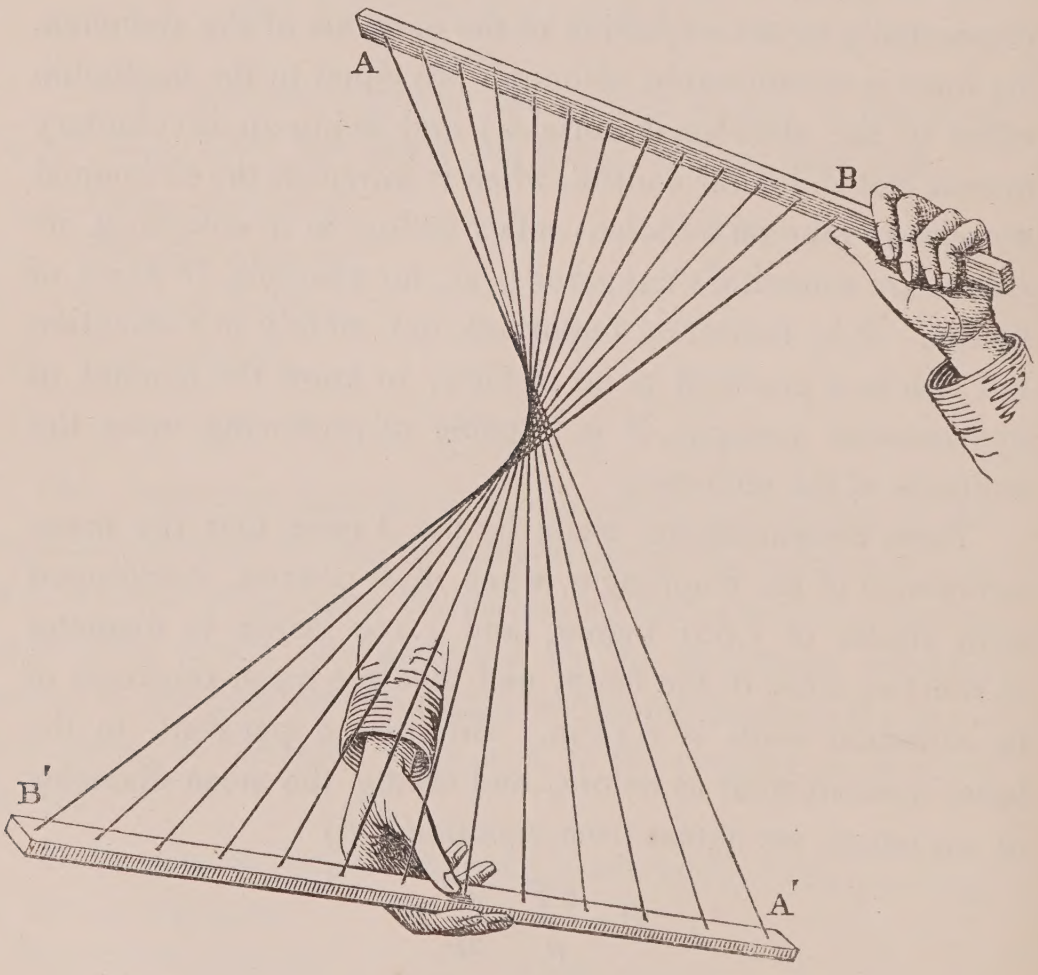


Fig. 54

bones not lying in the same plane, and each divided into an equal number of parts, such that the *anharmonic* ratio of any four points on one bone is equal to the *anharmonic* ratio of the

four corresponding points on the other bone; and the corresponding points on the two bones are supposed to be joined by straight muscular fibres capable of contraction.

The surface formed by these muscular fibres is a skew surface, of a simple kind, called the *Hyperboloid of one sheet.* It may be regarded as formed from the *Quadrilateral* muscle, p. 195, by placing the origin and insertion of the muscular fibres AB and $A'B'$, in different planes, so as to form a skew quadrilateral instead of a plane quadrilateral.

A skew surface, of which the *Hyperboloid of one sheet* is one of the simplest examples, may be readily imagined from the shape of a saddle; and at each point it possesses curvatures which are not all in one direction, as in the convex or ellipsoidal surface. If a tangent plane be drawn to the skew surface at any point, this plane, instead of touching the surface at a single point, will cut the surface along two intersecting right lines, which divide the surface at the point of contact into two regions, in one of which the curvatures are convex, and in the other concave, while along the right lines themselves, as is evident, there is no curvature at all.

If a plane be drawn parallel to the tangent plane, and very near it, it will intersect the surface in a hyperbola and not an ellipse. Let this hyperbola, or *indicatrix*, be represented in Fig. 55, its asymptotes XY and AB, are parallel to the two right lines, along which the tangent plane intersects the surface, and the indicatrix plane meets the surface inside the angles XOA and YOB, but does not meet the surface inside the angles XOB and YOA. If, however,

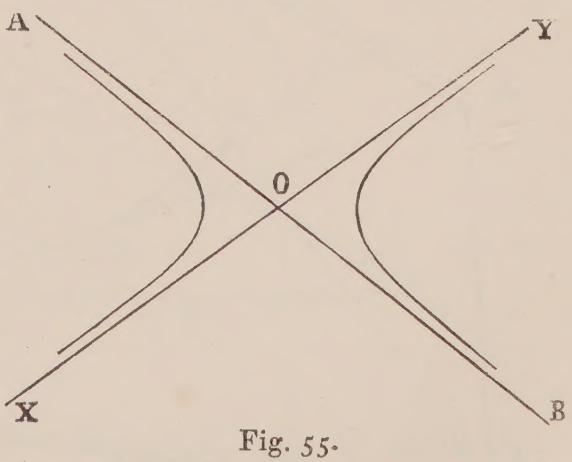


Fig. 55.

a second plane be drawn very near the tangent plane, and at the other side of it, this plane will intersect the surface as shown in Fig. 56, not meeting the surface within the angles XOA and YOB, but cutting it along the hyperbola contained in the angles XOB and YOA. As before, XY and AB are asymptotes to the hyperbola, and parallel to the right lines along which the tangent plane intersects the surface. The curvatures of the surface, in the two angular regions marked out in Figs. 55, 56, &c., are in opposite directions, and, therefore, the perpendicular pressures acting on the muscular sheets in these two

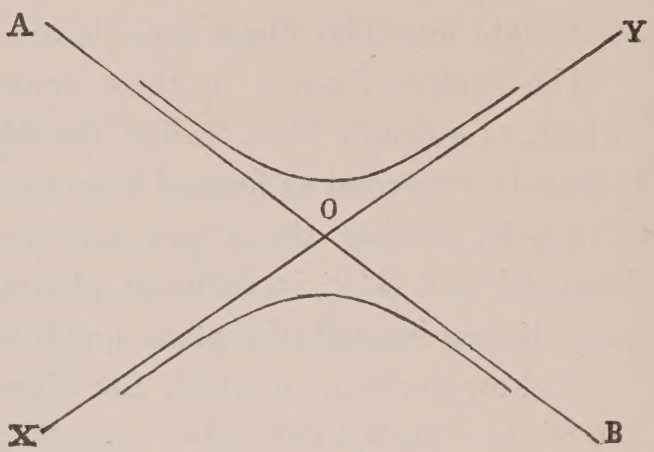


Fig. 56.

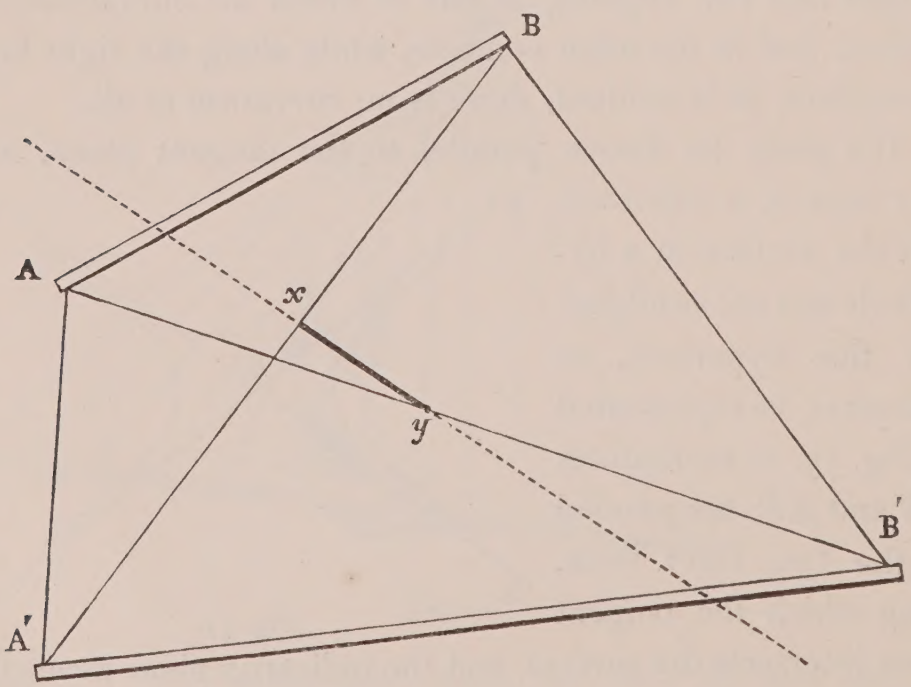


Fig. 57.

regions are also in opposite directions, while along the lines

XY and AB, there is no curvature and no force perpendicular to the muscular fibres.

In the simplest case of muscular skew surface that occurs in nature, let AB and $A'B'$, Fig. 57, represent straight bones which are the origin and insertion of the rectilinear muscular fibres joining them. The curved surface formed by the muscular fibres (*vide* Fig. 54) is a surface of the second order, known as the *Hyperboloid of one sheet*, the direction of whose centre may be found readily by means of the following simple construction.

Draw the diagonals AB' and $A'B$ of the skew quadrilateral $ABB'A'$, and bisect each diagonal in x and y, and join x and y, producing indefinitely both directions; the right line xy passes through the centre of the hyperboloid.

This construction may be thus proved—Fig. 58. Since

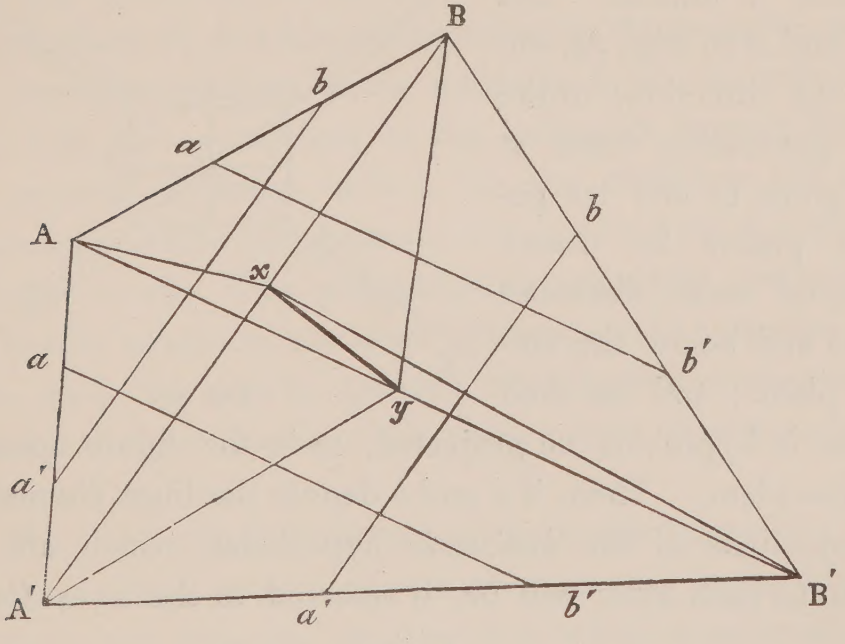


Fig. 58.

the bone AB, and the muscular fibre AA' both lie upon the surface, the plane BAA' is a tangent plane at the point A; and in like manner, the plane $A'B'B$ is the tangent plane at the point B'; and similarly the tangent plane at A' is the

plane of the bone $A'B'$, and of the muscular fibre $A'A$; and the tangent plane at B is the plane ABB'. Draw any number of chords ab', ab', &c., parallel to the diagonal AB'; since these are all chords of the common surface, their points of bisection all lie upon a diametral plane ByA' of that surface, and passing through y, the point of bisection of the diagonal AB'. In like manner, if a number of chords $a'b$, $a'b$, &c., be drawn parallel to the diameter $A'B$, their points of bisection will also lie upon another diametral plane AxB', and passing through x, the point of bisection of the diameter $A'B$. The common line of intersection xy of these two diametral planes ByA' and AxB', will, therefore, pass through the centre of the skew surface, or hyperboloid.

We are now in a position to determine the mechanical conditions of equilibrium of such a muscle. Let XY and AB, Fig. 59, denote the directions of the two generating lines at any point O, and let parallel planes be drawn at equal small distances above and below the tangent plane; and let their *indicatrix* hyperbola be projected, as in the figure upon the tangent plane. Then, if a and b denote the lines Oa and Ob, the equations of the *indicatrix* hyperbolas, which are conjugate to each other will be, if referred to the axes Oa and Ob,

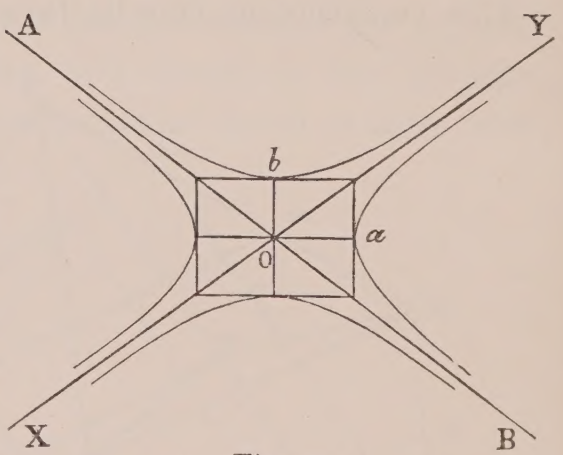


Fig. 59.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \pm 1. \qquad (42)$$

All that has been previously stated respecting the tangent cone and *indicatrix* ellipse, in convex muscular surfaces, holds

true in the skew surface for its tangent cone and *indicatrix* hyperbola. Thus, the tensile strain in the tangent plane at O, will vary in any azimuth as the square of the perpendicular let fall upon the tangent to the hyperbola, or as the radius of curvature of the section of the surface passing through that perpendicular. Let us trace the variation in the tensile strain round the point O. In the direction Oa the strain is proportional to a^2, and diminishes from Oa to OY, varying always as the square of the perpendicular let fall from O upon the tangent to the hyperbola YaB; in the direction OY, this perpendicular vanishes because OY is the asymptote, and the tensile strain and curvature vanish with it. When we pass OY, the tensile strain increases from zero to b^2, in going from OY to OB; varying now according to the square of the perpendicular let fall upon the tangent to the hyperbola YbA, from the centre O, and so on. Thus the tensile strain has two maxima, a^2, and b^2, corresponding to the axes Oa and Ob, and vanishes between these maxima, in the directions XY and AB, the asymptotes of the indicatrix hyperbolas. In the convex muscular surface, on the contrary, the tensile strain ranged from a maximum a^2 to a minimum b^2, and never vanished at all. The perpendicular pressures, produced by the tensile strains in the skew surface, are in opposite directions inside the angular spaces containing the two hyperbolas, because the curvatures of the surface have opposite signs in these two regions.

The joint effect of the tensile muscular forces in a closed convex surface is to compress the contents enclosed within it; but in the muscular skew surface, which is an open surface and cannot be made to enclose a space, the joint effect of a contraction of all its fibres results in an effort made by the surface to destroy its own curvature and return into the condition of a plane surface. Thus, if one of the bones AB be

fixed, and $A'B'$ free to move (Fig. 54), the contraction of the fibres will bring $A'B'$ into the same plane with AB, making $ABA'B'$ a plane quadrilateral; this reduction of the skew quadrilateral to a plane quadrilateral being accompanied by a peculiar twisting motion of the entire sheet of muscle varying with the nature of its curvatures at each point.

Let us now consider, in detail, the action of a quadrilateral skew muscle, in which one bone, AB, or $A'B'$ is fixed, and the other is either altogether free, or subject to various conditions in its motion, the most usual of which is, that it is compelled to turn always round some fixed socket or joint, whose position, as well as that of the fixed bone, is known and given. As the muscle may be supposed to perform its work by two distinct motions, one of which reduces the skew quadrilateral to a plane, and the other is confined altogether to that plane, it will be convenient to divide the propositions relative to skew quadrilateral muscles into two parts, viz.—

1. Action of a quadrilateral muscle in a plane.
2. Reduction of a skew quadrilateral muscle to the corresponding plane muscle.

The following postulate is necessary in order to solve the problems that present themselves, and will be admitted by some readers as an *a priori* truth, and by others as an inductive truth demonstrable by instances.

Postulate.

The Framer of the Universe (Δημιουργός τοῦ κοσμοῦ) *has constructed all muscles on the principle that each shall perform the maximum of Work possible for it under the given external conditions.*

From this postulate, the following results appear:—

Proposition A. (Problem.)

Given a fixed bone $A'B'$, and a perfectly free bone AB, lying in the same plane, forming the origin and insertion of a

quadrilateral muscle; it is required to find the motion of AB, when the muscle contracts.

The most general possible movement of AB may be represented by a movement of translation parallel to itself, and a movement of rotation round a point.

Produce AA' and BB' to meet at O, and draw OX, the bisector of the vertical angle. I say that no work is done by a

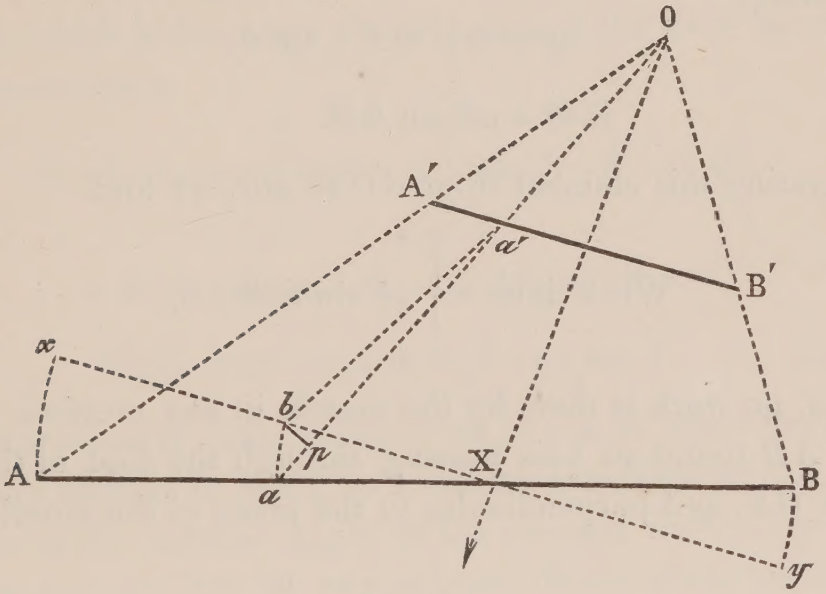


Fig. 61.

rotation of the bone AB round an axis passing through X, the foot of the bisector, and perpendicular to the plane of the muscle, and, therefore, that the muscle by its own contraction can never produce such a rotation. For, let xy be the new position of the bone AB, when turned through a small angle of rotation, $xXA = yXB = \omega$.

Let $aa'O$ be the original direction of any muscular fibre, and let ab be the path described by the point a during the rotation; draw bp perpendicular to aa' and join $a'b$. Let

$$Oa = l$$
$$Oa' = l'$$
$$aOX = \theta$$
$$BXO = \beta$$
$$OX = b.$$

It is plain that ap denotes the change in length of the fibre aa' during the rotation, and the work done by it is

$$\delta l d\theta = ap \times d\theta,$$

but

$$ap = ab \times \sin(\beta - \theta)$$

$$ab = \omega \times aX = \omega \times OX \times \frac{\sin\theta}{\sin(\beta - \theta)}.$$

Therefore,

$$ap = \omega \times OX \times \sin\theta,$$

and

$$\delta l d\theta = \omega b \sin\theta d\theta.$$

Integrating this element from AO to BO, we find

$$\text{Work done} = \int_{-\theta}^{+\theta} \omega b \sin\theta d\theta = 0.$$

Hence, no work is done by the muscle in any rotation of the bone AB round an axis passing through the foot of the bisector OX, and perpendicular to the plane of the muscle.

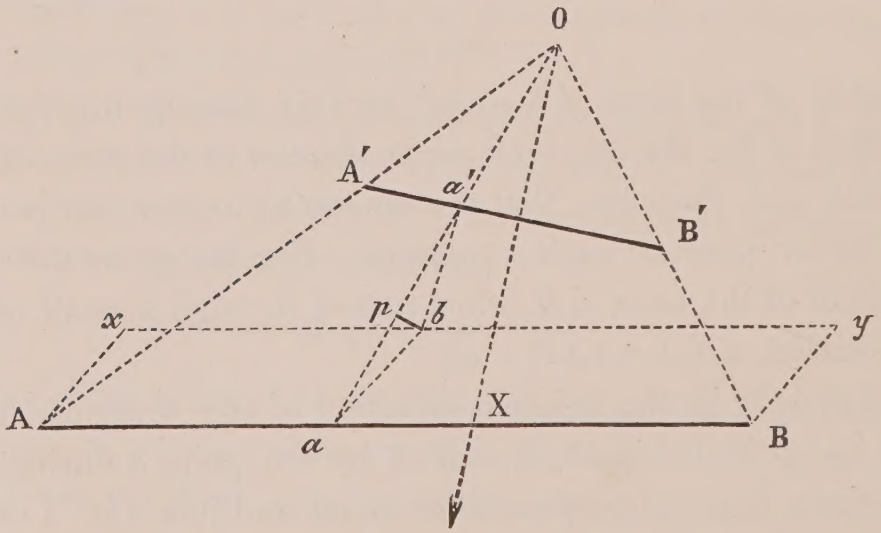


Fig. 61.

Let us now suppose the bone AB, Fig. 61, to be moved parallel to itself into the position xy, and let

$$xAB = \phi;$$

it is required to find the conditions necessary to produce the maximum amount of work done by such a movement.

Let aa' be any fibre, and let ab be the line traversed by the point a; and draw bp perpendicular to aa'. Then

$$\delta l = aa' - a'b = ap = ab \times \cos(pab) \text{ but } pab = \beta - \theta - \phi.$$

Therefore,

$$\delta l d\theta = ab \times \cos(\beta - \theta - \phi)\, d\theta.$$

Let ab, which is the same for all points of the bone be called Δ, then we have

$$\text{Work done} = \int \delta l d\theta = \Delta \int_{-\theta}^{+\theta} \cos(\beta - \theta - \phi)\, d\theta,$$

$$= \Delta\{\sin(\beta - \phi + \theta) - \sin(\beta - \phi - \theta)\}.$$

or

$$\text{Work done} = 2\Delta \cos(\beta - \phi) \sin\theta. \qquad (43)$$

This expression is a maximum when $\beta = \phi$, or when the bone is moved parallel to itself, in a direction parallel to OX, the bisector of the vertical angle of the triangle formed by producing AA' and BB' the two extreme fibres of the Quadrilateral muscle. Therefore,

If there be given a fixed bone $A'B'$, and a perfectly free bone AB lying in the same plane, forming the origin and insertion of a Quadrilateral muscle; when that muscle contracts, the foot of the bisector of the vertical angle OX must move towards O, but the bone AB may have any position whatever, subject to the foregoing condition.

From the preceding investigation it appears that the motion of the bone AB is indeterminate, but subject to a certain condition, viz., that the point X of the bone shall move towards the point O; a condition which agrees with what has been already stated, pp. 181, 195. In nature, however, nothing is indeterminate, and conditions are always added, which completely fix the actual motion in every case. One of the most common additional conditions in nature is the

following restriction on the free motion of the bone AB. Let $A'B'$ (Fig. 62) be the fixed bone, and AB the moveable bone, which is subject to the condition of always turning round a

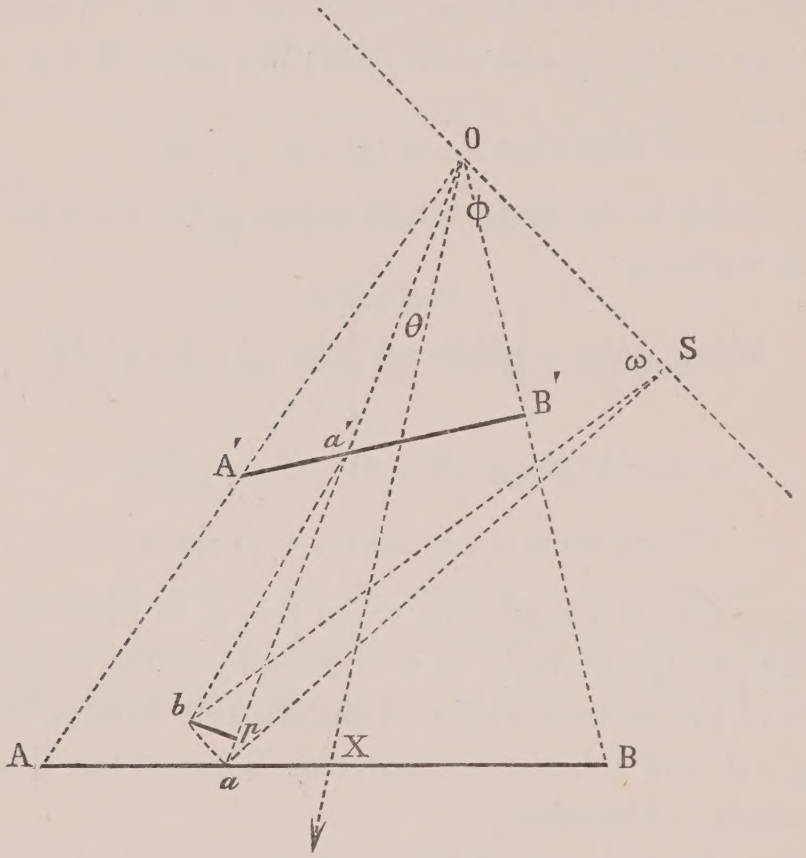


Fig. 62.

joint, or socket, placed at the point S. Hence, we have the following problem.

Proposition B. (Problem.)

Given a fixed bone $A'B'$, and a moveable bone AB lying in the same plane, subject to the condition of always turning round an axis passing through a certain point S, and perpendicular to the plane of the bones AB and $A'B'$; it is required to find the locus of the joint S, so that the work done by the muscle, in contracting, shall be a maximum.

Let aa' be any fibre, and let the bone AB receive a small

rotation $\delta\omega$, round the socket S; and let the point a move to the point b; and let OX be the bisector of the vertical angle AOB. Also, let

$$OSa = \omega,$$
$$SOX = \phi, \quad OS = k,$$
$$XOa = \theta, \quad Sa = A.$$

Draw $a'b$, and let fall bp perpendicular upon aa'. We now have

$$\delta(l - l') = ap = ab \times \cos(baa') = ab \times \sin(SaO),$$

but

$$ab = A\delta\omega,$$

and

$$\sin(SaO) = \sin(\omega + \phi + \theta);$$

therefore,

$$\delta(l - l') = A\delta\omega \sin(\omega + \phi + \theta);$$

but

$$A = k \frac{\sin(\phi + \theta)}{\sin(\omega + \phi + \theta)}.$$

Therefore, finally,

$$\delta(l - l') = k\delta\omega \sin(\phi + \theta).$$

The work done by the muscle during the rotation through $\delta\omega$, is, therefore,

$$\int \delta(l - l')d\theta = k\delta\omega \int_{-\theta}^{+\theta} \sin(\phi + \theta)\, d\theta.$$

The integration of this expression gives us for the work done

$$\int \delta(l - l')\, d\theta = 2k\delta\omega \sin\phi \sin\theta. \qquad (44)$$

This result will be a maximum when $\phi = 90°$. Hence we obtain the following solution of the problem proposed:

Produce the extreme fibres AA' and BB', to meet in the point O, and draw OX, the bisector of the vertical angle of the triangle

so formed; it is necessary, in order to produce the maximum work due to a rotation round the socket S, that this socket shall be placed somewhere upon a right line drawn through the vertex of the triangle, and perpendicular to the internal bisector of that angle.

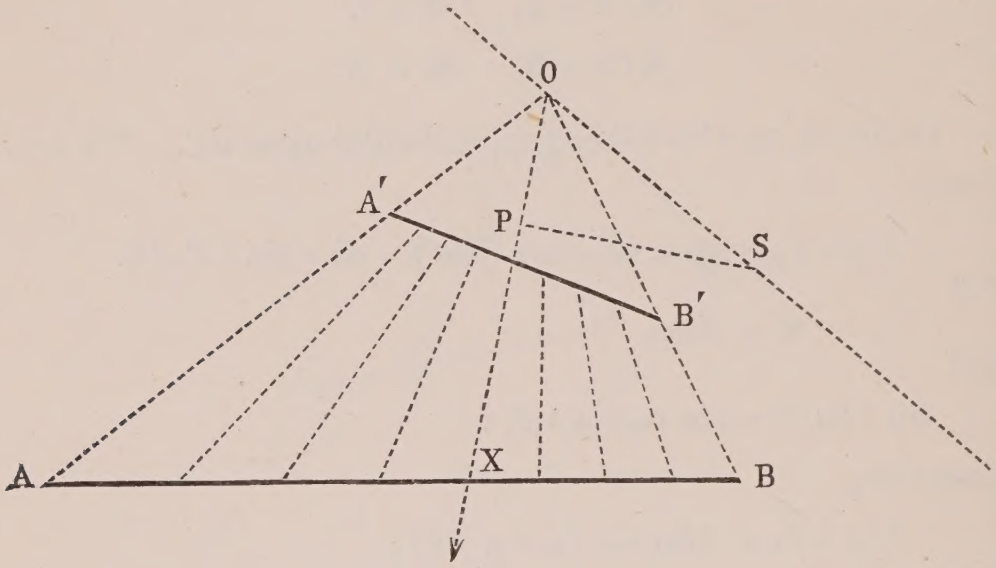


Fig. 63.

The same result may be thus obtained without integration, from the principles laid down in p. 181. Let AB, $A'B'$, Fig. 63, be the quadrilateral muscle, and let OX be the bisector of the vertical angle AOB.

$$OS = k,$$
$$SOX = \phi,$$
$$AOX = \theta.$$

The resultant of all the forces of the muscular fibres lies in the right line OX, and its moment, in turning the bone AB round the socket S, will be a maximum when the perpendicular SP, let fall from the socket upon OX the direction of the force, is a maximum. The resultant force (p. 181), is

$$2f \sin \theta;$$

and from the figure,

$$SP = k \sin \phi.$$

Hence, the moment of the force round the socket will be

$$\text{Moment} = 2kf \sin\theta \sin\phi,$$

which agrees with equation (44).

If we suppose the angle θ to be given, the work done will be a maximum when $k \sin\phi = SP$ is a maximum; and if k, the distance of the socket from the vertex, be also given, then $\phi = 90°$, or OS perpendicular to OX, will be the arrangement capable of producing the maximum effect.

Illustrations of Prop. B.—On examining, in nature, the actual relative positions of the socket of the joint, and vertex of the triangular or quadrilateral muscle, we find them always placed in accordance with the result just found; so that, at the moment at which the muscle makes its most vigorous contraction, the direction of its resultant is perpendicular to the line SO, and, therefore, produces the maximum effect in turning the bone round the socket of the joint.

(*a*). *Latissimus dorsi in Man.*—In Fig. 64, let S represent the socket of the shoulder joint, and O the insertion of the *latissimus dorsi*, whose extreme fibres make with each other an angle of 35° (p. 191). The bisector OX of the vertical angle of the muscle passes, very nearly, through the spinous process of the first lumbar vertebra. Draw SH horizontal, and place the arm SO in such a position that the angle SOX shall be a right angle. The following measurements are readily made:—

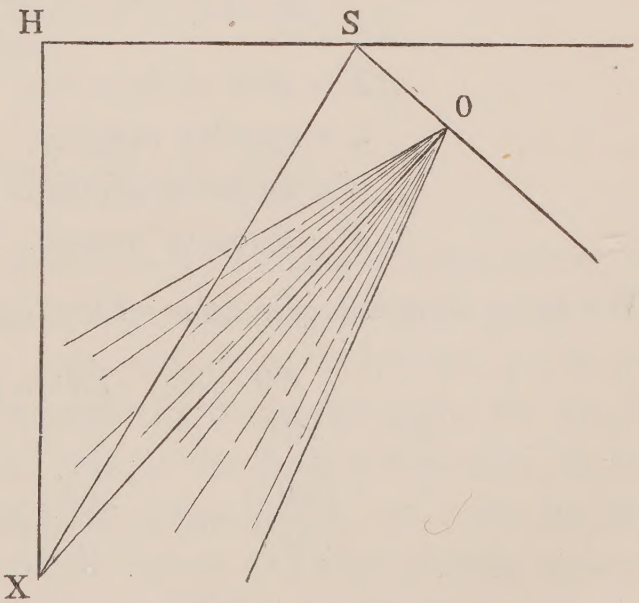


Fig. 64.

$$HS = 8 \text{ inches.}$$
$$HX = 10 \quad ,,$$
$$SO = 2.16 \quad ,,$$

From which we find

$$SX = 12.8 \text{ inches.}$$
$$HSX = 51^\circ\ 20'.$$
$$XSO = 80^\circ\ 17'.$$

Adding together the two angles, we find

$$HSO = 131^\circ\ 37';$$

or, that the arm is depressed $48^\circ\ 23'$ below the horizontal line, when SO is perpendicular to OX. In giving a back sword cut, or back-handed stroke to a racket-ball, this is the angle of depression at which the moment of the force of the *latissimus dorsi* round the socket S is a maximum.

The angular velocity produced by the contraction of the muscle is thus found. Let

$SO = k.$ $\qquad$ $XSO = \theta.$
$XO = x.$ $\qquad$ $XOS = \phi.$
$SX = a.$

ω = angular velocity.
I = moment of inertia of arm.
F = force of muscle.

We have, from the principles of Dynamics,

$$\frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \frac{Fk \sin \phi}{I}$$

$$\left(\frac{d\theta}{dt}\right)^2 = \omega^2 = \frac{2kF}{I}\int \sin \phi \, d\theta,$$

but

$$\sin \phi = \frac{a}{x} \sin \theta$$

$$x = \sqrt{a^2 + k^2 - 2ak \cos \theta}.$$

Hence, we have

$$\omega^2 = \frac{F}{I} \frac{2ak \sin \theta d\theta}{\sqrt{a^2 + k^2 - 2ak \cos \theta}}.$$

$$\omega^2 = \frac{2F}{I} \sqrt{a^2 + k^2 - 2ak \cos \theta} + \text{const.}$$

$$\omega^2 = \frac{2F}{I} (x_1 - x_2).$$

or

$$\omega^2 = \frac{2F}{I} \delta x. \qquad (45)$$

where δx denotes the total change of length of OX in passing from the angle θ_1 to the angle θ_2. Hence the angular velocity of the arm produced by the contraction of the *latissimus dorsi* varies as the square root of that contraction; and it is important, being supposed given the amount of the contraction, to find the conditions necessary to produce the resulting angular velocity with a minimum angle of rotation. This may be thus determined,

$$x^2 = a^2 + k^2 - 2ak \cos \theta.$$

Hence,

$$x\delta x = ak \sin \theta \delta\theta,$$

$$\delta\theta = \frac{x\delta x}{ak \sin \theta} = \frac{\delta x}{k \sin \phi}.$$

Hence, $\delta\theta$ will be a minimum when $\sin \phi$ is a maximum, or when $\phi = 90°$, which coincides with our previous result.

When the arm is depressed 48° 23′ below the horizontal line, so as to make the angle XOS a right angle, the length of OX is 12.62 inches; and if the total contraction of the muscle be one-eighth part of its length, we have for the minimum angle of rotation, before and after passing through the position of the arm, where XOS is a right angle,

$$\delta\theta = \frac{\delta x}{k} = \frac{79}{216} = 21°.$$

Whenever, therefore, the arm is depressed by the *latissimus dorsi*, from 27° 23′ to 69° 23′, passing through an arc of 42°, a greater angular velocity will be attained, in passing through a given angle, than is possible for any other motion of the arm. Racket players, cavalry soldiers, and others who require to give a swift back stroke with the arm, instinctively learn the proper range of depression, and the relations of position of the muscle, socket, and arm, are all arranged in strict conformity with Proposition B.

In the action here described, the muscles which extend the forearm upon the arm act in unison with those which depress the humerus, and it is interesting to compare their relative forces. I determined the weights of the two groups of muscles in a male and female subject, with the following results :—

Depressors of Arm.

	Male.	Female.
Latissimus dorsi, ...	7·55 oz.	4.58 oz.
Teres major, ...	4.09 oz.	1.67 oz.
	11.64 oz.	6.25 oz.

Extensors of Forearm.

	Male.	Female.
Triceps longus, ,, *internus*, ,, *externus*, . . .	13.10 oz.	6.59 oz.

Ratio of Depressors of Arm to Extensors of Forearm.

Male, ...	$\frac{1164}{1310}$	0.89
Female, ...	$\frac{625}{659}$	0.95
	Mean, . .	0.92

From this it appears that the two groups of muscles are nearly of equal weights, and therefore contain nearly equal quantities of work stored up; so that their simultaneous action is highly advantageous. It may be noticed that in the case of a ginglymoid joint, like the elbow, where the motion of the bone is circular, the condition required by Proposition B. is necessarily fulfilled in every position of the bone.

(*b*). *Latissimus dorsi of the Tiger and Lion.*—The position and relations of the *latissimus dorsi* in the Tiger are shown in Fig. 65, where *S* is the centre of the glenoid cavity, *C* is

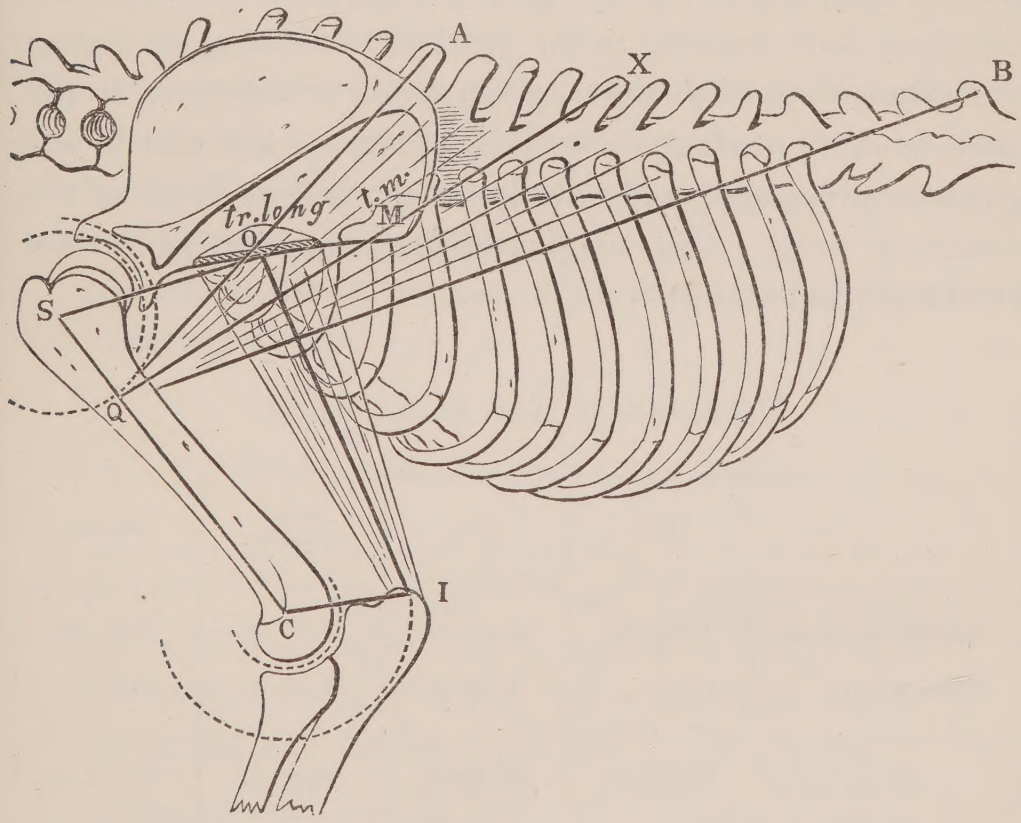


Fig. 65.

the centre of motion of the elbow joint, and *Q* is the position of the centre of the tendon of the *latissimus dorsi*, inserted on the inner side of the humerus. The line, *QA*, drawn to the fifth dorsal vertebra, represents the anterior fibres of the muscle, and *QB*, drawn to the third lumbar vertebra, represents its posterior fibres. The line, *QX*, bisecting the angle,

AQB, passes through the ninth dorsal vertebra, and is the line of force acting upon the humerus.

In the position of the Tiger's arm, represented in the figure, the angle, *SQX*, is 93°, and the force of the *latissimus dorsi* is exerted to flex the arm upon the body at nearly its position of maximum advantage.

It is important to observe that, when the angle *SQX* becomes a right angle, the *teres major* (whose origin is represented at the angle of the scapula, and which is inserted with the *latissimus dorsi*) acts with its entire force along the bisector *QX*, and at right angles to *SQC*; so, that when the *latissimus dorsi* is acting to the greatest advantage, producing a maximum velocity with a given angle of rotation; at the same time, its auxiliary muscle, *teres major*, acts also to the greatest advantage, and exactly in the line of force of the *latissimus dorsi*. The weights of these muscles in several Felidæ are as follows:—

Depressors of Arm in the Felidæ.

	Tiger.	Lion.	Jaguar (female).	Leopard.
Latissimus dorsi,	39.75 oz.	25.50 oz.	4.19 oz.	8.40 oz.
Teres major, .	11.45 ,,	9.50 ,,	1.68 ,,	2.65 ,,
Total force,	**51.20** oz.	**35·00** oz.	**5.87** oz.	**11·05** oz.

The *latissimus dorsi* and *teres major* act upon the humerus, tending to turn it round an axis perpendicular to the plane of the triangular sheet of muscle; but another important muscle assists in this rotation. This muscle is the *triceps longus*, which takes its origin on the line marked on the posterior edge of the scapula, and is inserted, in conjunction with

the other muscles that act upon the elbow joint, into the olecranon process. The right line *OI* represents the direction in which the force of this muscle is applied, and it will produce its maximum mechanical effect when the angle *SOI* becomes a right angle, in conformity with Proposition *B*.

In the position of the arm shown in the figure, the angles are as follows:—

$SQX = 93°$ (*latiss. dorsi and teres major*),
$SOI = 95°$ (*triceps*).

The arrangement of both groups of muscles is so contrived, that these angles pass through 90° together, thus enabling both sets of muscles to act at the same moment with maximum effect upon the humerus.

The *triceps* and its associated muscles in the Felidæ have the following weight:—

Extensors of Forearm in the Felidæ.

	Tiger.	Lion.	Jaguar (female).	Leopard.
Triceps longus, . .	30.65 oz.	—	4.20 oz.	7.27 oz.
Triceps internus, .	7.10 ,,	—	1.48 ,,	1.79 ,,
Triceps externus, .	14.05 ,,	—	1.54 ,,	3.45 ,,
Triceps accessorius, .	2.95 ,,	—	0.20 ,,	0.67 ,,
Anconæus, . . .	3.05 ,,	—	0.33 ,,	0.60 ,,
Total	**57.80** oz.	**43.50** oz.	**7.75** oz.	**13.78** oz.

If we compare these weights, which represent the work done

in each contraction of the group of muscles, with those already given, we find—

Ratio of Depressors of Arm to Extensors of Forearm in the Felidæ.

Tiger.	Lion.	Jaguar (female).	Leopard.
0.88	0.80	0.76	0.80.

With respect to the action of the *triceps* and its assistants upon the elbow joint, the following considerations are of interest. The *triceps* will produce, in conformity with Proposition B. its maximum effect upon the forearm, turning round the centre *C* (Fig. 66), when the quadrilateral *OSCI* comes into the position in which the line of force of the *triceps* *OI* is at right angles to the line *CI*. In this position of the quadrilateral, the right line *CI*, drawn from the point *C* to meet the circle *I'I''*, becomes a tangent to that circle.

It is evident from the figure, that the angle *OIC* becomes a right angle at the same time as the angles *SQX* and *SOI*; and therefore, that in the position of maximum effect of all the muscles acting on the shoulder and elbow joints, the lines *SO* and *CI* become parallel to each other, and are both at right angles to the direction of the triceps *OI*.

From the preceding facts, it is plain that the *triceps* is intended to co-operate with the *latissimus dorsi* and *teres major;* and in point of fact, in the Tiger it co-operates with these muscles, when both they and it are placed in their relative positions of maximum effect upon the rotations, round both the shoulder and elbow joints.

I have repeatedly watched the back stroke of the arm in all the large Felidæ, and am unable to perceive the slightest difference in the time of action of the elbow and shoulder joints in the Tiger, Jaguar, or Panther; in the Lion it appears

to me that the extension of the forearm precedes the backward motion of the arm by a very short interval.*

In Fig. 66, I have drawn to scale the relative positions of the various sockets, and origin and insertion of muscles in the

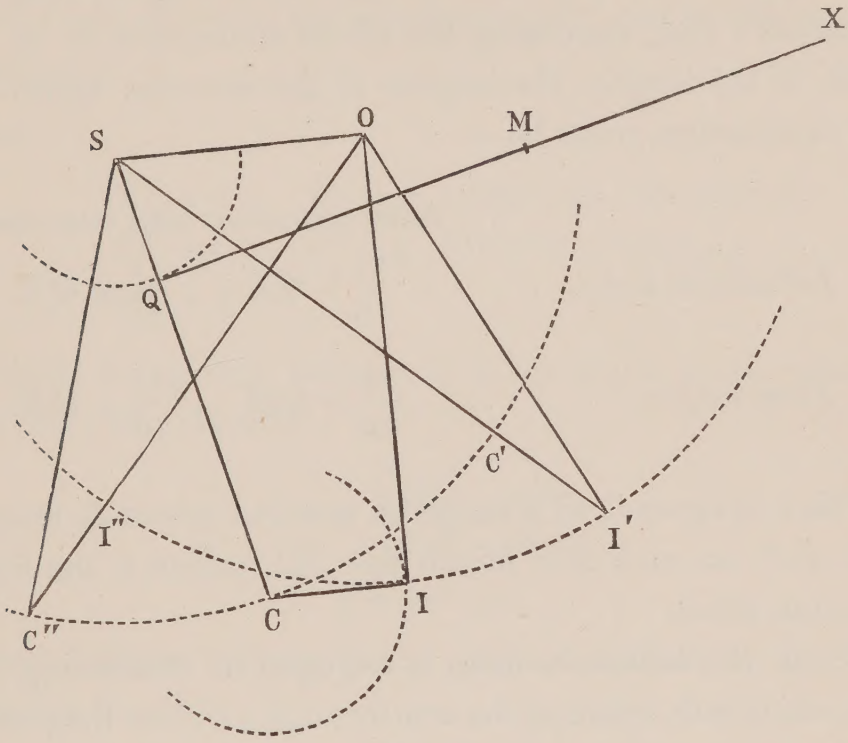


Fig. 66.

arm of the Bengal Tiger. In this figure, *S* is the centre of the shoulder joint, *O* is the centre of origin of *triceps longus*, *M* is the centre of origin of *teres major*, *X* is the extremity of the bisector of vertical angle of *latissimus dorsi*, *C* is the centre of the elbow joint, *I* is the centre of insertion of the *triceps longus*.

The measurements of the fixed lines in the Tiger are as follows:—

SO = 5.62 inches.	*SM* = 9.50 inches.
SC = 10.50 ,,	*SX* = 16.12 ,,
CI = 3.00 ,,	*SQ* = 2.89 ,,

* In the action of a Horse, walking or trotting, the forearm always moves before the arm, by a perceptible interval.

The point of insertion, Q, of the *latissimus dorsi* and *teres major* describes a circle, of which S is the centre, and the fibres of these muscles must be contrived of such a length, that XQ and MQ shall be equal to their respective lengths, when half contracted; and, supposing the whole contraction to be one-eighth of the length, the lengths of the muscles, before and after contraction, must be—

	Before contraction.	After contraction.
Latissimus dorsi, . . .	$\frac{17}{16} \times QX$	$\frac{15}{16} \times QX$
Teres major, ...	$\frac{17}{16} \times QM$	$\frac{15}{16} \times QM$

This corresponds to a range of rotation round S, amounting to 21°, at each side of the line SC, which is the line of maximum effect.

While the *latissimus dorsi* is engaged in shortening itself by one-sixteenth, rotating the arm through 21°, into the position SC, the *triceps longus* also shortens itself by one-sixteenth, and causes the forearm to rotate round C, through an angle of 9° 30′, into the position of maximum effect, in which OI and CI are at right angles; and in the entire complex downward and backward stroke of the Tiger's paw, the arm passes through 42°, while the forearm passes through 19°.

It is important to observe that the condition fulfilled by the Tiger's arm, by which the angles SQX, SOI, and CIO become right angles at the same time, and so allow all the muscles of the shoulder and elbow to co-operate together, each acting to the greatest advantage, is not a random condition attained by chance; but must have been the result of deliberate forethought and contrivance. This can be proved by showing that the condition involves rigorous geometrical consequences, which might or might not have been fulfilled

by bones and muscles selected at random. Let us suppose that S, O, Q, M, and X, or everything relating to the shoulder joint, be given; it is required to contrive for the Tiger an *olecranon* and *triceps*, which shall enable all the muscles of shoulder and elbow to act simultaneously, and to the greatest advantage.

Let

$$SO = a \qquad CI = x \text{ (unknown olecranon).}$$
$$SC = b \qquad OI = y \text{ (unknown triceps).}$$
$$OSC = \phi$$

The angle ϕ is given, because it is the angle whose cosine is found from the relation

$$\cos \phi = \frac{SQ}{SX} = \frac{289}{1612}$$
$$\phi = 79^\circ\ 40',$$

because XQ is a tangent to the circle described by Q, round the centre S.

It is easy to see that the *olecranon* and *triceps* are given by the equations

$$x = a - b \cos \phi$$
$$y = b \sin \phi.$$

If the values of a, b, and ϕ, above given, be substituted in these equations, they will determine the lengths of *olecranon* and *triceps*, which (and which only) will enable all the muscles of the shoulder and elbow in the Tiger to act simultaneously to the greatest advantage.

Whenever, as in the case of the *triceps longus*, a muscle passes over two joints, a considerable amount of rotation round both joints is possible, without requiring the fibres of the muscles to be lengthened or shortened. Thus, in the quadrilateral figure $OSCI$, the sides OS, SC, and CI, are absolutely constant; and it may be shown that the arm may

be rotated through a considerable angle, without altering the length of the fourth side, *OI*; and, consequently, without lengthening or shortening the *triceps longus*.

In the forward direction, the arm *SC* may be moved into the position, *SC''*, such that *OI* plus *IC* shall be equal to the right line, *OI''C''*; and in the backward direction, the arm may be moved into the position, *SC'I'*, such that *SC* plus *CI* shall be equal to the right line, *SC'I'*. While the arm is moved from *SC'* to *SC''*, through an angle of 65°, the fibres of the *triceps* muscle, *OI*, are neither lengthened nor shortened; if the arm be brought farther forward than *SC''*, its fibres will be stretched, and the muscle provoked to contract itself; and the arm cannot be brought farther backward than the position *SC'*, unless the fibres of the *triceps* be shortened.

(*c*). *Latissimus dorsi* (*Albatross*).

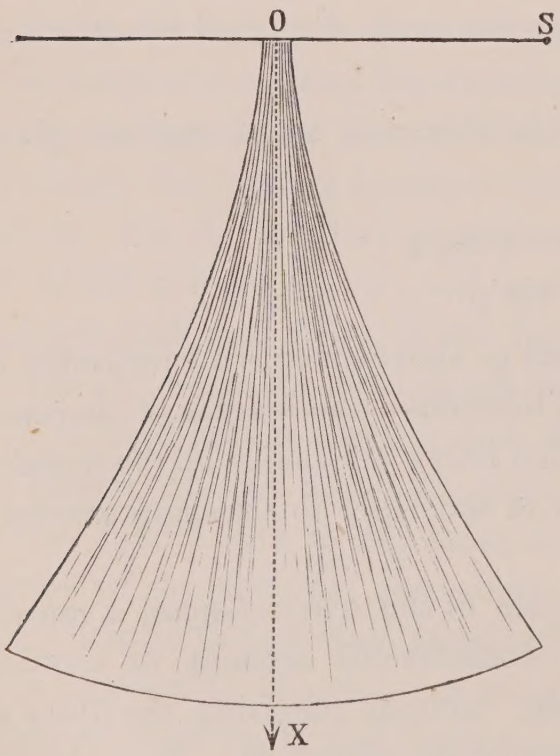


Fig. 67.

The *latissimus dorsi* of the wing of the *Albatross*, (one half natural size) is shown in Fig. 67, in which *S*, the socket of the humerus, is 2.20 inches distant from *O*, the point of insertion of the muscle, which is triangular in shape, its extreme fibres making an angle of 51° with each other. In the mean position of the wing, in the dead bird, the angle *SOX*, contained between *OS* and *OX*, the bisector of the musular fibres is 86°. During life,

the contraction of the muscle always swings the wing through the position corresponding with $\phi = SOX = 90°$, so as to produce, in striking the air, a given angular velocity, with a minimum angle of rotation of the wing. Assuming, as before, the contraction of the muscular fibres to be one-eighth of their length, we readily obtain the following result:—Since OX is equal to 4.62 inches—

$$\delta\theta = \frac{\delta x}{R \sin \phi} = \frac{462}{16 \times 220} = 7°31'.$$

Hence the wing of the Albatross, when acted upon by the *latissimus dorsi*, will sweep through an angle of 15°, making the angle SOX a right angle in the course of the rotation, and producing a maximum velocity of stroke, for a given angle of rotation.

(*d*). *Hamstring Muscles of Tiger and Lion.*

The next example of Proposition *B* that I shall give is taken from the hamstring muscles of the great Felidæ, which constitute one of the most wonderful instances of well-devised mechanism to be found in the entire range of the Animal Kingdom.

In Fig. 68 are shown the pelvis and leg of the Bengal Tiger, in which O is the tuber ischii and origin of the *biceps femoris* muscle, whose extreme fibres are OA, to the middle of femoral fascia, and OB wound round the *tendo Achillis*, and directed towards the calcaneum. The angle AOB is 50°, and the bisector OX passes through the top of the fibula, and is the direction of the resultant force of all the fibres of the *biceps*. This great muscle, instead of being inserted into a single point, is spread out, so as to act upon several joints at the same time; its upper fibres act on the hip joint only; its middle fibres act upon both the hip and knee joints, and its lower

fibres influence, in addition, the ankle joint. We shall, however, regard it as a whole, represented by its resultant force acting in the direction OX. The other hamstring muscles,

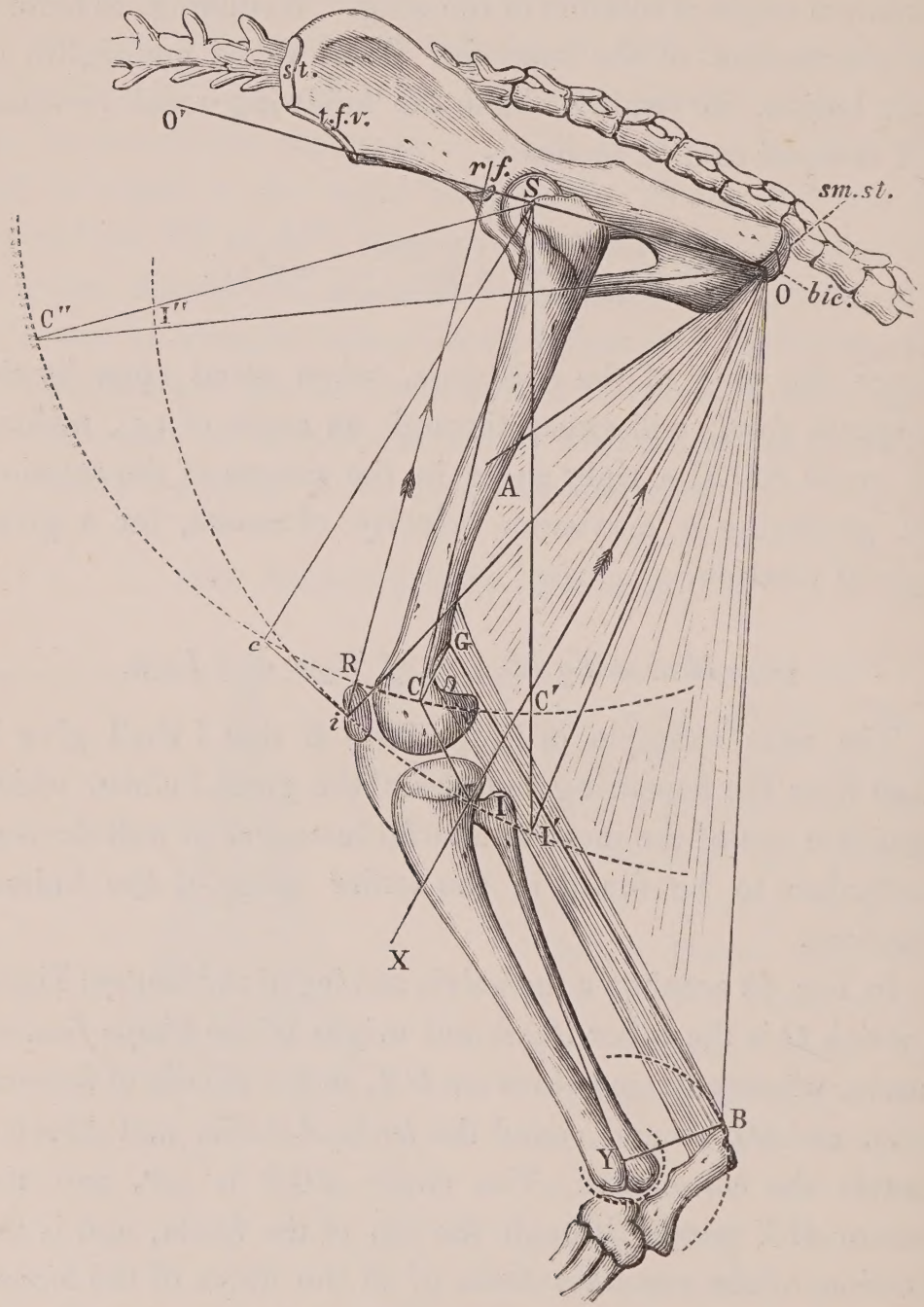


Fig. 68.

semi-tendinosus and *semimembranosus*, take origin from the tuber ischii, immediately beside the *biceps*, and are inserted into the tibia, in a position corresponding with the position

of the line OX; so that our conclusions, with respect to the *biceps*, will apply to all the hamstring muscles.

Let S be the centre of the acetabulum, O the tuber ischii, C the centre of motion of the knee joint, and I the top of the fibula. I obtained the following measurements from the male Tiger, already mentioned at p. 127 :—

OS = 5.49 in.	CI = 2.63 in.
SC = 11.75 „	OI = 14.00 „

The distances OS, SC, and CI are absolutely fixed; but the distance OI may be shortened by the action of the *biceps* and other hamstring muscles. I have taken the length of OI at 14 inches, before the contraction of the muscles.

Let us first inquire what range of motion the leg can have, before the line OI is shortened or extended. This question gives us the quadrilateral figure $OSCI$, whose four sides are given; and we are required to move the quadrilateral about OS regarded as fixed, so as to find the extreme limits possible for its construction. With O as centre, describe the circle $I'II''$; and with S as centre, describe the circle $C'CC''$.

The leg of the Tiger may be moved forward, or flexed upon the body, until the lines OI and IC form one continued right line, in the position $OI''C''$, where $I''C''$ is equal to IC; the leg cannot be moved forward beyond this position without lengthening the muscle OIX, and thus provoking it to contract. The leg of the Tiger may be moved backwards, or extended upon the body, until the lines SC and CI form one continued right line, in the position $SC'I'$, where $C'I'$ is equal to CI; and the leg cannot be moved farther backwards without a shortening of the muscle OIX. Produce the line OS to O', and measure the angles $O'SC''$ and $O'SI'$, which represent the extreme limits of the position of the leg, compatible with the supposition, that the muscle

OIX has been neither lengthened nor shortened. We find—

$$O'SC'' = 31°$$
$$O'SC' = 105°$$

Showing a range of 74° of motion round the socket S, through which the *biceps* and other hamstrings are neither lengthened nor shortened.

When the leg attains the position $C'I'$, the angle SOI' is 83°, and shortly afterwards passes through 90°, when the hamstring muscles contract. If we examine the angle CIO, at the point I', when the angle SOI' is approaching a right angle, and when the hamstring muscles are producing their maximum effect with respect to the hip joint S, we shall see that $C'I'O$ becomes acute, and that it is far removed from the position in which its maximum effect would be produced upon the knee joint C. In fact, the position of the leg, in which the hamstring muscles would produce their maximum effect upon the knee joint, corresponds with the point c, from which a tangent ci, to the circle $I'II''$, would be equal to CI. In this position of the leg, the angle ciO is 90°, while the angle SOi, corresponding to it, is only 64°. From this it follows, that the knee joint is not related to the hip joint, in the same way that the elbow joint is related to the shoulder joint; and that it is not intended that the muscles acting upon each should attain their positions of maximum effect at the same time.

In the Tiger's leg, it is the ankle joint that bears to the hip joint, the same relation that the elbow bears the shoulder joint, in the arm; for, let Y be the centre of the ankle joint, and join YB to the calcaneum; and let G denote the origin of the external *gastrocnemius* muscle, which acts from G to B. This powerful muscle, and the corresponding internal *gastrocnemius*, will produce their maximum effect upon the ankle joint, when the angle YBG becomes a right angle, in con-

formity with Proposition *B*; and it is easy to see from the figure, that the angles *SOI* and *GBY* will pass through right angles at nearly the same time. The *gastrocneme* muscles, in the hind leg, therefore, stand in the same relation (mechanically) to the hamstring muscles, that the *triceps longus* and its allies bear to the *latissimus dorsi* and *teres major*, in the fore leg.

I have found the following weights of the hamstring and gastrocneme muscles in the Felidæ :—

Hamstring Muscles in the Felidæ.

	Tiger.	Lion.	Jaguar (female).	Leopard.
Biceps femoris, . .	47.60 oz.	27.75 oz.	3.77 oz.	9.66 oz.
Bicipiti accessorius .	1.28 ,,	—	0.14 ,,	0.18 ,,
Semitendinosus, . .	13.70 ,,	10.00 ,,	1.58 ,,	3.28 ,,
Semimembranosus, .	23.40 ,,	26.25 ,,	2.61 ,,	5.36 ,,
Gracilis, ...	13.35 ,,	9.00 ,,	0.92 ,,	2.41 ,,
Total	**99.33** oz.	**73.00** oz.	**9.02** oz.	**20.89** oz.

Gastrocneme Muscles in the Felidæ.

	Tiger.	Lion.	Jaguar (female).	Leopard.
Gastrocnemius externus,	9.80 oz.	—	2.84 oz.	4.93 oz.
Gastrocnemius internus,	9.50 ,,	—		2.38 ,,
Plantaris, ...	7.35 ,,	—	—	—
Total	**26.65** oz.	—	**2.84** oz.	**7.31** oz.

It will be observed, that, while the hamstring muscles are attaining their position of maximum effect upon the hip joint,

by the angle *SOI* becoming a right angle, and the gastrocneme muscles are acquiring their maximum effect upon the ankle joint, by the angle *GBY* becoming a right angle, that both these muscles act at a considerable disadvantage upon the knee joint; for the angles *CIO* and *CGB* become very acute, when the angles *SOI* and *GBY* pass through right angles. Still, both hamstring and gastrocneme muscles tend to flex the knee joint, in every position of the legs; an effect, which would be disastrous to the combined movement of the muscles of the hip and ankle, when the Tiger walks, runs, or springs upon its prey. Therefore, the flexing effect of these muscles upon the knee, is always counteracted by the extending effect of the *rectus femoris*, and its allies, upon the same joint; and these antagonist muscles are called into action, to an extent necessary to keep the knee joint from bending, or, in some motions of the leg, to an extent that actually extends the leg upon the thigh.

It is very interesting, therefore, to observe that the *rectus femoris*, which is the natural ally of the hamstring and gastrocneme muscles, acting in the line *R.rf.*, is at right angles to the lever *cR*, at the same time that the angles *SOI* and *GBY*, of the hamstring and gastrocneme muscles, pass through 90°.

In Propositions *A* and *B*, we have considered the case of two bones, lying in the same plane, and united by a quadrilateral sheet of muscular fibres; one bone being supposed fixed, and the other bone compelled to turn round an axis of rotation perpendicular to the plane. We have now to examine the more complex case in which the moveable bone turns round some axis lying in the plane of the two bones; an action, by which the plane quadrilateral becomes a gauche quadrilateral, and the muscular fibres form the surface known as an hyperboloid of one sheet.

I shall first suppose the axis of rotation to be perpendi-

cular to the bisector of the muscular fibres, when lying in the same plane, and investigate the relations between the bones which are necessary to produce a maximum effect, when the muscle contracts.

In Fig. 69, AB and $A'B'$ are the two bones, $A'B'$ being supposed fixed, and AB moveable, round an axis SK, passing through the fixed socket S, and perpendicular to OX,

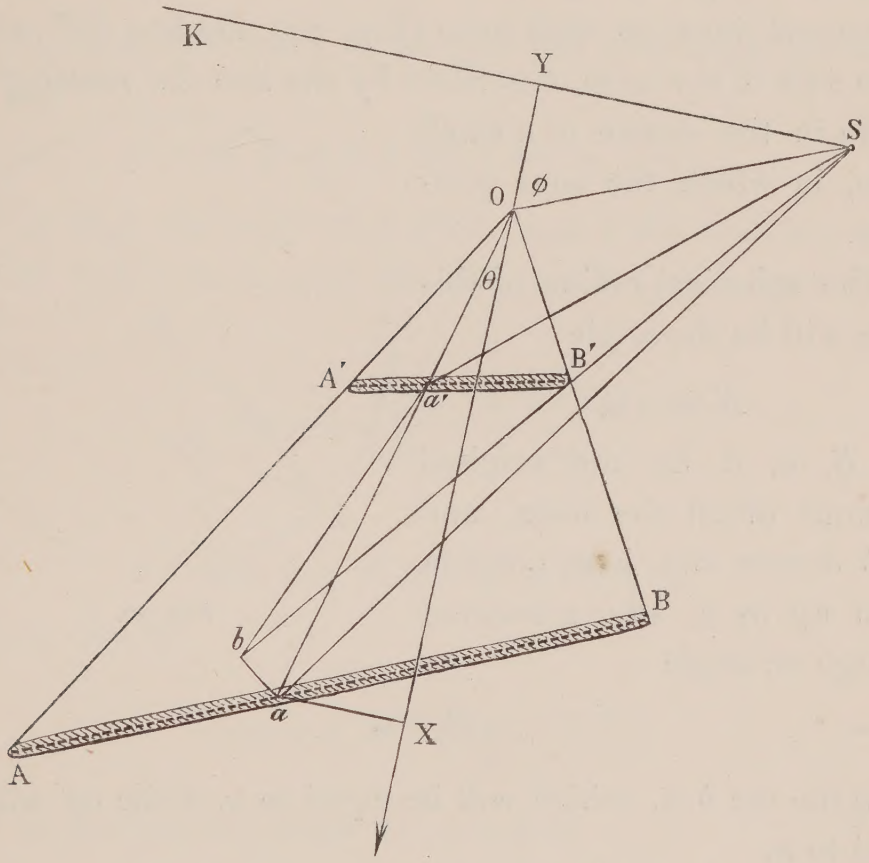


Fig. 69.

the bisector of the muscular fibres. Let aa' be any fibre which takes the position $a'b$, when a small rotation occurs round the axis SK, moving the point a to b, which lies outside the plane $ABA'B'$.

In the triangle aa'S, let

$$aa' = aO - a'O = l - l' = p$$

$$aS = A \qquad a'S = A' \qquad aSa' = x.$$

Hence we have

$$p^2 = A^2 + A'^2 - 2AA' \cos x.$$

In the triangle bSa', let

$$ba' = p' \quad bS = A \text{ (by hypothesis) } a'S = A' \quad bSa' = y.$$

Hence we have

$$p'^2 = A^2 + A'^2 - 2AA' \cos y.$$

If we imagine a sphere described round S, intersecting the several lines, we shall have (Fig. 70), the line SK, which is the axis of the cone, described by the line Sa, meeting the sphere in the centre of a small circle, in which the cone pierces it.

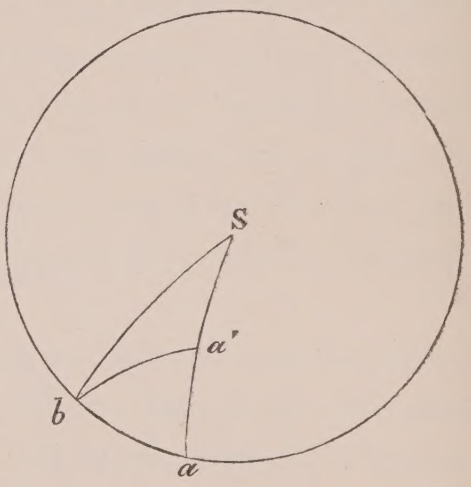


Fig. 70.

The spherical radius of this circle will be the angle

$$KSa = \sigma.$$

Let S, a, a' be the original positions of all the lines, and let b denote the new position taken up by a, after a rotation through an angle

$$aSb = \omega$$

Draw the arc $b\,a'$, which will be equal to y, while aa' will be equal to x.

In the spherical triangle $ba'S$, we have

$$bS = \sigma,\ ba' = y,\ Sa' = \sigma - x,\ bSa' = \omega.$$

Therefore

$$\cos y = \cos \omega \sin \sigma \sin (\sigma - x) + \cos \sigma \cos (\sigma - x),$$

and when ω is a moderately small angle

$$\cos \omega = 1 - \frac{\omega^2}{2},$$

which reduces the preceding equation to

$$\cos y = \{\sin \sigma \sin (\sigma - x) + \cos \sigma \cos (\sigma - x)\}$$

$$- \frac{\omega^2}{2} . \sin \sigma \sin (\sigma - x) ;$$

or, finally,

$$\cos y = \cos x - \frac{\omega^2}{2} \sin \sigma \sin (\sigma - x).$$

Substituting this value of $\cos y$ in the preceding equations, we find

$$p'^2 = p^2 + \omega^2 . AA' \sin \sigma \sin (\sigma - x) \qquad (46)$$

In Fig (69), the angle σ is KSa; and if we draw a perpendicular aX from a, upon the bisector YX, it is easy to see that

$$\sin \sigma = \frac{YX}{aS} = \frac{YO + OX}{aS}.$$

Let

$$OS = a, \ YOS = \phi, \ aOX = \theta,$$

and we obtain

$$\sin \sigma = \frac{a \cos \phi + l \cos \theta}{A};$$

and, in like manner, since $\sigma - x = KSa'$

$$\sin (\sigma - x) = \frac{a \cos \phi + l' \cos \theta}{A'}.$$

Hence, equation (46) becomes

$$p'^2 = p^2 + \omega^2 (a \cos \phi + l \cos \theta) (a \cos \phi + l' \cos \theta),$$

or,

$$p' = p \sqrt{1 + \omega^2 \frac{(a \cos \phi + l \cos \theta) (a \cos \phi + l' \cos \theta)}{p^2}};$$

expanding the root, and remembering that ω is moderately small, we obtain

$$p' = p\left(1 + \frac{\omega^2}{2}\cdot\frac{(a\cos\phi + l\cos\theta)\,(a\cos\phi + l'\cos\theta)}{p^2}\right)$$

and finally, substituting for p, its value $l - l'$, we have

$$p' - p = \delta p = \frac{\omega^2}{2}\cdot\frac{(a\cos\phi + l\cos\theta)\,(a\cos\phi + l'\cos\theta)}{l - l'}. \quad (47)$$

This expression denotes the shortening or lengthening of a single fibre aa', produced by the rotation (ω) round the axis KS; and the work done by the entire muscle, in a single contraction, will be,

$$\text{Work done} = \int \delta p\, d\theta = \frac{\omega^2}{2}\int\frac{(a\cos\phi + l\cos\theta)\,(a\cos\phi + l'\cos\theta)\,d\theta}{l - l'};$$

and we are required to investigate the conditions, which shall render this work a maximum.

Using the same notation, as in Prop. B, we have

$$l = \frac{b\sin\beta}{\sin(\beta - \theta)}, \qquad l' = \frac{b'\sin\beta'}{\sin(\beta' - \theta)}.$$

Hence,

$$\int \delta p\, d\theta =$$

$$\frac{\omega^2}{2}\int\frac{\{a\cos\phi\sin(\beta-\theta) + b\sin\beta\cos\theta\}\,\{a\cos\phi\sin(\beta'-\theta) + b'\sin\beta'\cos\theta\}\,d\theta}{b\sin\beta\,.\sin(\beta'-\theta) - b'\sin\beta'\,.\sin(\beta-\theta)}.$$

If we write the denominator,

$$\Delta = b\sin\beta\sin(\beta' - \theta) - b'\sin\beta'\sin(\beta - \theta);$$

we shall find, by expanding the numerator,

$$\int \delta p\, d\theta = \quad (48)$$

$$\frac{\omega^2}{2}\sin\beta\sin\beta'(a\cos\phi + b)\,(a\cos\phi + b')\int\frac{\cos^2\theta\, d\theta}{\Delta}$$

$$+\frac{\omega^2}{2} a^2 \cos^2\phi \cos\beta \cos\beta' \int \frac{\sin^2\theta d\theta}{\Delta}$$

$$-\frac{\omega^2}{2} a \cos\phi \times$$

$$\{(a\cos\phi + b)\sin\beta\cos\beta' + (a\cos\phi + b')\sin\beta'\cos\beta\} \times$$

$$\int \frac{\sin\theta\cos\theta\, d\theta}{\Delta}.$$

Previous to integrating the general expression for the work done by the skew quadrilateral muscle (48); it will be interesting to discuss the simpler case of a triangular muscle. which is deduced from the general expression by reducing the bone $A'B'$ to a point coinciding with the vertex O. This may be best done by combining the general expression for the work done, viz.,

$$\int \delta p d\theta = \frac{\omega^2}{2} \int \frac{(a\cos\phi + l\cos\theta)\,(a\cos\phi + l'\cos\theta)\, d\theta}{l - l'}.$$

with the condition, $l' = 0$.

Hence, we obtain,

$$\int \delta p d\theta = \frac{\omega^2}{2} \int \frac{a\cos\phi\,(a\cos\phi + l\cos\theta)\, d\theta}{l},$$

which becomes, by substituting for l, its value

$$l = \frac{b\sin\beta}{\sin(\beta - \theta)},$$

after a few reductions

$$\int \delta p d\theta = \frac{\omega^2}{2} \cdot \frac{a\cos\phi\,(a\cos\phi + b)}{b} \int \cos\theta d\theta. \qquad (49)$$

$$-\frac{\omega^2}{2} \cdot \frac{a^2\cos^2\phi\cot\beta}{b} \int \sin\theta d\theta.$$

Integrating this expression from $+\theta$ to $-\theta$, the second term will disappear, and we find for the work done,

$$\text{Work done} = \int \delta p d\theta = \omega^2 \sin\theta \cdot \frac{a\cos\phi\,(a\cos\phi + b)}{b} \qquad (50)$$

This work will be a maximum, when $\phi = 0$, when it becomes

$$\int \delta p d\theta = \omega^2 \sin \theta \, . \, \frac{a(a+b)}{b}.$$

Therefore, when a the distance OS (Fig. 69), of the socket joint from the vertex of the triangle is given, the maximum effect will be produced when the socket lies on the bisector of the vertical angle of the triangular muscle, that is, on a line perpendicular to the locus of socket for maximum effect, for axis perpendicular to the plane of the muscle considered in Prop. B. Hence, we see that the conditions for maximum effect are essentially different for the two axes of rotation, and that it is not possible for a given muscle to produce the maximum effect, by its rotation of the bone AB round both axes.

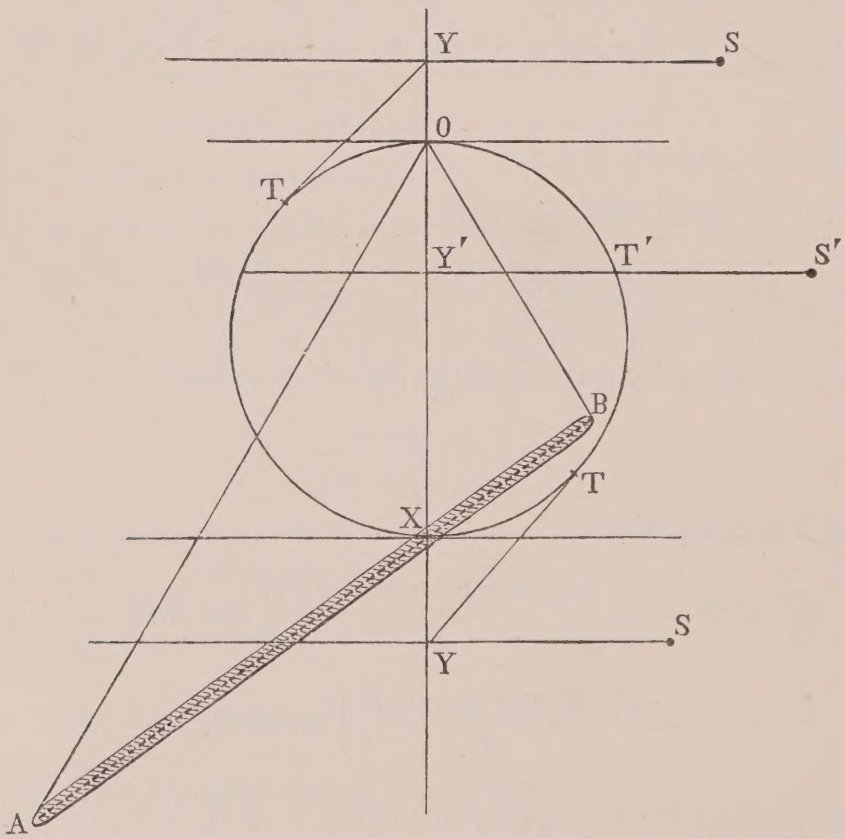


Fig. 71.

This is a remarkable result, and will lead us to important consequences.

The following geometrical construction for the work done for any position of the axis SY, Fig. 71, is easily remembered, and applies to every case. Describe the circle XTO, whose diameter is $OX = b$, the bisector of the angle made by the muscular fibres. From Y, the point where the axis of rotation intersects the bisector YOX, draw YT a tangent to the circle XTO; I say, that the work done by the muscle during a small rotation of the bone AB round the axis SY, is proportional to the square of the tangent YT. For, the work done is proportional to

$$a \cos \phi \, (a \cos \phi + b),$$

and,

$$YO = a \cos \phi,$$

$$YX = a \cos \phi + b,$$

also,

$$(YT)^2 = YO \times YX. \qquad \text{Euc. iii. 36.}$$

Therefore,

Work done, varies as $(YT)^2$. Q. E. D.

If the axis of rotation be supposed to move parallel to itself, the work done will be greater, the farther off the point Y is from the circle, either beyond O, or beyond X. When it passes through either O or X, the work done becomes zero; and for positions like $S'Y'$ lying between O and X, the work done becomes *negative*, and is proportional to the square of the ordinate $Y'T''$; and the negative work done will be a maximum, when the axis of rotation passes through the centre of the circle XTO; or through the point of bisection of the bisector of the vertical angle AOB.

The mechanical interpretation of the preceding facts, is—

1°. That if the axis of rotation, round which the bone AB is compelled to turn, be placed beyond O or X in either direction, no amount of contraction of the muscle can alter its position, for any change of position would be equivalent to a *total lengthening* of the fibres, which is impossible.

2°. That if the axis of rotation pass through either O or X (the vertex of triangular muscle or foot of bisector of vertical angle), the bone AB will remain, indifferently, in any position in which it is placed, for no work is done in altering its position.

3°. That if the axis of rotation lies between O and X, a contraction of the muscular fibres will immediately cause a motion of the bone round the given axis, for any change of position will be accomplished by a *total shortening* of the fibres, which is possible.

4°. The triangular muscle will exert a maximum effect in displacing the bone AB, round the axis passing through the socket joint, when that axis of rotation bisects the line OX, because in this case the *total shortening* of the fibres will be a maximum.

The positions of the axis of rotation, beyond O or X, which correspond to positive values of the work done, denote positions of *stable* equilibrium, and the work done by the muscle consists in forcing back the bone into the plane containing itself and the axis of rotation.

The positions of the axis of rotation, passing through O or X, correspond with positions of *neutral* equilibrium, when the bone and muscle are indifferent to the position in which they are placed, and no work is done by the contraction of the muscle, tending to rotate the bone round the axis.

The positions of the axis of rotation, lying inside O and X, which correspond to negative values of the work done, denote positions of *unstable* equilibrium, and the work done by the muscle consists in pulling the bone out of the plane containing itself and the axis of rotation.

All the preceding results may be verified, geometrically, by drawing the cone described by the bone AB round the axis of rotation, and considering in detail, the lengthening or

shortening of the muscular fibres joining the bone AB with the vertex O.

The preceding investigation leads to the following :—

Proposition C.

If, in a triangular muscle AOB, the moveable bone AB rotate round an axis, perpendicular to OX, the bisector of the angle AOB :—

1°. *The contraction of the muscle will tend to restore the bone to its original position, if the axis of rotation intersect the bisector OX, beyond either O or X; and its restoring power will increase with the distance of the intersection of the axis from O or X.*

2°. *If the axis of rotation pass through O or X, the contraction of the muscle can have no effect in moving the bone.*

3°. *The contraction of the muscle will tend further to displace the bone, if the axis of rotation intersect the bisector OX, between O and X; and its displacing power will be a maximum when the point of intersection of the axis bisects the bisector OX.*

In addition to the construction shown in Fig. 71, for the work done by the muscle, whether positive or negative, the following construction may be used, and is interesting, because it is a particular case of the construction belonging to the general equation (48).

Let us suppose the work done to be represented by the square of the ordinate of a curve, whose abscissa is $a \cos \phi = x$. Then, by equation (50), we have, for the triangular muscle,

$$y^2 = x(b + x). \qquad (51)$$

When y^2 is positive, this is the equation of an equilateral hy-

perbola, whose axis major is OX, the bisector of the angle made by the muscular fibres.

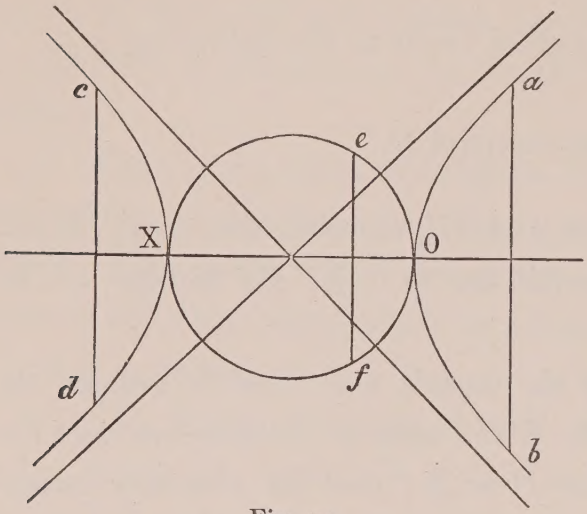


Fig. 72.

Let this hyperbola be aOb, cXd, Fig. 72; then the square of any chord ab, cd, will be proportional to the work done, by the contraction of the muscle, in rotating the bone round the chord considered as an axis.

When y^2 is negative, equation (51) denotes a circle described on the bisector OX as diameter; and the square of the chord ef is proportional to the work done, by the contraction of the muscle, in rotating the bone round the chord ef.

The denominator Δ, used in equation (48), is

$$\Delta = b \sin \beta \sin (\beta' - \theta) - b' \sin \beta' \sin (\beta - \theta);$$

or

$$\Delta = A \cos \theta + B \sin \theta,$$

where

$$A = (b - b') \sin \beta \sin \beta'.$$

$$B = b \sin \beta \cos \beta' - b' \sin \beta' \cos \beta.$$

If $B = 0$, the equation (48) becomes much simplified, and is reduced to the following:—

$$\int \delta p d\theta = \frac{\omega^2}{2} . \sin \beta \sin \beta' (a \cos \phi + b) (a \cos \phi + b') \int \frac{\cos^2 \theta d\theta}{A \cos \theta} + \frac{\omega^2}{2} . a^2 \cos^2 \phi . \cos \beta \cos \beta' \int \frac{\sin^2 \theta d\theta}{A \cos \theta} \qquad (52)$$

The third term of equation (48) disappears altogether, for it consists of pairs of elements, of the following form:—

$$\frac{\sin \theta \cos \theta d\theta}{A \cos \theta + B \sin \theta} - \frac{\sin \theta \cos \theta d\theta}{A \cos \theta - B \sin \theta}$$

or,

$$\frac{-2B \sin\theta \cos\theta d\theta}{A^2 \cos^2\theta - B^2 \sin^2\theta}$$

and each such pair of elements is zero, when $B = 0$.

The geometrical condition involved in the equation

$$B = 0$$

is shown in Fig. 73. Let the bones AB and $A'B'$ be produced to meet in the point Q.

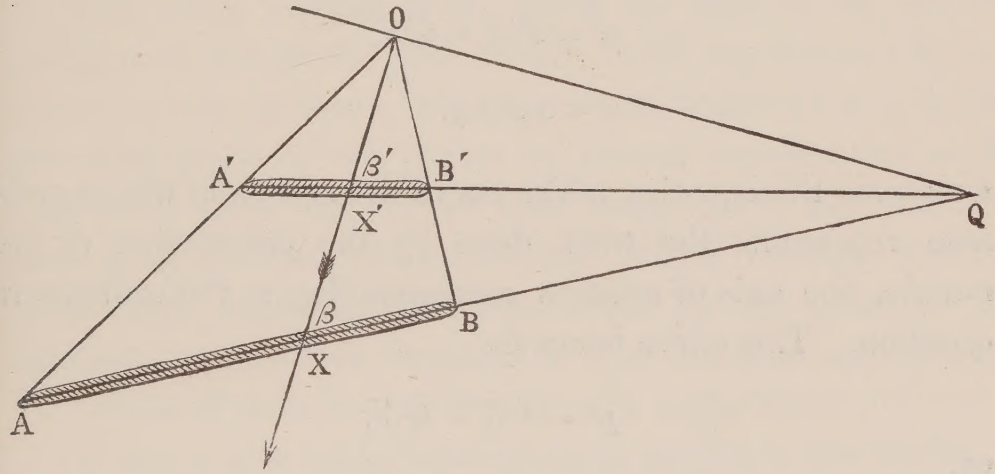


Fig. 73.

If the condition $B = 0$ be granted, then the point of intersection, Q, must lie somewhere on the line OQ drawn through the vertex O, perpendicular to OX, the bisector of the angle made by the muscular fibres. For

$$OQ = OX \tan\beta = OX' \tan\beta';$$

or,

$$b \tan\beta - b' \tan\beta' = 0,$$

which is equivalent to

$$B = b \sin\beta \cos\beta' - b' \sin\beta' \cos\beta = 0.$$

When this condition is fulfilled, equation (52) becomes, writing,

$$X = \int \frac{\cos^2\theta d\theta}{\cos\theta} \qquad Y = \int \frac{\sin^2\theta d\theta}{\cos\theta}$$

$$P = \sin\beta \sin\beta' (b + a\cos\phi)(b' + a\cos\phi),$$

$$Q = \cos\beta \cos\beta' . a^2 \cos\phi^2,$$

$$\int \delta p d\theta = (PX + QY) \left(\frac{\omega^2}{2 (b - b') \sin\beta \sin\beta'} \right).$$

If, as before, we write

$$y^2 = PX + QY,$$

$$x = a\cos\phi,$$

we obtain the equation of the curve the square of whose ordinate represents the work done by the contraction of the muscle, the axis of rotation corresponding to the ordinate in question. This curve becomes

$$y^2 = PX + QY,$$

or,

$$y^2 = X \sin\beta \sin\beta' (b + x) (b' + x) + Y \cos\beta \cos\beta' x^2. \quad (53)$$

This is the equation of a central conic, whose major axis lies on the bisector OX, and whose centre is situated at a distance from O towards X represented by

$$\delta = -\frac{1}{2} \cdot \frac{(b + b') X \sin\beta \sin\beta'}{X \sin\beta \sin\beta' + Y \cos B' \cos\beta''} \quad (54)$$

Since X and Y are essentially positive, and since the condition $B = 0$, requires β and β' to be both acute, δ will be essentially negative, and the centre of the conic will lie inside the vertex of the triangle AOB.

The positions of the axis on the bisector of AOB, which correspond to neutral equilibrium are found by equating (53)

to zero, which gives the following quadratic whose roots determine the positions of neutral equilibrium :—

$$X \sin\beta \sin\beta' (b + x)(b' + x) + Y \cos\beta \cos\beta' x^2 = 0.$$

The condition for real roots in this quadratic is, that

$$X \sin\beta \sin\beta' \{(b - b')^2 X \sin\beta \sin\beta' - 4bb' Y \cos\beta \cos\beta'\} \quad (55)$$

shall be positive.

If this condition be fulfilled, and the points corresponding to neutral equilibrium be taken on the bisector of AOB; for all axes lying outside those limits the equilibrium will be stable, and the work done positive, and represented by the square of the ordinate of an *hyperbola* constructed with the intercept between the points of neutral equilibrium as its transverse axis, and for all axes lying inside the points of neutral equilibrium, the equilibrium will be unstable, and the work done negative, and represented by the square of the ordinate of an *ellipse* constructed with the intercept between the points of neutral equilibrium as its major axis.

Hence a geometrical construction, similar to that shown in Fig. (72), may be made to represent the work done; provided we replace the circle and equilateral hyperbola, by an ellipse and hyperbola with unequal axes.

If the equation of the central conic (53) be referred to its centre, as origin of co-ordinates, it will have the form

$$\pm \frac{y^2}{n^2} = \frac{x^2}{m^2} - 1 ; \quad (56)$$

where

$$n^2 = \frac{X \sin\beta \sin\beta' \{(b - b')^2 X \sin\beta \sin\beta' - 4bb' Y \cos\beta \cos\beta'\}}{4 \{X \sin\beta \sin\beta' + Y \cos\beta \cos\beta'\}}$$

$$m^2 = \frac{X \sin\beta \sin\beta' \{(b - b')^2 X \sin\beta \sin\beta' - 4bb' Y \cos\beta \cos\beta'\}}{4 \{X \sin\beta \sin\beta' + Y \cos\beta \cos\beta'\}^2} \quad (57)$$

These values for m^2 and n^2 are essentially positive when the

quantity (55) is positive, which is the condition necessary for the existence of real positions of unstable equilibrium.

When the equilibrium is stable, y^2 is positive, and the equation becomes

$$\frac{x^2}{m^2} - \frac{y^2}{n^2} = 1 ;$$

which represents an *hyperbola*, whose axes are m and n, the real axis being m, and situated on the bisector of the angle AOB.

When the equilibrium is unstable, y^2 is negative, and the equation becomes

$$\frac{x^2}{m^2} + \frac{y^2}{n^2} = 1 ;$$

which represents an *ellipse*, whose axis are m and n; the axis m being situated on the bisector of the angle AOB; and the maximum value of the negative work, corresponding with the position of maximum instability of the muscle, determined by equation (54), will be represented by n^2.

From the preceding equations it appears that

$$\frac{n^2}{m^2} = X \sin\beta \sin\beta' + Y \cos\beta \cos\beta' ;$$

or, that the conjugate axis will be greater or less than the transverse axis, according as this function is greater or less than unity.

There are three distinct cases to be considered according as the expression (55) is positive, zero, or negative.

1°. If the expression (55)

$$(b - b')^2 X \sin\beta \sin\beta' - 4bb' Y \cos\beta \cos\beta'$$

be positive, then m^2 and n^2, in the equations (56, 57), are both

positive, and these equations will represent, for all values of y^2, positive or negative, an ellipse and hyperbola, having the same axes.

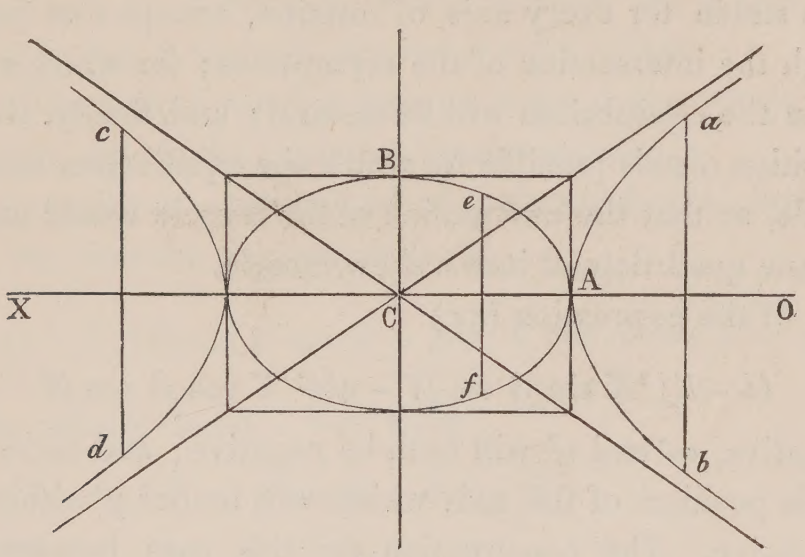


Fig. 74.

Let C be the common centre of the ellipse and hyperbola, Fig. (74), found by the equation (54), by measuring off $OC = \delta$. Take $CA = m$ and $CB = n$, and construct the ellipse and hyperbola corresponding to these lines as semi-axes, the line OX being the real axis of the hyperbola. Then, as before, the work done by the quadrilateral muscle in rotating the moveable bone round any axis, ab, cd, or ef will be proportional to the square of these chords; and if the chord of rotation intersect the hyperbola, the muscular contraction will restore the skew quadrilateral to the plane; but if the chord of rotation intersect the ellipse, the contraction of the muscle will convert the plane quadrilateral into a skew muscle; and finally, the contraction of the muscle will produce no effect at all, if the axis of rotation pass through the extremity of the axis of the ellipse and hyperbola.

2°. If the expression (55)

$$(b - b')^2 X \sin\beta \sin\beta' - 4bb' Y \cos\beta \cos\beta'$$

be equal to zero; the semi-axes m and n will both become

zero also; and the hyperbola will be reduced to its asymptotes while the ellipse disappears altogether. Hence, in this case, the equilibrium of the plane quadrilateral muscle will be always stable for every axis of rotation, except that passing through the intersection of the asymptotes; for which axis of rotation the equilibrium will be neutral; and, finally, there is no position of axis possible for which the equilibrium becomes unstable, so that the contraction of the muscle would convert the plane quadrilateral into a skew muscle.

3°. If the expression (55)

$$(b - b')^2 X \sin\beta \sin\beta' - 4bb' Y \cos\beta \cos\beta'$$

be negative, m^2 and n^2 will both be negative; and there is no possible position of the axis which will render y^2 either zero or negative. The construction for this case becomes the following (Fig. 75):—

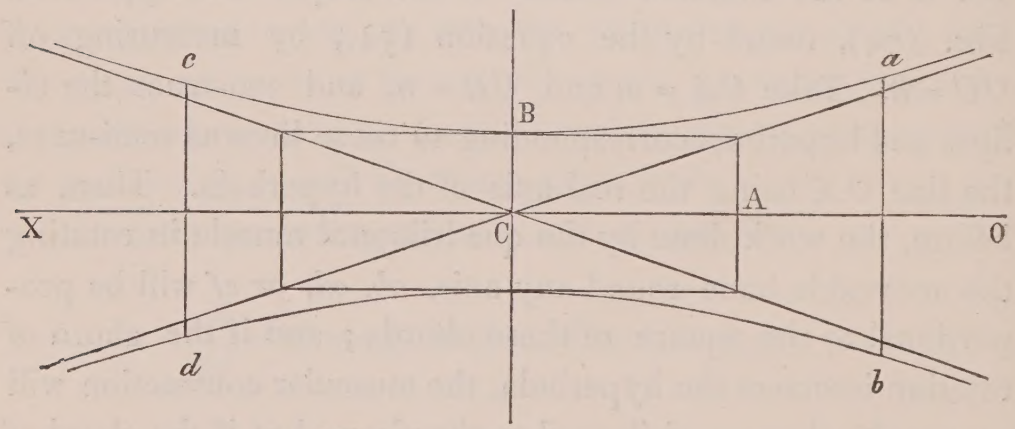


Fig. 75.

Construct, with the same axes and asymptotes as before, the conjugate hyperbola, whose real axis is $CB = n$, on the line CB, perpendicular to OX, the bisector of the angle made by the muscular fibres. If a rotation take place round any chord ab, cd, the work done by the muscle will be proportional to the square of the chord of rotation, and its contraction will always restore the skew muscle to the plane, and the equilibrium will never become neutral or unstable.

In the three cases now considered, we have :

1°. The axis of rotation passing through the centre of the conic (56, 57), is the axis of *maximum instability*.

2°. The axis of rotation passing through the centre of the conic (56, 57), is the axis of neutral equilibrium.

3°. The axis of rotation passing through the centre of the conic (56, 57), is the axis of *minimum stability*.

The condition contemplated in the second of the foregoing cases, viz. that the expression (55)

$$(b - b')^2 X \sin\beta \sin\beta' - 4bb' Y \cos\beta \cos\beta' = 0,$$

may be readily found. For, since

$$b \tan\beta = b' \tan\beta',$$

we have

$$\tan\beta' = \frac{b}{b'} \tan\beta,$$

which reduces (55) to the following :

$$\tan^2\beta = \frac{4b'^2}{(b - b')^2} \frac{Y}{X},$$

or

$$\tan\beta = \frac{2b'}{b - b'} \sqrt{\frac{Y}{X}}. \qquad (58)$$

When the angle β exceeds the value determined by equation (58), the expression (55) will be positive, and a position of axis of unstable equilibrium will be possible ; but when β is less than the value assigned by equation (58), the equilibrium will, in all cases, be stable, and there is no position of axis of rotation which will render it unstable.

We shall now consider the general expression (48) for the work done by a quadrilateral muscle revolving round an axis in its plane, and perpendicular to the bisector of the angle AOB, contained between its extreme fibres.

If we write

$$X = \int \frac{\cos^2\theta d\theta}{\Delta} \quad Y = \int \frac{\sin^2\theta d\theta}{\Delta} \quad Z = \int \frac{\sin\theta\cos\theta d\theta}{\Delta}$$

equation (48) becomes, writing as before, $x = a\cos\phi$,

$$\frac{2}{\omega^2}\int \delta p d\theta = X\sin\beta\sin\beta'(b+x)(b'+x) + Y\cos\beta\cos\beta' x^2 - Zx\{\sin\beta\cos\beta'(b+x) + \sin\beta'\cos\beta(b'\pm x).$$

If $$y^2 = \frac{2}{\omega^2}\int \delta p d\theta$$

we obtain, for the geometrical representation of the work done, the following conic:

$$y^2 = \{X\sin\beta\sin\beta' + Y\cos\beta\cos\beta' - Z\sin(\beta+\beta')\}x^2 + \{(b+b')X\sin\beta\sin\beta' - Z(b\sin\beta\cos\beta' + b'\sin\beta'\cos\beta)\}x + bb' X\sin\beta\sin\beta'. \quad (59)$$

The centre of the conic,

$$y^2 = Lx^2 + Mx + N,$$

is situated at a distance δ from the origin, determined by the equation

$$2Lx + M = 0,$$

or

$$\delta = -\tfrac{1}{2}\cdot\frac{(b+b')X\sin\beta\sin\beta' - Z(b\sin\beta\cos\beta' + b'\sin\beta'\cos\beta)}{X\sin\beta\sin\beta' + Y\cos\beta\cos\beta' - Z\sin(\beta+\beta')} \quad (60)$$

Conclusions, similar to those obtained from equation (53), may be drawn from equation (59).

1°. The equilibrium of the plane quadrilateral muscle will be stable when y^2 is positive.

2°. The equilibrium will be neutral when $y^2 = 0$.

3°. The equilibrium of the plane muscle will be unstable when y^2 is negative.

4°. The stability of the equilibrium increases indefinitely as the axis of rotation becomes more and more remote from O.

5°. The instability of the muscle reaches a maximum at a position of the axis of rotation passing through the centre of the conic determined by equation (60).

The definite integrals, X, Y, Z, are readily found as follows. We have

$$X = \int \frac{\cos^2\theta d\theta}{A\cos\theta + B\sin\theta},$$

$$Y = \int \frac{\sin^2\theta d\theta}{A\cos\theta + B\sin\theta},$$

$$Z = \int \frac{\sin\theta\cos\theta d\theta}{A\cos\theta + B\sin\theta},$$

where

$$A = (b - b')\sin\beta\sin\beta'$$

$$B = b\sin\beta\cos\beta' - b'\sin\beta'\cos\beta.$$

Assume

$$R\sin\alpha = A, \qquad R\cos\alpha = B;$$

then we have

$$R = \sqrt{A^2 + B^2},$$

and

$$X = \frac{1}{R}\int \frac{\cos^2\theta d\theta}{\sin(\alpha + \theta)}$$

$$Y = \frac{1}{R}\int \frac{\sin^2\theta d\theta}{\sin(\alpha + \theta)}$$

$$Z = \frac{1}{R}\int \frac{\sin\theta\cos\theta d\theta}{\sin(\alpha + \theta)}.$$

Integrating these expressions between $+\theta$ and $-\theta$, we obtain, after some reductions, the following values, which may be readily computed, for given values of A and B:—

$$X = \frac{1}{R}\left\{2\sin a \sin\theta + \cos^2 a \log_e \frac{\tan\frac{1}{2}(a+\theta)}{\tan\frac{1}{2}(a-\theta)}\right\}$$

$$Y = \frac{1}{R}\left\{-2\sin a \sin\theta + \sin^2 a \log_e \frac{\tan\frac{1}{2}(a+\theta)}{\tan\frac{1}{2}(a-\theta)}\right\} \qquad (61)$$

$$Z = \frac{1}{R}\left\{2\cos a \sin\theta - \sin a \cos a \log_e \frac{\tan\frac{1}{2}(a+\theta)}{\tan\frac{1}{2}(a-\theta)}\right\}$$

The best examples, in Nature, of the rotation, by means of a quadrilateral muscle, of limbs round an axis passing through a fixed socket, and perpendicular to the bisector of the angle made by the extreme fibres, are to be found in the wings of birds, which are depressed by the contraction of the great pectoral muscle, in the manner here considered. I shall give a few examples, to illustrate the general principles just given.

(*a*). *Wing of the Albatross.*—In Fig. 76, the bone AB represents the origin of the pectoral muscle, from the *furculum*, A, and *sternum*, B, and $A'B'$ represents the insertion of the muscle into the *humerus ;* both bones being placed in the same plane, when the wing is extended, previous to the contraction of the muscle. The centre of the shoulder-joint is shown at S, and the bone $A'B'$ is rotated, by the action of the pectoral muscle, round the axis ST, perpendicular to OX, the bisector of the angle AOB.

We are required to calculate, from the preceding theory the position of the axis of rotation, corresponding with the maximum work done, when the system is in a position of *unstable* equilibrium ; or, in other words, we are required to find the centre of the conic (59).

The theory just laid down, supposes the bones AB and $A'B'$ to be straight bones; and before applying it to the wing of the Albatross, it is necessary to calculate the magnitude

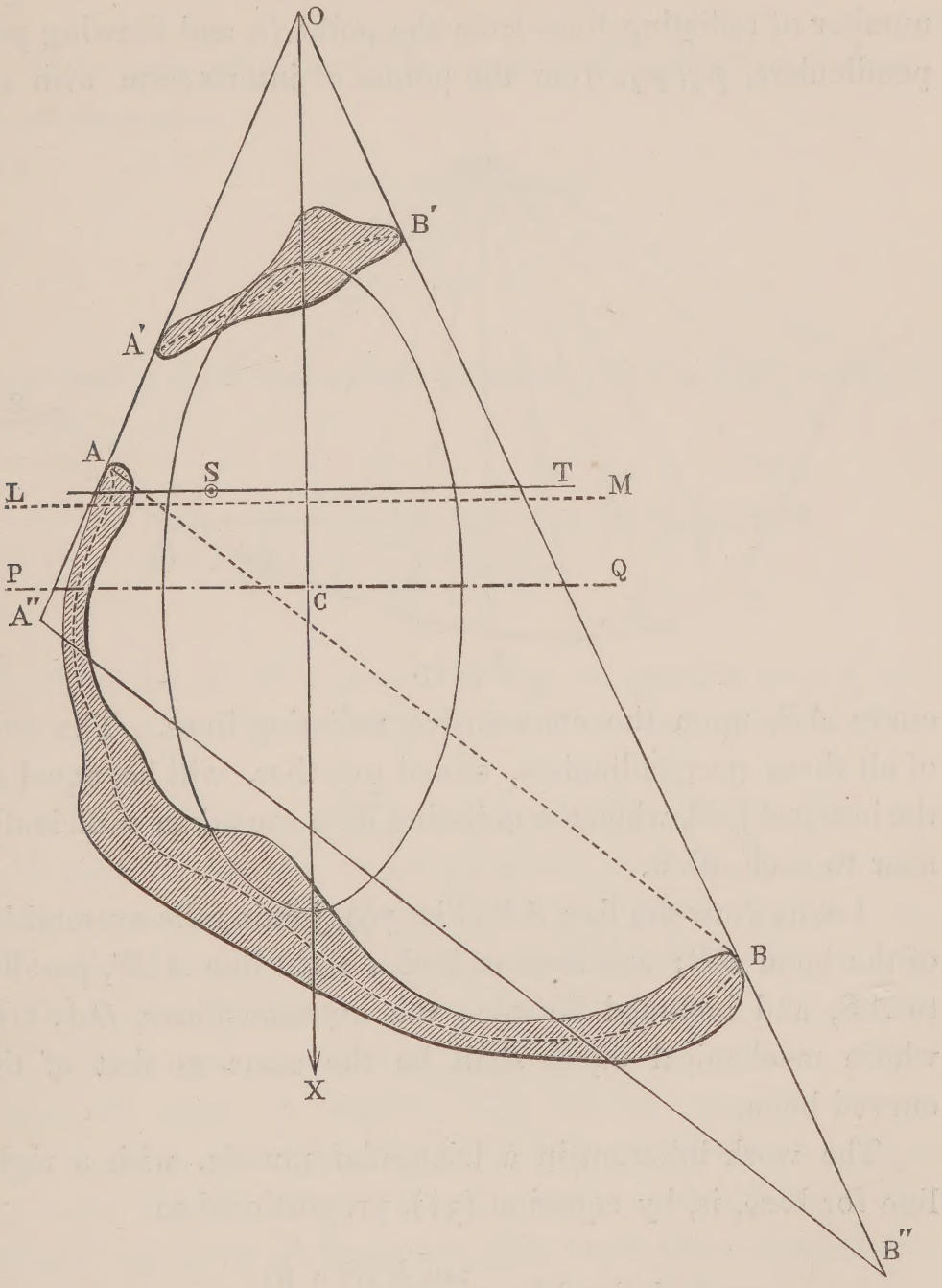


Fig. 76.

and position of a straight bone $A''B''$, which would be mechanically equivalent to the curved bone AB. Let AB be

any curved origin (Fig. 77) of a muscle, whose fibres converge to the point O. The work done by this muscle is represented by $\int l d\theta$, where l is the length of each fibre—and this integral is readily found by construction, by drawing a number of radiating lines from the point O, and drawing perpendiculars, pq, pq, from the points of intersection with the

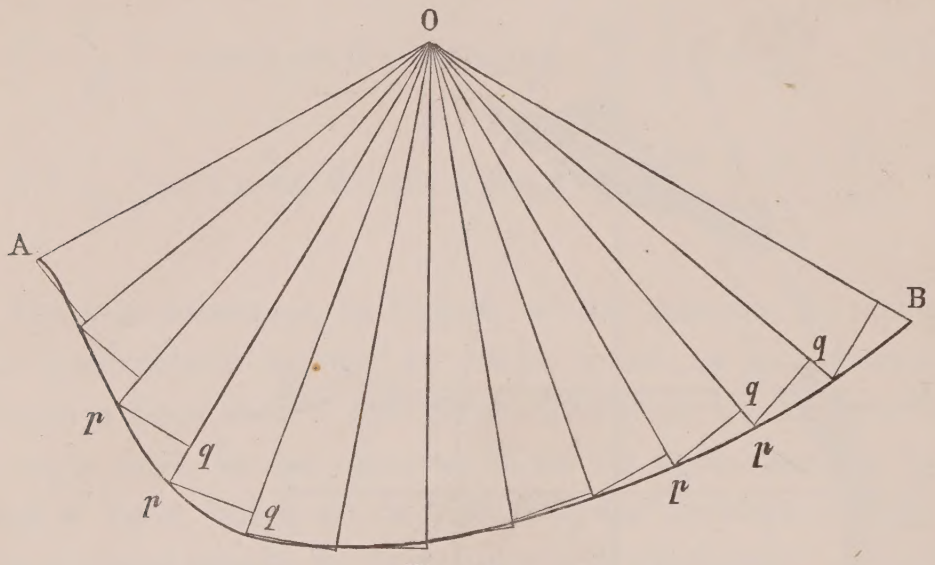


Fig. 77.

curve AB, upon the consecutive radiating lines. The sum of all these perpendiculars, added together, will be equal to the integral $\int l d\theta$, when the radiating lines are taken sufficiently near to each other.

Let us draw the line AB (Fig. 76), joining the extremities of the bone AB; and seek to find a right line $A''B''$, parallel to AB, and included between the extreme fibres, OA, OB, whose mechanical effect shall be the same as that of the curved bone.

The work inherent in a triangular muscle, with a right line for base, is, by equation (31), proportional to

$$b \sin \beta \log_e \frac{\tan \frac{1}{2}(\beta + \theta)}{\tan \frac{1}{2}(\beta - \theta)};$$

where b is the bisector of the vertical angle, and β is the angle made by it, with the base.

Hence we have

$$\int l d\theta = b \sin \beta \log_e \frac{\tan \frac{1}{2}(\beta + \theta)}{\tan \frac{1}{2}(\beta - \theta)}. \qquad (62)$$

In the case of the pectoral muscle of the Albatross, I found by measurement—

$$\int l d\theta = 7.17 \text{ inches.}$$
$$2\theta = 47°.$$
$$\beta = 129°.$$

Introducing these values into equation (62), and solving for b, we find

$$b = \frac{7.17}{\sin (129°) \log_e \frac{\tan 76° 15'}{\tan 52° 45'}} = 8.1317 \text{ inches.}$$

The right line $A''B''$ is drawn, in Fig. 76, parallel to AB, and intersecting the bisector OX, at the distance just found, from the vertex O.

A straight bone, $A''B''$, acting as the origin of the quadrilateral pectoral muscle, would produce upon the wing bone, $A'B'$, a mechanical effect similar to that of the curved bone, AB; but it would, obviously, cause much inconvenience by its awkward shape and length. Hence we see the reason for the curved outlines of many bony origins of muscles, which frequently occur in Nature. The curved bone fits into a smaller space, gives rounded outlines to the structure, and at the same time produces all the mechanical effects of the straight bones, which mathematicians use in their calculations; and we are obliged, in order to interpret Nature, first to translate the expressive language of her beautiful and symmetrical curves, into the less perfect language of the rigid lines and bars used by geometers.

Collecting together all the measurements of the wing of the Albatross, we have

$$b = 8.13 \text{ inches.} \qquad \beta = 129°$$
$$b' = 2.67 \quad ,, \qquad \beta' = 68° \qquad 2\theta = 47°.$$

From these we readily obtain—

$$A = (b - b') \sin \beta \sin \beta' = 3.934$$

$$B = b \sin \beta \cos \beta' - b' \sin \beta' \cos \beta = 3.925$$

$$R = \sqrt{A^2 + B^2} = 5.55$$

$$a = \tan^{-1}\left(\frac{A}{B}\right) = 45°$$

$$\log_e \frac{\tan \frac{1}{2}(a + \theta)}{\tan \frac{1}{2}(a - \theta)} = 1.2770$$

$$X = \frac{1.2024}{5.55} = + 0.2166$$

$$Y = \frac{0.0746}{5.55} = + 0.0138$$

$$Z = - Y = - 0.0138.$$

The centre of the conic (59) is found from equation (60), which is equivalent to

$$\delta = - \tfrac{1}{2} \frac{(b + b') X - Z (b \cot \beta' + b' \cot \beta)}{X + Y \cot \beta \cot \beta' - Z (\cot \beta + \cot \beta')}.$$

We have, by the preceding values,

$$(b + b') X = 10.80 \times 0.2166 = + 2.3393$$
$$- Z (b \cot \beta' + b' \cot \beta) = 0.0138 \times 1.12 = + 0.0154$$
$$\textit{Numerator}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad + 2.3547$$
$$X = \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad + 0.2166$$
$$Y \cot \beta \cot \beta' = 0.0138 \times - 0.327 = - 0.0045$$
$$- Z (\cot \beta + \cot \beta') = 0.0138 \times - 0.406 = - 0.0056$$
$$\textit{Denominator}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad + 0.2065$$

Therefore, finally,

$$\delta = -\tfrac{1}{2}\frac{2{\cdot}3547}{0.2065} = -5.70 \text{ inches.}$$

The negative sign of δ denotes, that the axis of rotation of maximum work lies at the near side of the vertex O. Draw PQ perpendicular to OX, so that

$$OC = 5.70 \text{ inches,}$$

then C is the centre of the conic (59), and its minor axis, PQ, is the axis of rotation, which would produce the maximum work in the motion of the wing.

In order to construct the conic, let us divide equation (59) by $\sin\beta \sin\beta'$, and we obtain

$$\frac{y^2}{\sin\beta \sin\beta'} = Lx^2 + Mx + N; \qquad (63)$$

where

$$L = X + Y \cot\beta \cot\beta' - Z(\cot\beta + \cot\beta');$$

$$M = (b + b')X - Z(b \cot\beta' + b' \cot\beta);$$

$$N = bb' X.$$

Substituting, in these expressions, the values already given, we obtain

$$\frac{y^2}{\sin\beta \sin\beta'} = 0.206\, x^2 + 2.355\, x + 4.702.$$

This equation gives us, when $y = 0$,

$$\left.\begin{matrix} x_1 \\ x_2 \end{matrix}\right\} = -5.700 \pm 3.138 \text{ inches,}$$

from which it appears, that the semi-axis major of the ellipse, whose centre is C (Fig. 76), is 3.138 inches.

Equation (63) denotes an ellipse or hyperbola, according as y^2 is negative or positive;—that is, according as the equilibrium of the bones is unstable or stable. Making y^2 negative, we have

$$y^2 = -\sin\beta \sin\beta' (Lx^2 + Mx + N),$$

and this expression will become a maximum, and equal to the square of the semi-axis minor, when

$$x = \delta = -\frac{M}{2L} = -5.70 \text{ inches},$$

which corresponds with C, the centre of the conic.

Writing the equation (63)

$$-y'^2 = \frac{y^2}{\sin\beta \sin\beta'} = Lx^2 + Mx + N,$$

we have, for the semi-axis minor of the ellipse formed by (y', x),

$$y' = \sqrt{-(L\delta^2 + M\delta + N)} = 1.424 \text{ inches}.$$

The ellipse (y', x) drawn to scale is shown in Fig. 76; and its equation, referred to C as origin of coordinates, is

$$\left(\frac{x}{3.138}\right)^2 + \left(\frac{y}{1.424}\right)^2 = 1. \qquad (64)$$

The work done by the muscle, causing the bone $A'B'$ to rotate round any two chords PQ and ST, is proportional to the squares of those chords; and we may, therefore, compare the *actual work* done round the chord, ST, with the *calculated maximum* round the chord PQ.

The point S is the observed centre of the glenoid cavity of the shoulder joint, and ST is the actual axis round which the wing rotates. This axis of rotation is placed at a distance of 4.83 inches from the vertex O; and, as we have just seen, the axis PQ, corresponding to the imaginary straight bone $A''B''$, lies at a distance from O, equal to 5.70 inches. The distance, therefore, between the actual axis, ST, and the calculated axis, PQ, is 0.87 inch. By substituting in equation (64), for x, the intercept 0.87 between ST and PQ, we find

$$\left(\frac{y}{1.424}\right) = 0.9232,$$

or

$$y^2 = 1.872;$$

but

$$n^2 = 2.028.$$

Therefore

$$\frac{n^2 - y^2}{n^2} = \frac{156}{2028} = \frac{1}{13};$$

or, the work done by the rotation of the wing of the Albatross round its observed socket differs by $\frac{1}{13}$th part from the calculated possible *maximum* derivable from the imaginary straight bone $A''B''$.

In Proposition C, we have seen that, in a triangular muscle, the position of maximum effect of the axis of rotation corresponds with the point of bisection of the bisector of the vertical angle; and it may be seen, from the preceding calculation, that, in a quadrilateral muscle with straight bones, the axis of rotation of maximum effect *nearly* corresponds to the point of bisection of the portion of the bisector included within the quadrilateral. For the distance from O to this point of bisection is

$$\delta = \frac{b + b'}{2} = \frac{8.13 + 2.67}{2} = 5.40 \text{ inches};$$

which may be regarded as a useful approximation to the true position of the maximum axis, which is 5·70 inches.

The method of finding a quadrilateral muscle, with straight bones, equivalent to a given muscle with curved bones which has just been given, leads to difficult calculations; and, moreover, does not seem to bring the calculated axis PQ, sufficiently near to the actual axis ST, to satisfy the requirements of exact science; it may, therefore, be useful to give another mode of computation, somewhat less laborious, and which may be made to approximate as near as we please to the conditions imposed by the actual curves formed by the bones of origin and insertion.

The general expression (47),

$$\int \delta p d\theta = \frac{\omega^2}{2} \int \frac{(a \cos \phi + l \cos \theta)\ (a \cos \phi + l' \cos \theta)}{l - l'},$$

for the work done during a rotation (ω) round an axis perpendicular to the bisector may be thus written, making $a \cos \phi = x$:

$$\frac{2}{\omega^2} \int \delta p d\theta = \int \frac{(x + l \cos \theta)\ (x + l' \cos \theta)}{l - l'}$$

or

$$\frac{2}{\omega^2} \int \delta p d\theta = y^2 = Lx^2 + Mx + N, \qquad (65)$$

where

$$L = \int \frac{d\theta}{l - l'}, \quad M = \int \frac{(l + l') \cos \theta d\theta}{l - l'}, \quad N = \int \frac{ll' \cos^2 \theta d\theta}{l - l'}.$$

Now, if the bones of the wing be carefully laid down to scale, and a number of radii drawn through the point O, the definite integrals L, M, N, may be found, by construction, to any degree of approximation, as shown by the following example of the Albatross, in which the measurements are made in twenty-fourths of an inch:—

Wing of Albatross.

Angle.	$l - l'$.	$l \cos \theta$.	$l' \cos \theta$.
0°	31	109	81
5	88	162	77
10	120	190	73
15	136	204	69
20	148	213	66
25	170	231	61
30	186	239	58
35	195	246	57
40	198	245	56
45	194	231	55

From these measurements, the following values are calculated :—

Wing of Albatross.

Angle.	$\frac{1}{l - l'}$	$\frac{(l + l') \cos \theta}{l - l'}$	$\frac{ll' \cos 2\theta}{l - l'}$
0°	0.03225	6.1219	284.80
5	0.01136	2.7160	141.75
10	0.00833	2.1917	115.58
15	0.00735	2.0074	103.50
20	0.00675	1.8851	94.98
25	0.00588	1.7164	82.83
30	0.00538	1.5968	74.53
35	0.00512	1.5506	71.76
40	0.00505	1.5202	69.29
45	0.00515	1.4742	65.49
Sum,	**0.0962**	**22.7803**	**1104.51**

Hence we obtain, approximately,

$$L = \int_{-\theta}^{+\theta} \frac{d\theta}{l \quad l'} = 0.0962.$$

$$M = \int_{-\theta}^{+\theta} \frac{(l + l') \cos \theta d\theta}{l - l'} = 22.7803.$$

$$N = \int_{-\theta}^{+\theta} \frac{ll' \cos^2 \theta d\theta}{l - l'} = 1104.51.$$

The centre of the conic,

$$y^2 = Lx^2 + Mx + N, \tag{66}$$

is found by the equation,

$$\delta = -\tfrac{1}{2} \frac{M}{L} = -\tfrac{1}{2} \cdot \frac{22.7803}{0.0962} = -118.4$$

$$= -4.93 \text{ inches.}$$

The actual distance of the axis passing through the socket S, from the vertex O, is 4.83 inches; and the distance calculated from our former approximation was 5.70 inches. The result of the present calculation is 4.93 inches.

If m and n denote the semi-axis major and minor of the ellipse (66) we have

$$m = \sqrt{\frac{M^2 - 4LN}{2L}} = 50.37; \qquad n = m\sqrt{L} = 15.62. \tag{67}$$

When expressed in inches, these values are

$$m = 2.099 \text{ inches};$$
$$n = 0.651 \quad ,,$$

The line LM, Fig. 76, is the minor axis of the conic (66), and it is distant from the actual axis ST, by only one-tenth of an inch.

If we substitute one-tenth of an inch for x, in the equation

$$\left(\frac{x}{2.099}\right)^2 + \left(\frac{y}{0.651}\right)^2 = 1,$$

we find $$\left(\frac{y}{0.651}\right)^2 = 0.9773;$$

$$y^2 = 0.4142.$$

This represents the amount of work done round the axis ST; and the work done round the axis LM is

$$n^2 = 0.4238.$$

Hence we find $$\frac{n^2 - y^2}{n^2} = \frac{96}{4238} = \frac{1}{44},$$

or the work done by the rotation of the wing of the Albatross round its observed socket differs by only $\frac{1}{44}$th part from the calculated possible maximum work. This degree of agreement between calculation and observation is sufficient to satisfy all reasonable expectations.

(*b.*) *Wing of the Wood Pigeon.*—The next example that I shall select is the wing of the Wood Pigeon, shown in Fig. 78. The origin of the pectoral muscle is shown at AB, and its insertion at $A'B'$. The angle AOB is 32°, and the socket of the wing is placed at S; the line ST being the axis round which the wing-bone revolves.

Dividing the angle AOB into six equal parts, I found the following measurements, made in thirty-seconds of an inch:—

Wing of Wood Pigeon.

Angle.	$l - l'$	$l \cos \theta$.	$l' \cos \theta$.
0°.00′	11	59	49
5.20	76	121	45
10.40	96	138	39
16.00	115	153	38
21.20	129	163	35
26.40	143	161	33
32.00	156	180	30

From these measurements the following values are calculated:—

Wing of Wood Pigeon.

Angle.	$\frac{1}{l-l'}$	$\frac{(l+l')\cos\theta}{l-l'}$	$\frac{ll'\cos^2\theta}{l-l'}$
0°.00′	0.09091	9.8182	262.82
5.20	0.01317	2.1843	71.64
10.40	0.01041	1.8437	56.06
16.00	0.00869	1.6609	50.55
21.20	0.00775	1.5349	44.22
26.40	0.00699	1.3567	37.15
32.00	0.00641	1.3462	34.61
Sum,	**0.14433**	**19.7449**	**557.05**

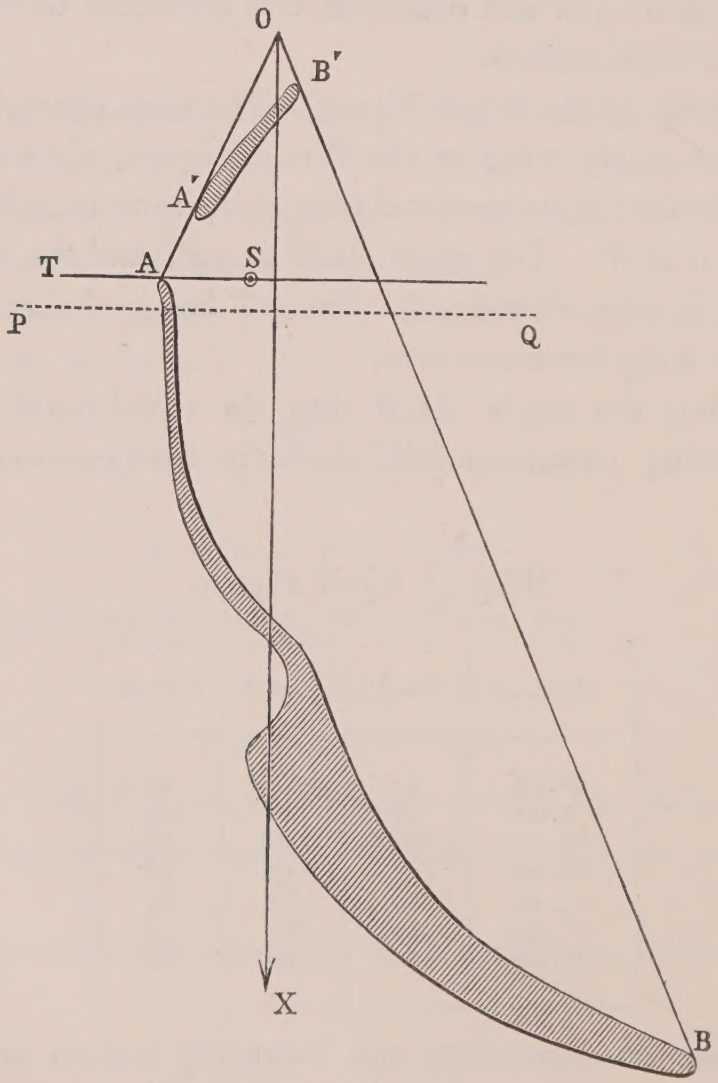


Fig. 78.

Hence, we have, for the centre of the ellipse (66) —

$$\delta = -\frac{M}{2L} = -\frac{1}{2} \cdot \frac{19{\cdot}7449}{0{\cdot}14433} = 68{\cdot}4.$$

The distance of the actual axis of the wing, ST, from O is 64, while the distance of the calculated axis of maximum work is 68.4.

(*c.*) *Wing of the Common Heron.*—The wing of the Heron is shown in Fig. 79, where AB is the origin, and $A'B'$ the

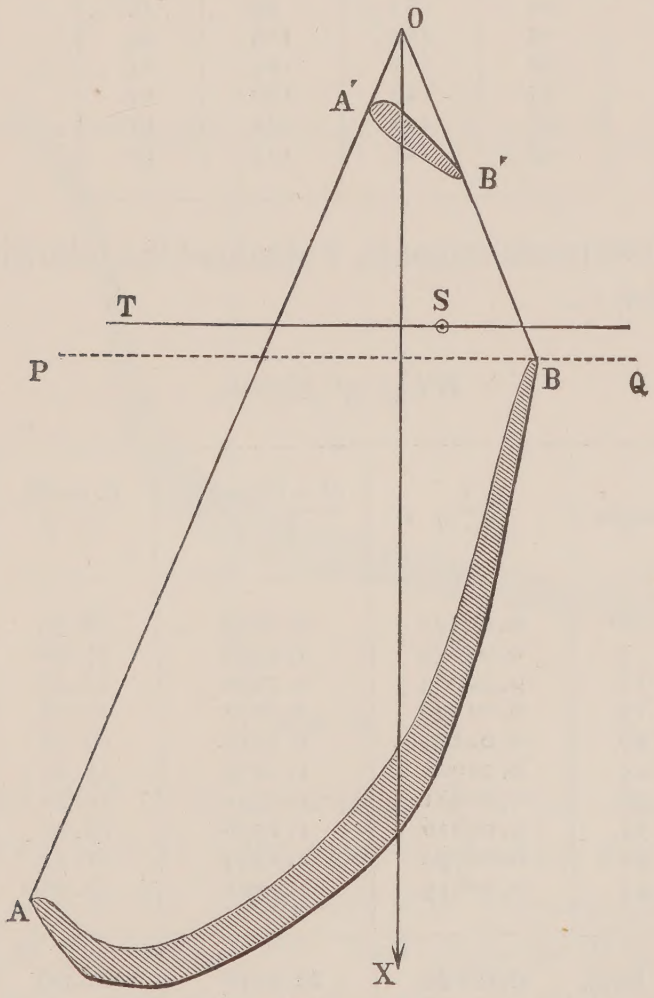


Fig. 79.

insertion of the pectoral muscle ; S is the socket of the wing-bone, and ST perpendicular to OX, the bisector of the angle AOB, is the axis round which the bone $A'B'$ rotates. The angle AOB is 45°, and, when divided into nine equal parts

of 5° each, gives the following measurements in thirty-seconds of an inch :—

Wing of Heron.

Angle.	$l - l'$.	$l \cos \theta$.	$l' \cos \theta$.
0°	28	47	22
5	38	55	19
10	48	65	18
15	65	81	17
20	83	99	16
25	100	113	14
30	111	124	13
35	121	130	12
40	126	132	11
45	122	123	10

From these measurements, I obtained the following results, by calculation :—

Wing of Heron.

Angle.	$\frac{1}{l - l'}$	$\frac{(l + l') \cos \theta}{l - l'}$	$\frac{ll' \cos^2 \theta}{l - l'}$
0°	0.03571	2.4649	36.93
5	0.02632	1.9474	27.50
10	0.02083	1.7292	24.37
15	0.01539	1.5077	21.18
20	0.01205	1.3855	19.08
25	0.01000	1.2700	15.82
30	0.00901	1.2342	14.52
35	0.00826	1.1736	12.00
40	0.00794	1.1349	11.52
45	0.00819	1.0901	10.08
Sum,	**0.15370**	**14.9375**	**193.90**

Hence we find, for the centre of the conic (66),

$$\delta = -\tfrac{1}{2} \cdot \frac{14.9375}{0.1537} = -48.60.$$

The axis PQ is drawn, in Fig. 79, at this distance from the point O; while the measured distance of the actual axis ST from the point O is 44.

(*d.*) *Wing of the Macaw.*—The wing of the Macaw is shown in Fig. 80, where AB is the origin of the pectoral

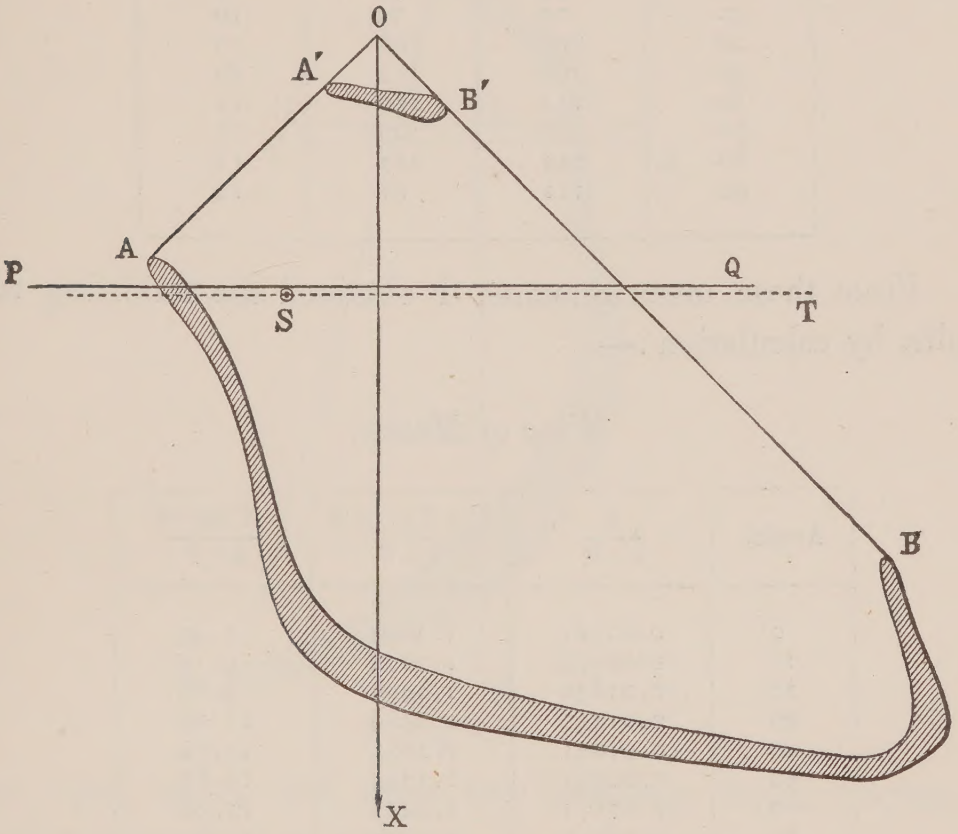


Fig. 80.

muscle, and $A'B'$ is its insertion. S is the centre of the glenoid cavity, and ST is the axis round which the wing-bone revolves. PQ is the calculated axis, corresponding to the minor axis of the ellipse (66). The angle AOB is 90°, which I divided into parts of 10° each, with the following results, measured in thirty-seconds of an inch :—

Wing of Macaw.

Angle.	$l - l'$.	$l \cos \theta$.	$l' \cos \theta$.
0°	42	39	8
10	46	47	9
20	54	58	9
30	70	77	10
40	96	106	10
50	106	115	10
60	114	120	11
70	127	126	11
80	143	127	11
90	112	91	12

From these measurements, I obtained the following results, by calculation :—

Wing of Macaw.

Angle.	$\frac{1}{l - l'}$	$\frac{(l + l') \cos \theta}{l - l'}$	$\frac{ll' \cos^2 \theta}{l - l'}$
0°	0.02381	1.1191	7.43
10	0.02174	1.2174	9.19
20	0.01852	1.2407	9.66
30	0.01415	1.2429	11.00
40	0.01041	1.2084	11.04
50	0.00943	1.1793	10.85
60	0.00877	1.1491	11.58
70	0.00787	1.0787	10.91
80	0.00699	0.9650	9.76
90	0.00893	0.9196	9.75
Sum,	**0.13062**	**11.3202**	**101.17**

Hence we find, for the distance of the centre of the Conic (66), from the vertex O,

$$\delta = -\tfrac{1}{2} \cdot \frac{11.3202}{0.13062} = -43.333.$$

The actual distance of the axis ST, passing through the socket S, is 45.

(*e.*) *Wing of the Pheasant.*—In the wing of the Pheasant, Fig (81), the angle AOB is 45°, which I divided into

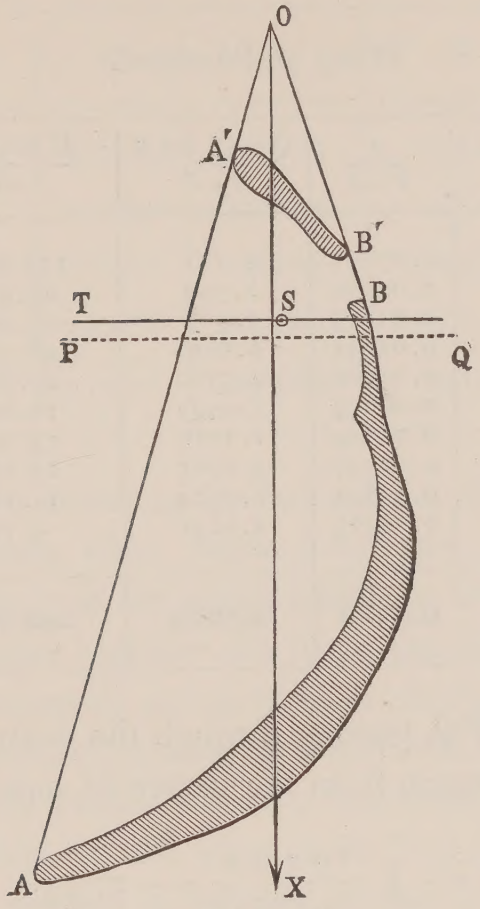


Fig. 81.

nine equal parts, and made the following measurements, in thirty-seconds of an inch :—

Wing of Pheasant.

Angle.	$l - l'$.	$l \cos \theta$.	$l' \cos \theta$.
0°	13	51	39
5	40	68	27
10	66	87	22
15	74	105	20
20	100	118	17
25	120	135	15
30	163	177	14
35	183	184	13
40	199	202	11
45	216	211	10

From these measurements, the following results were obtained by calculation :—

Wing of Pheasant.

Angle.	$\frac{1}{l-l'}$	$\frac{(l+l')\cos\theta}{l-l'}$	$\frac{ll'\cos 2\theta}{l-l'}$
0°	0.07692	6.231	153.00
5	0.02500	2.750	45.90
10	0.01515	1.515	29.00
15	0.01351	1.6892	28.38
20	0.01000	1.3500	20.06
25	0.00833	1.2500	16.88
30	0.00614	1.1717	15.20
35	0.00547	1.0765	13.07
40	0.00503	1.0704	11.16
45	0.00463	1.0231	9.77
Sum,	**0.17018**	**19.5805**	**342.42**

Hence, the axis PQ, passing through the centre of the Conic (66), lies at a distance from the vertex O, equal to—

$$\delta = -\tfrac{1}{2}\ \frac{19.5805}{0.17018} = -57.53.$$

The actual distance of the axis ST is 51.

In equation (66),

$$y^2 = Lx^2 + Mx + N,$$

the axis OX will not intersect the conic in real points unless

$$M^2 - 4LN$$

be positive. This, therefore, becomes the condition for the existence of axes of unstable equilibrium, and for the existence of the *ellipse of unstable equilibrium*. We are, therefore, bound to ascertain that this condition is fulfilled in the wings of the birds already investigated. The following Table shows that they all satisfy this test :—

Condition of Unstable Equilibrium in the Wings of Birds.

Bird.	L	M	N	$M^2 - 4LN$
Albatross, . .	0.0962	22.78c3	1104.51	+ 93.91
Woodpigeon, .	0.14433	19.7449	557.05	+ 68.27
Heron, . . .	0.15370	14.9375	193.90	+ 103.90
Macaw, . . .	0.13062	11.3202	101.17	+ 75.27
Pheasant, . . .	0.17018	19.5805	342.42	+ 150.29

The positive values in the last column of this Table show, that in every case, there exists, a real axis of *maximum instability*, or tendency in the plane muscle to become skew.

If we compare together the preceding examples, and bear in mind the necessary errors of observation, and also that we have been compelled to employ summation instead of integration, so that our results are only approximations; there can be no doubt that, in the wing of the bird, the socket of the humerus is so placed, in relation to the origin and insertion of the pectoral muscle, as to produce the maximum amount of work possible by the contraction of that muscle.

From all the preceding cases and calculations, we are entitled to lay down the following:—

Proposition D.

When a quadrilateral muscle unites any two bones whatever, and motion takes place round an axis lying in their plane, and perpendicular to the bisector of the angle made by their extreme fibres; two points may be found on that bisector, such that—

1°. *If the axis of rotation lie outside these points, the contraction of the muscle will tend to replace the bones in the same plane, and their equilibrium will be stable.*

2°. *If the axis of rotation pass through either of these points,*

the contraction of the muscle will produce no effect, and the equilibrium of the bones will be neutral.

3°. *If the axis of rotation lie inside these points, the contraction of the muscle will tend to convert the plane quadrilateral into a skew muscle; and this effect will be a maximum, when the axis passes through the point midway between these points; and the equilibrium of the bones will be unstable.*

The Divine Architect of the Universe has placed the axis of rotation of the wings of birds in the position of maximum work, or so that a given amount of work shall be performed by the minimum amount of muscle; which is in conformity with the "Principle of Least Action" observable in all departments of Nature. Hence we may regard the Postulate of page 238 as proved for this class of muscles, and as thus having additional probabilities in its favour.

In Propositions A and B, we have considered the case of two bones, lying in the same plane, and united by a quadrilateral sheet of muscle, one bone being supposed fixed, and the other compelled to turn round an axis of rotation perpendicular to the plane.

In Propositions C and D, we have considered the same case, when the axis of rotation lies in the original plane of the quadrilateral muscle, and is perpendicular to the bisector of the muscular fibres.

In order to complete the consideration of the action of the muscle, when the bone rotates round any axis whatever, we shall now consider its action when the bone rotates round an axis perpendicular to both the former axes of rotation; that is to say, when the rotation of the moveable bone takes place round an axis parallel to the bisector of the muscular fibres.

Let AB and $A'B'$ (Fig. 82) be the bones, and let the same notation be used as before. Let S be the socket of the joint, and SZ the axis of rotation, parallel to OX, the bisector of the angle made by the fibres. Draw SY perpendicular to YOX, and let

$OS = a$ $\qquad$ $Oa = l$

$YOS = \phi$ $\qquad$ $Oa' = l'$

$aOX = \theta$

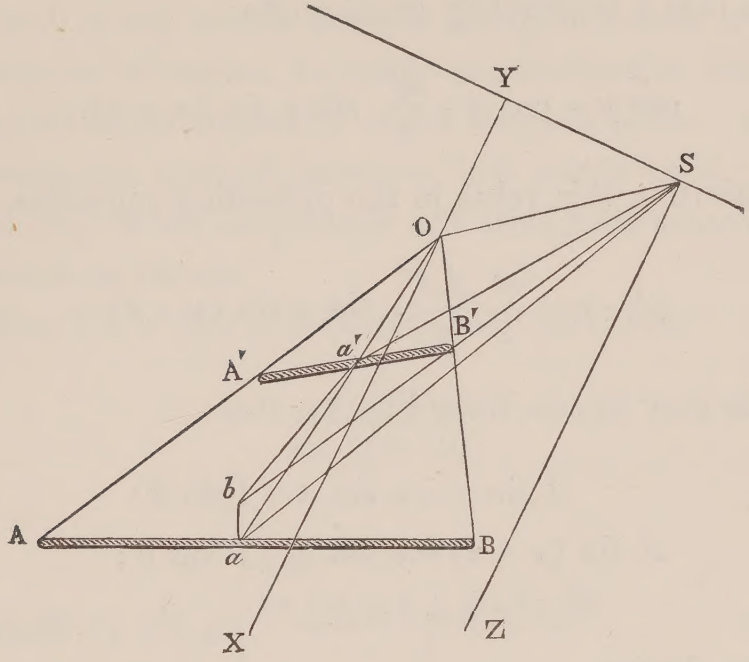


Fig. 82.

Let the point a be moved to the point b, by a rotation (ω) round the axis SZ. Then, if

$Sa = A$ $\qquad$ $bSa' = y$

$Sa' = A'$ $\qquad$ $aSZ = \sigma$

$aSa' = x$

we have, since A and A' are constant,

$$p = aa' = \sqrt{A^2 + A'^2 - 2AA' \cos x};$$

$$p' = ba' = \sqrt{A^2 + A'^2 - 2AA' \cos y}.$$

If we describe a sphere round S as centre, we shall have (Fig. 83),

$Sa = Sb = \sigma$

$aa' = x$

$ba' = y$

$bSa = \omega.$

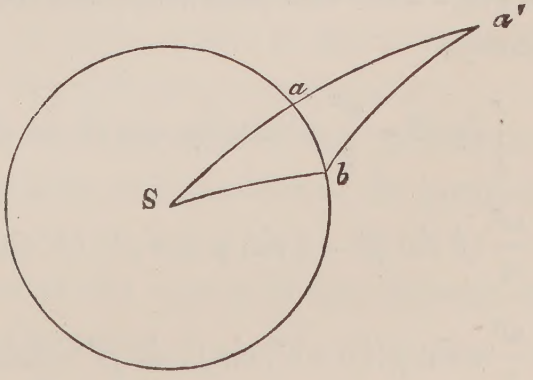


Fig. 83.

From the spherical triangle Sba', we find

$$\cos y = \cos \omega \sin \sigma \sin (\sigma + x) + \cos \sigma \cos \overline{\sigma + x};$$

or, when ω is a moderately small angle,

$$\cos y = \cos x - \frac{\omega^2}{2} \cdot \sin \sigma \sin (\sigma + x).$$

Substituting this value in the preceding equations, we find

$$p' - p = \frac{\omega^2}{2} \frac{AA'}{p} \cdot \sin \sigma \sin (\sigma + x).$$

Now, it is easy to see, from Fig. 82, that

$$A \sin \sigma = a \sin \phi + l \sin \theta;$$
$$A \sin (\sigma + x) = a \sin \phi + l' \sin \theta;$$
$$p = l - l'.$$

Hence we obtain

$$p' - p = \frac{\omega^2}{2} \cdot \frac{(a \sin \phi + l \sin \theta)(a \sin \phi + l' \sin \theta)}{l - l'}; \quad (68)$$

and, finally, for the work done, by a rotation round SZ, through the small angle ω—

$$\int \delta p d\theta = \frac{\omega^2}{2} \int \frac{(a \sin \phi + l \sin \theta)(a \sin \phi + l' \sin \theta)\, d\theta}{l - l'}. \quad (69)$$

If we substitute, in this equation, the values of l and l' given in page 266, and follow the method there employed, we obtain, finally,

$$\int \delta p d\theta = \frac{\omega^2}{2} a^2 \sin^2 \phi \sin \beta \sin \beta' \int \frac{\cos^2 \theta d\theta}{\Delta}. \quad (70)$$

$$+ \frac{\omega^2}{2} (b \sin \beta - a \sin \phi \cos \beta)(b' \sin \beta' - a \sin \phi \cos \beta') \int \frac{\sin^2 \theta d\theta}{\Delta}.$$

$$+ \frac{\omega^2}{2} a \sin \phi \{(b + b') \sin \beta \sin \beta' - a \sin \phi \sin (\beta + \beta')\} \int \frac{\sin \theta \cos \theta d\theta}{\Delta}.$$

This expression might be integrated in terms of the definite integrals, X, Y, Z (p. 282), and the whole theory of work done by plane quadrilateral muscles deduced from it; but we shall obtain results more in accordance with the actual circumstances of nature, by using the method of differences, as employed in the discussion of the wings of birds. Previous to adopting this method, however, it is worth while to apply equation (69) to the simple case of a triangular muscle, which often occurs in nature.

Making $l' = 0$, and substituting for l its value,

$$\frac{b \sin \beta}{\sin (\beta - \theta)},$$

we find

$$\frac{2}{\omega^2} \int \delta p d\theta = a \sin \phi \int_{-\theta}^{+\theta} \frac{(a \sin \phi + l \sin \theta)\, d\theta}{l}$$

$$= \frac{a \sin \phi}{b \sin \beta} \int_{-\theta}^{+\theta} \{a \sin \phi \sin \beta \cos \theta + (b \sin \beta - a \sin \phi \cos \beta) \sin \theta\}\, d\theta$$

$$= \frac{2a^2 \sin^2 \phi \sin \theta}{b};$$

or,

$$\int \delta p d\theta = \frac{\omega^2 a^2 \sin^2 \phi \sin \theta}{b}. \qquad (71)$$

From this expression for the work done by a triangular muscle, when rotated round an axis parallel to the bisector of its vertical angle, we obtain the following conclusions:—

1°. If the vertical angle and bisector of the muscle be given (θ, b), the work done will be independent of the position of the bone.

2°. The work done varies as the square of the distance of the axis of rotation from the bisector.

3°. For the bisector itself, taken as axis of rotation, the equilibrium of the muscle is *neutral*, and for all other positions it is *stable*, and never becomes *unstable*.

If we compare these results with the corresponding results for rotation round axes perpendicular to the bisector of the angle of the muscular fibres (pp. 269, 270), we see very important consequences, which may be thus stated:—

PROPOSITION E.

No triangular muscle, by the contraction of its fibres, can compel the bone that forms its base to revolve round an axis of rotation lying in the plane of the muscle, unless the socket of the joint lie between the perpendiculars drawn at each extremity of the bisector of the vertical angle of the muscle.

For, if the rotation be resolved round two rectangular axes, parallel and perpendicular to the bisector, the equilibrium of the muscle, with respect to the first component axis of rotation, is always either stable or neutral; and, with respect to the second component axis of rotation, the equilibrium is also either stable or neutral, unless the axis lie between the perpendiculars drawn at each extremity of the bisector.

I have already shown, in the *latissimus dorsi* of Man, of the Tiger, and of the Albatross (pp. 246, 249, 256), and in the hamstring muscles of the Tiger (p. 258), that the socket of the joint is placed upon the perpendicular at the vertex to the bisector of the muscle, and that the object of that arrangement is to produce the maximum amount of Work possible by a rotation round an axis perpendicular to the plane of the muscle. It follows from Prop. E, that these muscles can produce no rotation whatever round any axis lying in the plane of the muscle.

These muscles are, therefore, intended to cause rotation round one axis only, and to do so with a maximum amount of useful work.

This consideration affords us a glimpse of the reason why so many muscles are made to act in various ways round the shoulder and hip joints of animals ; each muscle is intended to produce a rotation round some axis or definite group of axes ; and, in order to enable it to do so with maximum effect, its power to cause rotations round other axes is taken away.*

We shall now consider, in detail, the case of rotation round an axis parallel to the bisector, with respect to the five birds already discussed.

If we expand equation (69), writing

$$x^2 = \frac{2}{\omega^2}\int \delta p d\theta,$$

$$y = a \sin \phi,$$

we find

$$x^2 = Py^2 + Qy + R; \qquad (72)$$

where

$$P = \int \frac{d\theta}{l - l'}; \quad Q = \int \frac{(l + l') \sin \theta}{l - l'}; \quad R = \int \frac{ll' \sin^2 \theta}{l - l'}.$$

Equation (72) represents a central conic, whose centre lies upon the axis of y, on the perpendicular drawn at O, to the bisector OX, and at a distance from O, determined by the equation

$$2Py + Q = 0. \qquad (73)$$

When the equation (72) is referred to axes of co-ordinates passing through the centre, it becomes

$$\pm \frac{x^2}{m^2} = \frac{y^2}{n^2} - 1, \qquad (74)$$

* May not the remarkable similarity in the bones and muscles of various animals, in corresponding parts of their structure, be the result of some such geometrical and mechanical necessity, foreseen by the Divine Contriver, instead of being the result of common descent from a remote ancestor possessing a similar arrangement of bones and muscles ?

where

$$m^2 = \frac{Q^2 - 4PR}{4P}; \qquad n^2 = \frac{Q^2 - 4PR}{4P^2}. \tag{75}$$

When x^2 is positive, equation (74) represents an hyperbola, and the equilibrium of the muscle is *stable;* and when x^2 is negative, equation (74) represents an ellipse, and the equilibrium of the muscle is *unstable.*

If P, Q, R, be positive, it is evident from (75) that m^2 and n^2 will have the same sign as

$$Q^2 - 4PR;$$

and if this quantity become negative, the ellipse represented by

$$-\frac{x^2}{m^2} = \frac{y^2}{n^2} - 1$$

will be imaginary; or, in other words, there will be no axis of *unstable* equilibrium possible.

(*a.*) *Wing of the Albatross.*—The following measurements, taken from the wing of the Albatross, are made, as before, in twenty-fourths of an inch:—

Wing of Albatross.

Angle.	$l - l'$	$l \sin \theta$	$l' \sin \theta$
0°	31	+ 48	+ 35
5	88	+ 54	+ 26
10	120	+ 45	+ 18
15	136	+ 30	+ 10
20	148	+ 12	+ 4
25	170	− 6	− 2
30	186	− 28	− 7
35	195	− 52	− 12
40	198	− 74	− 17
45	194	− 103	− 25

From these values we can calculate the following Table :—

Wing of Albatross.

Angle.	$\frac{1}{l-l'}$	$\frac{(l+l')\sin\theta}{l-l'}$	$\frac{ll'\sin^2\theta}{l-l'}$
0°	0.03225	+ 2.6774	54.20
5	0.01136	+ 0.9091	15.95
10	0.00833	+ 0.5250	6.75
15	0.00735	+ 0.2941	2.20
20	0.00675	+ 0.1081	0.33
25	0.00588	− 0.0471	0.07
30	0.00538	− 0.1882	1.05
35	0.00512	− 0.3282	3.20
40	0.00505	− 0.4596	6.35
45	0.00515	− 0.6598	13.27
Sum,	**0.0962**	**2.8308**	**103.37**

The sums here given are the quantities P, Q, R; and it will be observed that the elements of Q have opposite signs at each side of the bisector, so that its value is the difference of two groups of elements, and not the sum, as in the corresponding quantity—

$$M = \int \frac{(l+l')\cos\theta}{l-l'}.$$

Hence we are not surprised to find that

$$Q^2 - 4PR = -31.763,$$

from which we obtain, by equation (75),

$$n^2 = -(29.29)^2,$$
$$m^2 = -(9.08)^2.$$

As these values are both negative, equation (72) will represent an hyperbola, whose transverse axis is parallel to the axis OX, and whose centre lies upon OY, as shown in Fig. 84. The centre of this hyperbola is found from equation (73),

$$2Py + Q = 0;$$

or,

$$\delta = -\tfrac{1}{2}\ \frac{2.8308}{0.0962} = 14.71.$$

The actual distance of the socket S, from the line OY, is $\frac{22}{24}$ths of an inch.

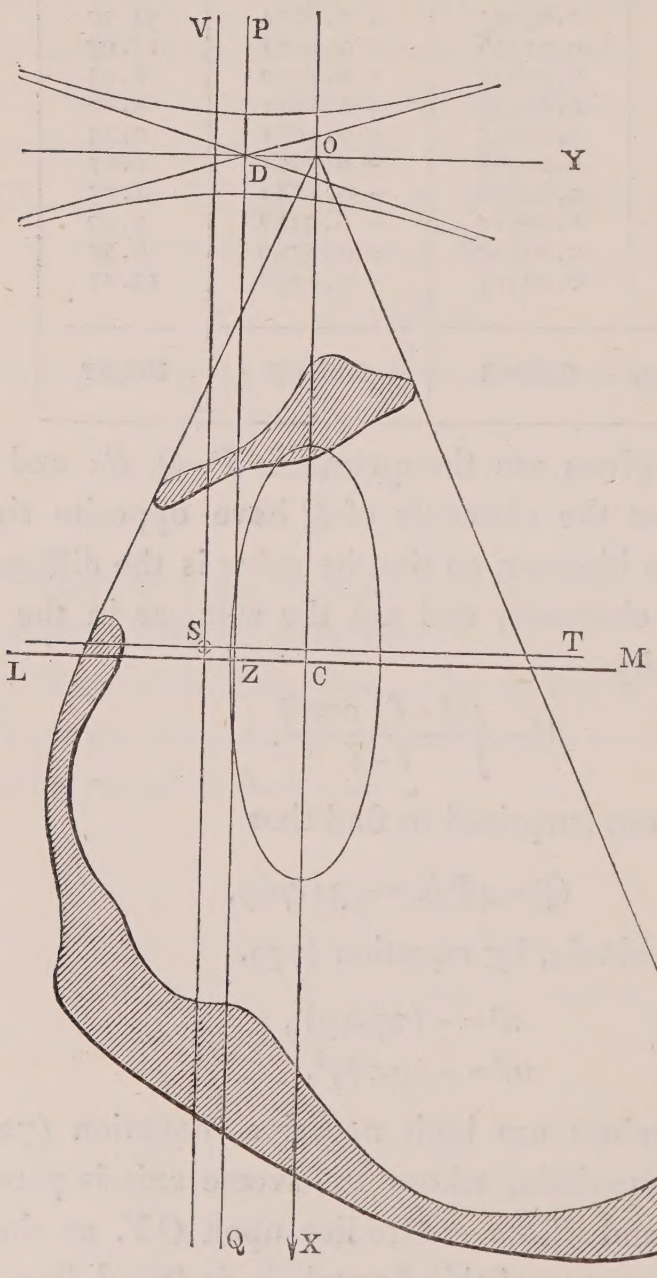


Fig. 84.

In Fig. 84, I have drawn carefully to scale, for the wing

of the Albatross, the two conics represented by the equations,

$$y^2 = Lx^2 + Mx + N;$$
$$x^2 = Py^2 + Qy + R,$$

referred to OX and OY as co-ordinate axes.

The ellipse whose centre is C may be called the *ellipse of instability*, and the axis of rotation, LM, passing through its centre, is the *axis of maximum instability*. The hyperbola whose centre is D may be called the *hyperbola of stability*, and the axis of rotation, PQ, passing through its centre, is the *axis of minimum stability*. The intersection of these two axes, Z, determines the exact position of the centre of the socket of the shoulder joint, which fulfils the conditions imposed by the principle of "least action;" because if the wing rotate round any axis passing through this point, and lying in the plane of the muscle, the contraction of the muscle will produce a *maximum* amount of *work done*, by the component rotation parallel to OX; and, although the contraction of the muscle cannot produce any rotation round an axis parallel to OY, yet, if any such rotation take place from external causes, the contraction of the muscle will restore the bones to their original positions, with the *minimum* amount of *work lost*.

The near approach of the calculated socket Z to the observed socket S is a proof of the truth of the Postulate (p. 238); and we shall see that the wings of the remaining birds confirm the conclusion here drawn.

(*b.*) *Wing of the Wood Pigeon.*—The following measurements on the Wood Pigeon and other birds are made in 32nds of an inch:—

Wing of Wood Pigeon.

Angle.	$l - l'$.	$l \sin \theta$	$l' \sin \theta$.
0°.00′	11	+ 19	+ 16
5 .20	76	+ 23	+ 8
10 .40	96	+ 13	+ 4
16 .00	115	0	0
21 .20	129	− 15	− 3
26 .40	143	− 31	− 5
32 .00	180	− 49	− 8

From these values the following Table is calculated :—

Wing of Wood Pigeon.

Angle.	$\frac{1}{l - l'}$	$\frac{(l + l') \sin \theta}{l - l'}$	$\frac{ll' \sin^2 \theta}{l - l'}$
0°.00′	0.09091	+ 3.1818	27.64
5 .20	0.01317	+ 0.4080	2.42
10 .40	0.01041	+ 0.1770	0.54
16 .00	0.00869	0.0000	0.00
21 .20	0.00775	− 0.1393	0.35
26 .40	0.00699	− 0.2517	1.08
32 .00	0.00641	− 0.3166	2.18
Sum,	**0·14433**	**3·0592**	**34·21**

From these values, we can readily calculate,

$$Q^2 - 4PR = - 10.391.$$

Since this quantity is negative, the equilibrium of the wing is always stable, with respect to axes of rotation parallel to OY.

The position of the axis of *minimum stability* is found as before,

$$\delta = \tfrac{1}{2} \frac{3 \cdot 0592}{0.14433} = 10.60.$$

The actual distance of the socket S from the line OY is $\frac{2}{32}$nds of an inch.

(*c.*) *Wing of the Heron.*—The following measurements were made upon the wing of the Heron, in 32nds of an inch.

Wing of Heron.

Angle.	$l - l'$.	$l \sin \theta$.	$l' \sin \theta$.
0°	28	+ 21	+ 9
5	38	+ 18	+ 6
10	48	+ 16	+ 4
15	65	+ 11	+ 3
20	83	+ 5	+ 1
25	100	− 4	− 1
30	111	− 16	− 2
35	121	− 28	− 3
40	126	− 42	− 4
45	122	− 50	− 5

From these measurements, the following Table is calculated :—

Wing of Heron.

Angle.	$\frac{1}{l - l'}$	$\frac{(l + l') \sin \theta}{l - l'}$	$\frac{ll' \sin^2 \theta}{l - l'}$
0°	0.03571	+ 1.0714	6.75
5	0.02632	+ 0.6316	2.84
10	0.02083	+ 0.4166	1.33
15	0.01539	+ 0.2154	0.51
20	0.01205	+ 0.0723	0.06
25	0.01000	− 0.0500	0.04
30	0.00901	− 0.1622	0.29
35	0.00826	− 0.2562	0 70
40	0.00794	− 0.3651	1.33
45	0 00819	− 0.4508	2.05
Sum,	**0·15370**	**1·1230**	**15·90**

From these values, we find,

$$Q^2 - 4PR = - 8.514,$$

showing that the equilibrium is always stable, and

$$\delta = \tfrac{1}{2}\frac{1.1230}{0.1537} = 3.65.$$

The actual distance of the socket from OY is $\frac{7}{32}$nds of an inch.

(*d.*) *Wing of the Macaw.*—The following measurements on the wing of the Macaw are made in 32nds of an inch :—

Wing of Macaw.

Angle.	$l - l'$	$l \sin\theta$	$l' \sin\theta$
0°	42	+ 39	+ 9
10	46	+ 33	+ 6
20	54	+ 27	+ 4
30	70	+ 21	+ 2
40	96	+ 9	+ 1
50	106	− 10	− 2
60	114	− 32	− 3
70	127	− 59	− 4
80	143	− 89	− 8
90	112	− 91	− 10

From these measurements, the following Table is calculated :—

Wing of Macaw.

Angle.	$\frac{1}{l - l'}$	$\frac{(l + l')\sin\theta}{l - l'}$	$\frac{ll' \sin^2\theta}{l - l'}$
0°	0.02381	+ 1.1428	8.36
10	0.02174	+ 0.8480	4.30
20	0.01852	+ 0.5741	2.00
30	0.01415	+ 0.3286	0.60
40	0 01041	+ 0.1042	0.09
50	0.00943	− 0.1132	0.19
60	0.00877	− 0.3070	0.84
70	0.00787	− 0.4960	1.86
80	0.00699	− 0.6783	5.00
90	0.00893	− 0.9018	8.12
Sum,	0·13062	0·5014	31·36

From these values, we find

$$Q^2 - 4PR = -16.134,$$

showing that the equilibrium is always stable; and

$$\delta = \tfrac{1}{2}\,\frac{0.5014}{0.13062} = 1.92.$$

The actual distance of the socket from the axis OY is $\frac{1}{3}\frac{5}{2}$nds of an inch.

(*e.*) *Wing of the Pheasant.*—The following measurements were made on the wing of the Pheasant, in 32nds of an inch:—

Angle.	$l - l'$.	$l \sin\theta$.	l s in θ'.
0°	13	+ 22	+ 16
5	40	+ 22	+ 9
10	66	+ 20	+ 5
15	74	+ 14	+ 3
20	100	+ 6	+ 1
25	120	− 5	− 1
30	163	− 23	− 2
35	183	− 42	− 2
40	199	− 63	− 3
45	216	− 85	− 4

From these measurements the following Table is constructed:—

Wing of Pheasant.

Angle.	$\frac{1}{l - l'}$	$\frac{(l + l')\sin\theta}{l - l'}$	$\frac{ll'\sin^2\theta}{l - l'}$
0°	0.07692	+ 2.9231	27.07
5	0.02500	+ 0.7750	4 95
10	0.01515	+ 0.3788	1.51
15	0.01351	+ 0.2300	0.57
20	0 01000	+ 0.0700	0 06
25	0.00833	− 0.0500	0.04
30	0.00614	− 0.1533	0.28
35	0.00547	− 0.2404	0.49
40	0.00503	− 0.3316	0.95
45	0.00463	− 0.4120	1.59
Sum,	0.17018	3.1896	37.51

From these values we find

$$Q^2 - 4PR = -15.361,$$

showing that the equilibrium is always stable, and

$$\delta = \tfrac{1}{2} \cdot \frac{3.1896}{0.17018} = 9.37.$$

The actual distance of the socket from the line OY is $\frac{4}{3}\frac{5}{2}$nds of an inch.

Collecting together into one Table the co-efficients of equation (72), we obtain—

Bird.	P	Q	R	$Q^2 - 4PR$
Albatross, . .	0.09620	2.8308	103.37	- 31.763
Wood Pigeon, .	0.14433	3.0592	34.21	- 10.391
Heron, . . .	0.15370	1.1230	15.90	- 8.514
Macaw, . . .	0.13062	0.5014	31.36	- 16.134
Pheasant, . .	0.17018	3.1896	37.51	- 15.361

From the foregoing investigation, it appears that Prop. E may be extended to the curved quadrilateral muscles which form the great pectoral in the wings of birds, and that the contraction of the muscle cannot produce a rotation of the wing round any axis lying in its plane, unless the socket of the joint be placed somewhere in that plane lying between the tangents drawn to the *ellipse of instability* at the extremities of its diameter on the line OX, which bisects the angle formed by the fibres of the muscle.

The exact position of this socket, required to fulfil the conditions of *maximum work done* and *minimum work lost*, is found by the intersection of the axes of *maximum instability* and *minimum stability*.

The difference between the calculated and observed positions of these two axes is shown in the following Table:—

Difference between Calculation and Observation in the position of the Socket in the Wings of Birds.

Bird.	Axis of maximum instability.	Axis of minimum stability.
Albatross, . . .	0.10 inch.	0.30 inch.
Wood Pigeon, . .	0.14 ,,	0.27 ,,
Heron, ...	0.14 ,,	0.10 ,,
Macaw, ...	0.05 ,,	0.41 ,,
Pheasant, . . .	0.20 ,,	0.15 ,,
Mean Error,	**0.126** inch.	**0.246** inch.

The differences between calculation and theory shown in the foregoing Table (one-eighth and one-quarter of an inch) are not greater than should be expected in such a class of observations. It may be noted, also, that the mean error in the calculation of the *axis of maximum instability* (which is the more important of the two axes) is only half the mean error in the calculation of the *axis of minimum stability.*

We have seen by Prop. E, applied to the shoulder and hip joints of various animals, that muscles intended to produce the maximum work by rotation round axes perpendicular to their plane are incapable of producing a rotation round any axis lying in their plane. Let us now inquire how muscles like those of the pectoral muscles of birds, intended to produce the maximum work round an axis lying in their plane, behave with regard to the axis of rotation perpendicular to their plane drawn through the socket of the shoulder joint.

It will be readily seen, by reference to the proof given of equation (44), Prop. B, that it is independent of the shape of the bones, whether straight or curved ; so that if k, θ, ϕ, be given, there are an infinite number of muscles capable of producing the same work. This equation—

$$\text{Work done} = 2k\delta\omega \sin\phi \sin\theta,$$

may therefore be applied directly to calculate the work done by the pectoral muscles of birds, round the axis passing through the socket of the shoulder joint, and perpendicular to the plane of the muscle. The maximum work possible would be obtained by placing the socket upon the perpendicular to the bisector drawn at the vertex O, and corresponds to

$$\text{Maximum work} = 2k\delta\omega \sin\theta.$$

Hence we have,

$$\frac{\text{Work done}}{\text{Maximum work}} = \sin\phi. \qquad (76)$$

By direct measurement of the angle ϕ, we obtain the following Table :—

Ratio of Work done by the Pectoral Muscle of Birds round the observed Axis perpendicular to its Plane, to the maximum possible Work.

Bird.	ϕ	sin ϕ
Albatross, . . .	11°.00′	0.190
Wood pigeon, . .	1 .55	0.033
Heron, ...	9 .10	0.159
Macaw, ...	19 .30	0.333
Pheasant, . . .	5 .00	0.087
Mean ratio, . . .		**0·1604**

If we take for the socket of the shoulder the calculated instead of the observed positions, we find by measurement of the angle ϕ the following Table :—

Ratio of Work done by the Pectoral Muscle of Birds, round the calculated Axis perpendicular to its Plane, to the maximum possible Work.

Bird.	ϕ	sin ϕ
Albatross, . . .	6°.55	0.120
Wood pigeon, . .	9 .35	0.166
Heron, ...	9 .20	0.162
Macaw, ...	2 .00	0.035
Pheasant, . . .	9 .15	0.161
Mean ratio, . . .		**0·1283**

From these results it appears that the rotation of the wing round the observed axis, perpendicular to the plane of the muscle, gives only $\frac{1}{6}$th part of the maximum work possible; and that a similar rotation round the calculated axis gives only $\frac{1}{8}$th of the maximum work possible.

This conclusion confirms generally that already arrived at in p. 307, and shows that each muscle is constructed mainly for the performance of a certain kind of work, and that it performs any other kind of work under great disadvantages.

The condition necessary, in order that a given quadrilateral muscle should be absolutely incapable of producing any rotation round an axis perpendicular to its plane is readily found. For, the expression (44) for the work done must vanish, which gives us sin $\phi = 0$; or that the socket must be placed upon the bisector OX (Fig. 84). Hence the centre D of the hyperbola must coincide with O, and in equation (72) the term involving y must disappear; hence we have,

$$Q = \int_{-\theta}^{+\theta} \frac{(l + l') \sin \theta}{l - l'} = 0, \qquad (77)$$

as the condition required, in order that rotation round an axis perpendicular to the muscle shall be impossible.

The quantity Q, as I showed before, consists of the difference of two sums, and there is no difficulty in supposing a case in which it should vanish altogether. For the birds already discussed, its values are—

Albatross, ...	2.83
Wood Pigeon, ...	3.06
Heron, ...	1.12
Macaw, ...	0.50
Pheasant, ...	3.19

Quadrilateral and skew muscles, in which the socket of the joint does not lie in the plane containing the origin and insertion of the muscle.

In page 262, I commenced the discussion of Quadrilateral muscles, in which the plane of the origin and insertion of the muscle includes the socket of the joint, and fully illustrated the action of this important class of muscles by means of the great pectoral of the wings of birds. We have now to examine the class of muscles in which the length of the fibres is such as not to permit the origin and insertion to come into the same plane as that which contains the socket of the joint.

In this class of muscles, the maximum extension of the fibres possible still leaves the plane passing through the socket and insertion distinct from the plane passing through the socket and origin of the muscle; and we are to suppose the two bones, each lying in its own plane, placed in the position of maximum extension of fibres, the work to be done being to compel one bone to turn round the other bone, the intersection of the planes containing each being the axis of rotation.

It is easy to show, as a preliminary condition, independent of any special arrangement of the muscular fibres connecting the two bones, that, in order to produce the maximum moment to turn the planes round their mutual intersection, regarded as an axis of rotation, that it is necessary that the planes passing

through the socket, and the origin, and insertion, respectively, must be at right angles to each other.

With the socket of the joint as centre, describe a sphere, and let AB and $A'B'$ (Fig. 85) be the spherical arcs, which are the projections of the bones upon the surface of the sphere. Let I be the intersection of the planes containing the two bones, and let AA' be the projection of any muscular fibre; let $IA = a$, $IA' = a'$, and let the angle between the planes be I. If the plane $IA'B'$ be made to revolve round the intersection of the two planes through a small angle ω, so that A' and B' assume the positions A'' and B''; then let

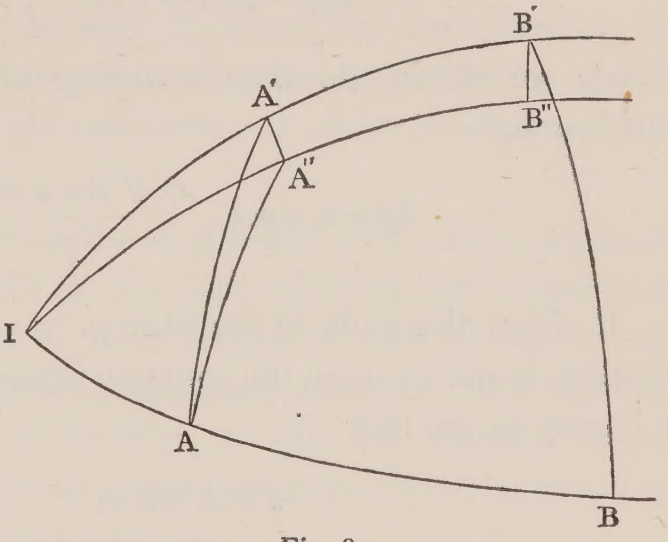


Fig. 85.

$$A'IA'' = \omega,$$
$$AA' = x,$$
$$AA'' = y.$$

If p be the original length of any fibre, and A, A', the distances of its extremities from the socket; and also, if p' be the length of the contracted fibre corresponding with the rotation through the angle ω, then we have

$$p^2 = A^2 + A'^2 - 2AA' \cos x;$$
$$p'^2 = A^2 + A'^2 - 2AA' \cos y;$$

but

$$\cos x = \cos I \sin a \sin a' + \cos a \cos a';$$
$$\cos y = \cos (I - \omega) \sin a \sin a' + \cos a \cos a'.$$

Hence (since ω is a small angle),

$$p^2 - p'^2 = 2AA' (\cos y - \cos x)$$
$$(p+p')(p-p') = 2AA'\omega \sin I . \sin a \sin a' ;$$

or,

$$p\delta p = AA'\omega . \sin I . \sin a \sin a'.$$

Hence we obtain, for the shortening of a single fibre, by a rotation ω,

$$\delta p = \omega \sin I . \frac{AA' \sin a \sin a'}{p}.$$

If, from the ends of the fibre p, we let fall two perpendiculars, π and π', upon the common intersection of the planes, it is easy to see that

$$\pi = A \sin a,$$
$$\pi' = A' \sin a' ;$$

from which we find

$$\delta p = \omega \sin I . \frac{\pi\pi'}{p},$$

and, summing together all the shortenings of all the fibres, we obtain

$$\Sigma (\delta p) = \omega \sin I \, \Sigma \left(\frac{\pi\pi'}{p}\right). \qquad (78)$$

It is evident that the work done by the muscle will be a maximum when

$$\Sigma (\delta p) = \text{maximum} ;$$

and this quantity will be a maximum independent of the sum, $\Sigma \left(\frac{\pi\pi'}{p}\right)$, (which depends on the special arrangement of the muscular fibres), when $I = 90°$.

Hence we obtain the following preliminary condition :—

CONDITION.

Whenever two bones, joined by muscular fibres, tend to turn each other round an axis of rotation formed by the intersection of two planes passing through the bones, the work done by the contraction of the fibres will be a maximum, when the axis of rotation is formed by the intersection of planes at right angles to each other.

Let us now consider the simple case of a single muscular fibre, of a given length p, the ends of which are required to remain in two planes at right angles to each other, in conformity with the foregoing preliminary condition ; it is required to find the position of the fibre, so that the moment of the force causing one bone to rotate round the line of intersection of the planes shall be a maximum.

Let OI (Fig. 86) be the line of intersection of the two

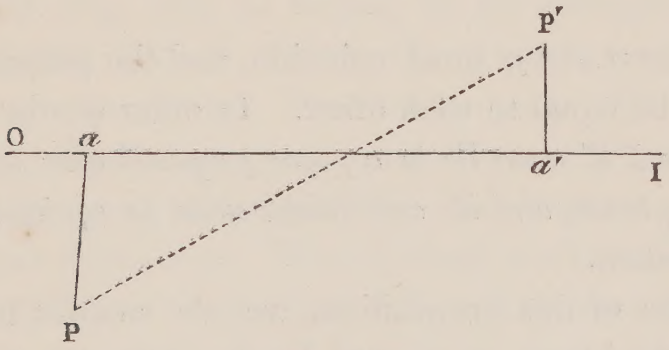


Fig. 86.

rectangular planes, and let P, P', be the ends of the fibre, whose length, $PP' = p$, is given. Draw Pa, $P'a'$, each in its own plane, perpendicular to OI; and let

$$PP' = p,$$
$$Pa = \pi,$$
$$P'a' = \pi',$$
$$aa' = u.$$

Since the planes are at right angles, we have

$$p^2 = \pi^2 + \pi'^2 + u^2,$$

and, by equation (78),

$$\pi\pi' = \text{maximum},$$

Differentiating these two equations, we find

$$\pi d\pi + \pi' d\pi' + u du = 0,$$
$$\pi d\pi' + \pi' d\pi = 0.$$

Eliminating $d\pi'$, we obtain

$$(\pi^2 - \pi'^2)\, d\pi + \pi u du = 0.$$

This equation of condition is satisfied by making

$$\pi = \pi',$$
$$u = 0;$$

or, the points a and a' must coincide, and the perpendiculars π, π', must be equal to each other. In other words—

The fibre PP' must lie in a plane perpendicular to both the intersecting planes, and its extremities must be equidistant from their intersection.

By means of this proposition, we are enabled to solve a number of problems, suggested by the *adductor* muscles of many animals.

If the muscle be triangular, then the resultant of all its fibres lies in the bisector of the vertical angle, which bisector is of a given length, and may be regarded as a single fibre, which is to be placed between the intersecting planes in conformity with the foregoing proposition.

If the muscle be a plane quadrilateral, we are then given two bones, AB and $A'B'$, which lie in two planes intersecting

at right angles, their extremities being joined by fibres, p, p', of given length, and we are required so to place the bones, that the work done by the contraction of the fibres in rotating one plane upon the other shall be a maximum.

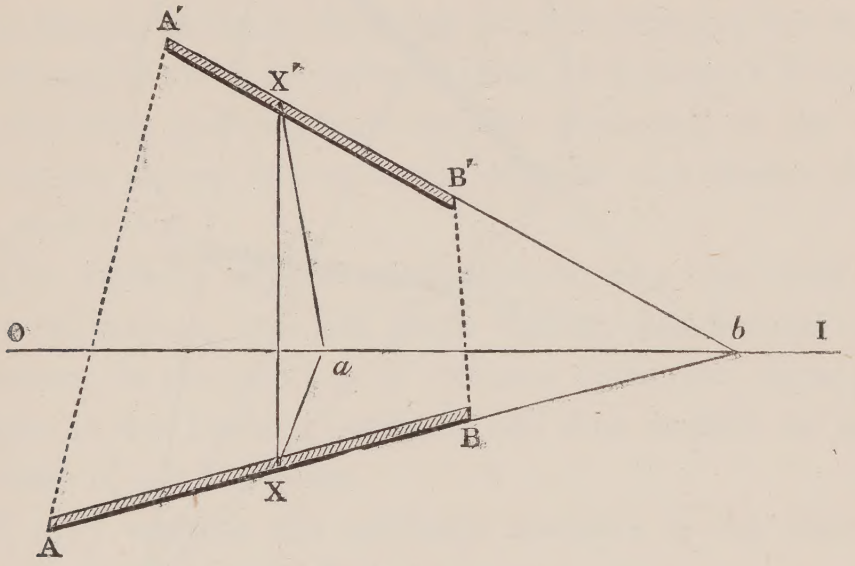


Fig. 87.

Let OI (Fig. 87), as before, be the intersection of the planes, and let AB and $A'B'$ be the given bones lying in the two planes, respectively. Produce the plane $ABA'B'$ to meet OI in the point b; and imagine the fibres AA' and BB' to be produced to meet, and that $X'X$ is the bisector of the angle under which they meet. The resultant of all the fibres joining AB and $A'B'$, lies in the line XX', which may be regarded as the only fibre with which we have to deal. The line XX' must, therefore, be placed in a plane perpendicular to the planes containing AB and $A'B'$, and the perpendiculars $Xa = \pi$ and $X'a = \pi'$, must be equal to each other. Now, in the triangles, Xab and $X'ab$, we have $Xab = X'ab = 90°$; $Xa = X'a$; and ab common to both. Hence we have $Xb = X'b$, and therefore $bXX' = bX'X$.

Let Fig. 88 represent the plane of the quadrilateral $ABA'B'$, in which YXX' is the bisector of the angle AYB.

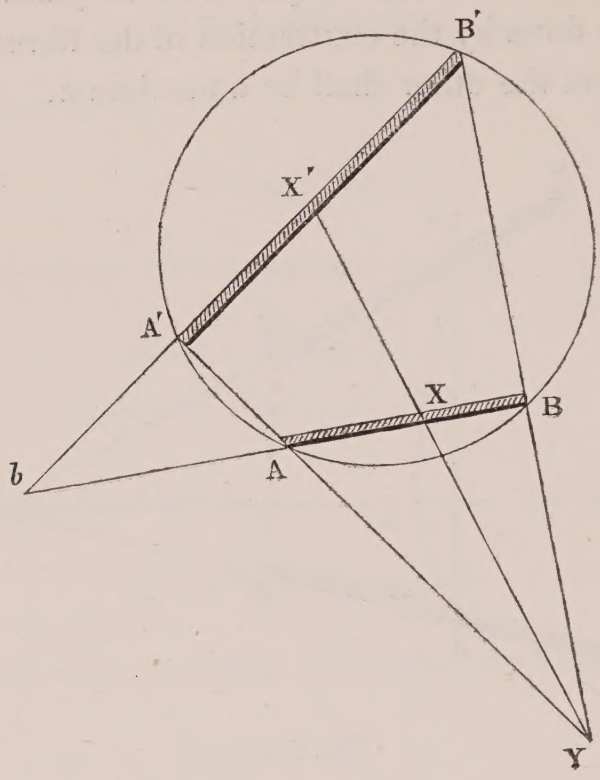


Fig. 88.

Hence, we have

$$BXX' = BYX + XBY$$
$$B'X'X = A'YX' + X'A'Y;$$

but $BYX = A'YX'$; therefore

$$XBY = X'A'Y;$$

or, in the quadrilateral $ABA'B'$, the external angle is equal to the opposite internal angle; or the sum of two opposite angles is equal to two right angles; hence the quadrilateral $ABA'B'$ is inscribable in a circle.

Hence,

If two bones, joined by muscular fibres forming a plane quadrilateral, lie in two planes at right angles to each other, the maximum work done in rotating the planes round their axis of

intersection, will occur when the bones are so placed as to form a quadrilateral inscribable in a circle.

This remarkable proposition enables us to determine readily the truth or falsehood of the Postulate, p. 238, in several cases. For, if the muscle be really inscribable in a circle, the rectangle under its diagonals must be equal to the sum of the rectangles under its opposite sides, by Ptolomy's Theorem; and the sides and diagonals may be measured on the fresh subject, by means of a compass, without any serious risk of any great error.

The *Adductor* muscles of all animals, as a rule, form skew sheets of surface, and not plane sheets; but several of the adductors, in the position of extreme abduction, form plane muscles in the limiting position, and thus enable us to test the truth of our Postulate.

I shall number the *adductor* muscles in the following order for convenience of reference:—

Adductor Muscles.	*Human Equivalents.*
1. *Adductor primus.*	1. The condyloid portion of *Adductor magnus.*
2. *Adductor secundus.*	2. The remaining portion of *Adductor magnus.*
α. Ischiadic sheet.	(α.) (β.)
β. Pubic sheet.	
3. *Adductor tertius.*	3. *Adductor brevis.*
4. *Adductor quartus.*	4. *Adductor longus.*
5. *Adductor quintus.*	5. *Pectinæus.*

The fourth and fifth of these adductors offer examples of plane quadrilateral muscles, whose origin and insertion lie in rectangular planes.

1. *Lion.*—In the Lion the fourth and fifth adductor muscles (*longus* and *pectinæus*) have origins and insertions of the same length, and lying beside each other on the bones.

The following measurements were made from a large adult lion :—

Pectinæus Muscle (*Lion*).

1. Length of origin, ...	1.70 in.	= *a*.
2. Length of insertion, ...	2.95 „	= *b*.
3. Length of anterior fibres, . .	4.52 „	= *c*.
4. Length of posterior fibres, . .	7.61 „	= *d*.
5. Length of first diagonal, . .	5.77 „	= *e*.
6. Length of second diagonal, .	6.76 „	= *f*.

Now, since *a*, *b* are opposite sides, and *c*, *d* are also opposite, and *e*, *f* are diagonals, we have, by Ptolomy's Theorem,

$$ab + cd = ef\,; \tag{79}$$

but

$$ab = 1.70 \times 2.95 = 5.01$$
$$cd = 4.52 \times 7.61 = 34.39$$
$$ab + cd = 39.40$$

and also,

$$ef = 5.77 \times 6.76 = 39.00.$$

These results are so close as to prove that, in the Lion, the *adductor quartus* and the *adductor quintus* are quadrilaterals, inscribable in a circle.

2. *Man*.—I made the following measurements, on a well developed male subject :—

Pectinæus Muscle (*Man*).

1. Length of origin, ...	1.82 in.	= *a*.
2. Length of insertion, ...	1.52 „	= *b*.
3. Length of anterior fibres, . .	4.70 „	= *c*.
4. Length of posterior fibres, . .	6.30 „	= *d*.
5. Length of first diagonal, . .	5.52 „	= *e*.
6. Length of second diagonal, .	5.66 „	= *f*.

Hence we have

$$ab = 2.76, \quad cd = 22.61$$
$$ab + cd = 32.37$$
$$ef = 31.24.$$

This result is sufficient to show that, in Man, the *Pectinæus* muscle is nearly inscribable in a circle.

3. *Llama.*—In the Llama, I obtained the following:—

Pectinæus Muscle (Llama).

1.	Length of origin, ...	1.91 in. = a.
2.	Length of insertion, ...	3.50 „ = b.
3.	Length of anterior fibres, . .	3.05 „ = c.
4.	Length of posterior fibres, . .	6.91 „ = d.
5.	Length of first diagonal, . . .	4.09 „ = e.
6.	Length of second diagonal, . .	6.77 „ = f.

Hence, we find

$$ab = 6.68, \quad cd = 21.07$$
$$ab + cd = 27.75$$
$$ef = 27.69.$$

It appears, to my mind, impossible to evade the conclusion, that the particular form of muscle employed in the preceding cases, was the result of design and foresight; for the chances, *a priori*, would be many millions to one against the occurrence of the particular quadrilateral which can be inscribed in a circle, if the formation of the muscle depended on slow random variations of external conditions, only.

I shall now proceed to consider the general case of a skew muscle, formed by fibres arranged in any manner joining two bones, AB, and $A'B'$, not lying in the same plane. Let IA, and $I'A'$ be the directions in which the bones AB and $A'B'$ are placed; I and I' being the points corresponding to II', which

is the shortest line between the directions, and perpendicular to both. Draw $I'X$ parallel to IA, and let

$$II' = 2c$$
$$A'I'X = 2\phi.$$

Take C, the centre of the line II', and draw Cx parallel to the bisector of $XI'A'$, and draw Cy perpendicular to Cx

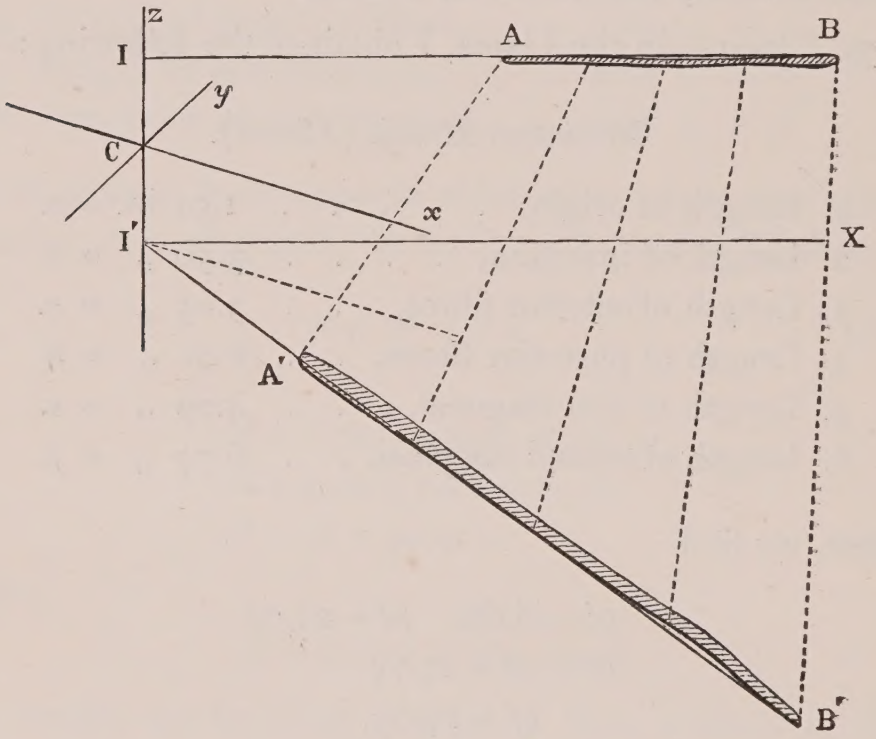


Fig. 89

and to II'; and take CI for the axis of z. Let Cx, Cy, Cz be the axes of co-ordinates, then the equations of the bones AB and $A'B'$ will be

$$\begin{aligned} y - mx &= 0 \qquad z - c = 0 \\ y + mx &= 0 \qquad z + c = 0. \end{aligned} \tag{80}$$

where $m = \tan \phi$.

Hence, the equations of any two planes passing through AB and $A'B'$, respectively, will be

$$\begin{aligned} (y - mx) + \lambda\,(z - c) &= 0 \\ (y + mx) + \lambda'\,(z + c) &= 0, \end{aligned} \tag{81}$$

where λ, λ' are arbitrary parameters.

I have already proved that the work done in rotating $A'B'$ round the intersection of these planes will be a maximum, when the angle between the planes is a right angle. Hence, in order to obtain the maximum effect, we must make the two planes (81) intersect at right angles. The angle θ, between the planes is found by the equation

$$\cos\theta = \frac{1 - m^2 + \lambda\lambda'}{\sqrt{\lambda^2 + m^2 + 1}\ \sqrt{\lambda'^2 + m^2 + 1}}, \qquad (82)$$

when $\theta = 90°$, $\cos\theta = 0$, hence we have the condition

$$\lambda\lambda' = m^2 - 1. \qquad (83)$$

If we now eliminate the parameters λ, λ', from the equations (81) and (83), we obtain the following locus of intersection of planes at right angles to each other, passing through the bones AB and $A'B'$,

$$(y^2 - m^2x^2) + (1 - m^2)(z^2 - c^2) = 0. \qquad (84)$$

This represents an Hyperboloid of one sheet, having its *Ellipse de Gorge* in the plane of y, z; and II' one of its axes; and the rectilinear generators of this hyperboloid are the intersections of the rectangular planes passing through the bones AB and $A'B'$.

In order, therefore, that the maximum work shall be done by the muscle $AB\,A'B'$, it is necessary that the axis of rotation shall coincide with some one or other of the generators of the hyperboloid (84); but this is not sufficient, for my Postulate (p. 238) requires, in addition, that of all the generators of the hyperboloid, that particular generator shall be chosen as axis of rotation, which shall give us the *maximum maximorum* of work done. We must, therefore, in the first place, discover which, of all the generators, possesses this property, and then, by measurements on the dead subject,

test whether or not the actual axis of rotation in the limb of the animal coincides with the *max. maximorum* axis.

Let $PP' = p$, be any fibre joining two points, (x, y, z) and (x', y', z') on the bones AB and $A'B'$. It has been already proved that the moment of this fibre, to turn $A'B'$ round the intersection of the planes passing through the two bones, is

$$\text{moment} = \frac{ff'}{p};$$

where f, f', denote the lengths of the perpendiculars let fall from each point upon the plane passing through the other point. If we express f, f', in terms of the coordinates, we obtain

$$\frac{ff'}{p} = \frac{4}{p} \cdot \frac{(mx + c\lambda')\,(mx' + c\lambda)}{\sqrt{\lambda^2 + m^2 + 1}\ \sqrt{\lambda'^2 + m^2 + 1}} \qquad (85)$$

or

$$\frac{ff'}{p} = \frac{4}{p} \cdot \frac{mc\,(x\lambda + x'\lambda') + \kappa}{\sqrt{\lambda^2 + m^2 + 1}\ \sqrt{\lambda'^2 + m^2 + 1}}, \qquad (86)$$

where

$$\kappa = m^2 xx' + c^2\,(m^2 - 1). \qquad (87)$$

Differentiating (86) with respect to λ and λ', and remembering the condition

$$\lambda d\lambda' + \lambda' d\lambda = 0.$$

we obtain the following equation of condition to determine the position of the generator which gives us the maximum or minimum amount of work,

$$\begin{aligned} &cm\,(\lambda^2 + m^2 + 1)\,(\lambda'^2 + m^2 + 1)\,(x\lambda - x'\lambda') \qquad (88)\\ &-(m^2 + 1)\,\{mc\,(x\lambda + x'\lambda') + \kappa\}\,(\lambda^2 - \lambda'^2) = 0 \end{aligned}$$

This is the condition required and expressed for a single fibre. The equation of condition for the entire muscle, com-

posed of any number of fibres, anyhow placed, may be thus deduced.

If we write

$$X = \Sigma\left(\frac{x}{p}\right) \quad X' = \Sigma\left(\frac{x}{p}\right)$$
$$K = m^2\Sigma\left(\frac{xx'}{p}\right) + c^2(m^2 - 1)\ \Sigma\left(\frac{1}{p}\right) \tag{89}$$

then we find

$$\text{Work} = \Sigma\left(\frac{ff'}{p}\right) = 4\,\frac{mc\,(X\lambda + X'\lambda') + K}{\sqrt{\lambda^2 + m^2 + 1}\ \sqrt{\lambda'^2 + m^2 + 1}} \tag{90}$$

and the equation of condition becomes

$$cm\,(\lambda^2 + m^2 + 1)\ (\lambda'^2 + m^2 + 1)\ \{X\lambda - X'\lambda'\} \tag{91}$$
$$= (m^2 + 1)\ \{mc\,(X\lambda + X'\lambda') + K\}\ (\lambda^2 - \lambda'^2).$$

This equation admits of a solution on inspection, viz.,

$$\lambda = \lambda',$$
$$X = X',$$

which implies that if the fibres were so arranged as to make $X = X'$, then the generator $\lambda = \lambda'$, which makes equal angles in space with the two bones AB and $A'B'$, is the required maximum. I have not, as yet, found any skew muscle whose fibres give $X = X'$, and therefore we must set aside the most obvious particular solution of our equation of condition, and seek its general solution. If we eliminate λ' by means of the relation

$$\lambda\lambda' = m^2 - 1,$$

and expand (91), arranging by powers of λ, we find, after a

few reductions, the following bi-quadratic equation of condition to determine the position of the axis of maximum work—

$$\begin{aligned} &- K(m^2 + 1)\lambda^4 \\ &+ 2c\{mX(m^4 + 1) - mX'(m^4 - 1)\}\lambda^3 \\ &+ 2c(m^2 - 1)\{mX(m^4 - 1) - mX'(m^4 + 1)\}\lambda \\ &+ (m^2 - 1)(m^4 - 1)K = 0. \end{aligned} \tag{92}$$

I am now required, in order to justify my Postulate, when applied to the complex case of skew muscles, to find the root of equation (92) for various muscles, and to show that the actual axis of rotation corresponds with that root. I select for examination the *adductor secundus* of various animals. This muscle, in all animals that I have dissected, forms a double sheet folded at the centre on itself, in this manner: the *posterior* fibres proceeding from the ischium are inserted the highest upon the femur, and gradually increase in length until we reach the most dependent point of the pubo-ischium; this portion of the muscle I shall call *adductor secundus* (α); in like, but reversed order, the anterior fibres proceeding from the pubes are inserted highest up on the femur, and gradually increase in length until they join the long fibres of (α), with which they coincide; this portion of the muscle I shall call *adductor secundus* (β). Thus, the two sheets of muscle, (α) and (β), are placed side by side, as if folded against each other on an axis corresponding to the longest and middle fibres of the muscle; and, although it is evident that the two sheets generally may act together, yet, mechanically considered, they are to be regarded as quite distinct muscles.

(1) *Adductor secundus* (β)—*Llama.*—Having dissected down upon the adductor muscles of a fine male Llama, after previously ascertaining the limits of abduction of the femur, before removing the skin and *gracilis* muscle; and having

carefully placed the thigh in the position of extreme abduction, I made the following measurements:—

Llama.

Lengths of equidistant fibres.

p = 4.27 in.
p_1 = 6.70 ,,
p_2 = 8.46 ,,
p_3 = 10.80 ,,

Lengths of origin and insertion.

a = 1.96 in. (origin).
a' = 6.00 ,, insertion.

Constants of Hyperboloid.

$2c$ = 2.38 in.
$2\phi = 49°15'$

The constants of the Hyperboloid were measured by fastening straight steel wires along the directions of origin and insertion, and then measuring the shortest distance between them, and the angle made by the two lines.

The positions of the points I and I', on the bones, may either be measured directly, like $2c$ and 2ϕ, or may be calculated from the other measurements. In order to check the whole calculation, I measured their position, and found

$$a = +\ 1.96 \text{ in.}$$
$$a' = +\ 4.40 \text{ ,, ;}$$

where a signifies the distance from I to the (*anterior*) fibre p, measured on the bone AB; and a' signifies the distance from I' to the same (*posterior*) fibre p, measured on the bone $A'B'$. It is to be noted that the fibre p is anterior on the pubes, and posterior on the femur; so that all the fibres cross each other, forming a well marked skew surface. The values of a and a' may be calculated as follows, from the measured lengths of any two fibres (p, p_3)—

$$p^2 = a^2 + a'^2 - 2aa' \cos 2\phi + 4c^2,$$
$$p_3^2 = (a - a)^2 + (a' + a')^2 - 2(a - a)(a' + a') \cos 2\phi + 4c^2;$$

from which we obtain—

$$\begin{aligned} p_3^2 - p^2 = &- 2\{a + a' \cos 2\phi\}\, a \\ &+ 2\{a' + a \cos 2\phi\}\, a' \\ &+ \{a^2 + a'^2 + 2aa' \cos 2\phi\} \end{aligned} \qquad (93)$$

Introducing the numerical values, I find

$$14.55a' = 11.72a + 43.28.$$

If we were to combine the four fibres in pairs, we should obtain from each pair a similar relation between a and a', in all of which it is easy to see that the co-efficients of a and a' remain the same, while the absolute term varies according to the several measurements. I shall write down the results of the six possible combinations:—

	Absolute Term.
$(p - p_3)$	43.28
$(p - p_2)$	43.23
$(p - p_1)$	61.65
$(p_1 - p_3)$	33.14
$(p_1 - p_2)$	25.29
$(p_2 - p_3)$	43.63
Mean, . . .	**41.70**

The value found from the fibres (p, p_3) is nearly the mean value, which is what we should expect; for the errors of observation in measurement will affect equation (93) least; when the differences of the fibres and the lengths of the intercepts between the points of application are greatest. Taking the mean value, we have

$$14.55\ a' = 11.72\ a + 41.70,$$

or

$$a' = 0.805\ a + 2.866. \qquad (94)$$

Introducing this value into the equation,

$$p^2 = a^2 + a'^2 - 2aa' \cos 2\phi + 4c^2,$$

we obtain

$$0.602\ a^2 + 0.888\ a^2 - 4.356 = 0; \qquad (95)$$

and, finally, remembering (94), we reach the following pairs of real values for a and a':—

$$a = \begin{cases} + 2.05 & \text{in. (observed = 1.96 in.)} \\ - 3.53 & \text{,,} \end{cases}$$

$$a' = \begin{cases} + 4.51 & \text{in. (observed = 4.40 in.)} \\ - 0.025 & \text{,,} \end{cases}$$

From the preceding agreement of calculation and observation, we may regard the true positions* of I and I' as having been accurately ascertained.

We are now in a condition to determine the quantities, X, X', K, which occur in the coefficients of the equation of condition (92); for

$$mX = \sin\phi\ \Sigma\left(\frac{a}{p}\right) \qquad mX' = \sin\phi\ \Sigma\left(\frac{a'}{p}\right)$$

$$K = \sin^2\phi\ \Sigma\left(\frac{aa'}{p}\right) + c^2\,(m^2 - 1)\,\Sigma\left(\frac{1}{p}\right) \qquad (96)$$

Using the measurements made in the freshly-dead muscle, we obtain the following:—

Llama—Adductor secundus (β).

$a = + 2.05$ in.	$a' = + 4.51$ in.
$a_1 = + 1.40$,,	$a'_1 = + 6.51$,,
$a_2 = + 0.75$,,	$a'_2 = + 8.51$,,
$a_3 = + 0.09$,,	$a'_3 = + 10.51$,,

* The second values of a and a', refer to the position of the bones, in which they make together an angle of 2ϕ reversed; and they have no relation to the present problem.

$$\frac{a}{p} = + 0.480 \qquad \frac{a'}{p} = + 1.056$$

$$\frac{a_1}{p_1} = + 0.209 \qquad \frac{a'_1}{p_1} = + 0.972$$

$$\frac{a_2}{p_2} = + 0.088 \qquad \frac{a'_2}{p_2} = + 1.006$$

$$\frac{a_3}{p_3} = + 0.009 \qquad \frac{a'_3}{p_3} = + 0.973$$

$$\Sigma\left(\frac{a}{p}\right) = + 0.786 \qquad \Sigma\left(\frac{a'}{p}\right) = + 4.007$$

$$\frac{aa'}{p} = + 2.165 \qquad \frac{1}{p} = 0.234$$

$$\frac{a_1a'_1}{p_1} = + 1.360 \qquad \frac{1}{p_1} = 0.149$$

$$\frac{a_2a'_2}{p_2} = + 0.754 \qquad \frac{1}{p_2} = 0.118$$

$$\frac{a_3a'_3}{p_3} = + 0.088 \qquad \frac{1}{p_3} = 0.092$$

$$\Sigma\left(\frac{aa'}{p}\right) = + 4.367 \qquad \Sigma\left(\frac{1}{p}\right) = 0.593$$

From these enumerations, we find, from equations (96),

$$mX = + 0.330$$
$$mX' = + 1.683$$
$$K = + 0.096.$$

Introducing these values into equation (92), we obtain

$$- 0.116\,\lambda^4 + 3.375\,\lambda^3 + 4.903\,\lambda + 0.042 = 0. \quad (97)$$

It is evident, from inspection, that equation (97) has a very small root and a very large root, both real; because the coefficients of the first and last terms are small, as compared with the intermediate coefficients; and the smallness of these coefficients depends on the small value of K, which is a factor of the first and last coefficients.

These small and large roots of equation (92) would become

$$\lambda = 0, \qquad \lambda = \frac{1}{0}; \tag{98}$$

in one or other of two cases, viz. :—

$$m = 1, \qquad K = 0.* \tag{99}$$

Having determined the value of λ from equation (97), we must introduce this value into equation (92), in order to calculate the actual value of the maximum work or moment of the muscle. This equation may be thus written, by elimination of λ' :—

$$\Sigma\left(\frac{ff'}{p}\right) = 4\,\frac{cmX\lambda^2 + K\lambda + cmX'(m^2-1)}{\sqrt{\lambda^2+m^2+1}\;\;\sqrt{(m^2+1)\lambda^2+(m^2-1)^2}}. \tag{100}$$

Since the denominator of this fraction does not change sign with the change of sign in λ, it is plain that the numerator equated to zero determines the position of the two neutral generators of the hyperboloid, with respect to which the skew muscle does no work, and that these generators divide all the generators of the hyperboloid into two groups, for one of

* It may be interesting to note that the geometrical meaning of $K = 0$ is, that the line joining X and X' is a tangent to the hyperboloid conjugate to the locus hyperboloid (84).

which the work done is *negative* and for the other *positive*. The equation to determine the neutral axes is

$$cmX \cdot \lambda^2 + K\lambda + cmX' (m^2 - 1) = 0. \qquad (101)$$

This equation becomes, in the case of the Llama,

$$0.393\ \lambda^2 + 0.096\ \lambda - 1.58 = 0.$$

The roots of this equation are

$$\lambda = \begin{cases} + 1.89 \\ - 2.13. \end{cases}$$

If we now solve the equation (97), we find two real roots, viz. :—

$$\lambda = - \ 0.0085,$$

$$\lambda = + 29.3.$$

The remaining roots of the equation are imaginary.

The small value of λ corresponds to the maximum *negative* work done by the muscle, as may be seen from the following Table, formed by introducing successive values of λ into equation (100) :—

Llama—Adductor secundus (β).

λ.	$\Sigma\left(\frac{ff'}{p}\right)$
+ 0.1000	− 5.6568
0.0000	− 7.2828
− 0.0085	− 7.2848
− 0.1000	− 7.2088

The large value of λ corresponds with the maximum *positive* work of the skew muscle.

Hence we see, by a process of reasoning similar to that employed in reference to the great pectoral muscle of the wing of the bird, that the axis of *maximum instability* corresponds to $\lambda = -0.0085$, and that the axis of maximum stability corresponds to $\lambda = +29.3$.

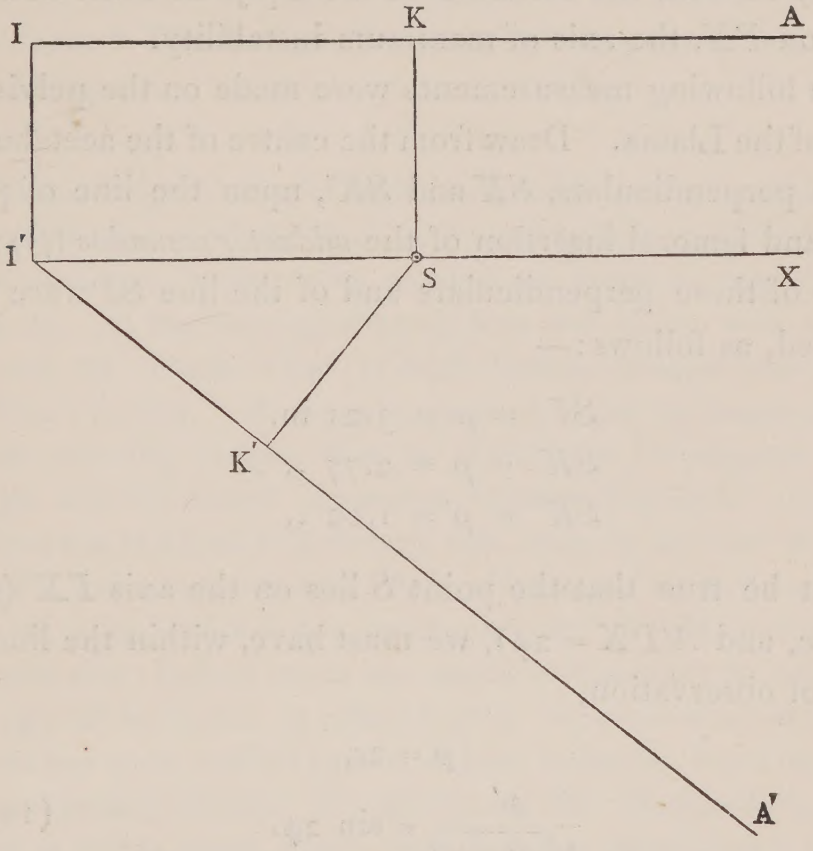


Fig. 90.

The value of λ corresponding to the axis of maximum instability is so nearly zero, that we may assume $\lambda = 0$, without sensible error. This value of λ corresponds with $\lambda' = \frac{1}{0}$, so that the equations (81) of the planes passing through the bones are reduced to

$$\begin{aligned} y - mx &= 0, \\ z + c &= 0. \end{aligned} \qquad (102)$$

The corresponding axis of rotation is shown in Fig. 90.

Let IA and $I'A'$ represent the directions of the bones AB and $A'B'$, and let II' be the shortest line joining these directions. Through the point I' draw $I'X$ parallel to IA ; then the axis IX represents equations (102).

It follows from this construction, that if the Postulate (p. 238) be true, the socket S' of the hip joint must lie somewhere on $I'X$, the axis of maximum instability.

The following measurements were made on the pelvis and femur of the Llama. Draw from the centre of the acetabulum, S, two perpendiculars, SK and SK', upon the line of pubic origin and femoral insertion of the *adductor secundus* (β); the lengths of these perpendiculars and of the line SI were then measured, as follows:—

$$\begin{aligned} SI &= \rho = 3.21 \text{ in.} \\ SK &= p = 2.77 \text{ ,,} \\ SK' &= p' = 1.02 \text{ ,,} \end{aligned}$$

If it be true that the point S lies on the axis $I'X$ (since $II' = 2c$, and $A'I'X = 2\phi$), we must have, within the limits of errors of observation,

$$\begin{aligned} p &= 2c, \\ \frac{p'}{\sqrt{\rho^2 - p^2}} &= \sin 2\phi. \end{aligned} \qquad (103)$$

Introducing the measured values, we obtain

$$\begin{aligned} p &= 2.77 \text{ in.} \\ 2\phi &= 50^\circ\, 58'. \end{aligned}$$

These values differ from the constants of the hyperboloid—

$$\begin{aligned} 2c &\text{ by } 0.39 \text{ of an inch,} \\ 2\phi &\text{ ,, } 1^\circ\, 43'. \end{aligned}$$

These differences are within the limits of errors of observation, and we may consider the Postulate to be proved true, when applied to the skew adductor of the Llama.

(2). *Adductor secundus* (β)—*Greyhound.*—The next example of a skew muscle that I shall give is the *adductor secundus* (β) in the Greyhound. The subject of my dissection was the celebrated Irish Greyhound, "Master Magrath," who is justly considered as the best and fastest dog that ever ran a course. The following brief notice of this wonderful dog, taken from "The Belfast News-letter," may be of interest to the reader:—

"'Master Magrath' (named after an orphan boy who reared him) was by 'Dervock,' out of 'Lady Sarah,' and made his first appearance at the Lurgan meeting in October, 1867, when he was about one year and eight months old. At that meeting, although little more than a week in training, he won the 'Visitors' Cup' (32 dogs), beating, amongst others, 'S.S.' and 'What's the Tip.' His second appearance was at the Creagh meeting the week following Lurgan, when he divided the 'Moneyglass Purse' (32 dogs) with his kennel companion, 'Master Nathaniel.' His next appearance was at Altcar, in February, 1868, when he won the 'Waterloo Cup.' His first course was with 'Belle of Scotland,' whom he outpaced both in the no-go and final spin, beating her very cleverly, indeed. His second trial with 'Kalista' might have ended unfortunately for him, as he got an ugly fall in the run up; but his great cleverness saved him, as he was on his legs again, and had puss in his jaws, before Mr. Kay's bitch had a chance of putting him out. His third course with 'Marionette' was such a hollow, as well as clever, victory, that there was immediately a rush to get out by those who had taken a liberty with the nomination. His coming against 'Brigade' in the next round was quite the event of the meeting, and, although the trial was not A 1, the speed and cleverness exhibited by Lord Lurgan's dog justly entitled him to the award. His next course with 'Lobelia' he made short by his extreme cleverness; and when, finally, he met 'Cock Robin,' it was, bar accidents, 'Lombard-street to a China orange,' and 4 and 5 to 1 was offered in vain. The deciding course was run under great difficulties, and, had not 'Master Magrath' been one of the cleverest greyhounds that ever went to slips, he, in all probability, as the course was run, would have been put out. At the Lurgan meeting of 1868 he divided the 'Brownlow Cup Stake' with Mr. Stooker's 'Sir William,' again beating 'S.S.' and three others. At the Waterloo meeting o 1869, he met and beat 'Borealis,' 'Hard Lines,'

'Charming May,' 'Randolph,' 'Lobelia,' and 'Bab-at-the-Bowster;' the style in which he beat the last two gaining for him, from good judges, the opinion that a faster or cleverer greyhound never was slipped. In 1870 he sustained his only defeat, which was from Mr. Trevor's 'Lady Lyon,' the course creating quite a consternation in all sporting circles; but, whatever the cause, it was not 'Magrath's' style of running. At the Lurgan meeting of 1870, he won the 'Brownlow Cup,' beating, in the last two courses, 'Smuggler' and 'Fritz.' His last appearance in public was at Altcar, in February, 1871, when he defeated 'Wharfinger,' 'Eyes of Fire' (drawn after an undecided course), 'Rocketer,' 'Letter T,' 'Black Knight,' and 'Pretender.' It will thus be observed that out of the thirty-seven courses which 'Master Magrath' ran in public, his colours were only lowered once."

"*Master Magrath.*"

Lengths of Equidistant Fibres.	Lengths of Origin and Insertion.
$p = 4.81$ in.	$a = 1.20$ in.
$p_1 = 6.11$,,	$a' = 3.16$,,
$p_2 = 8.20$,,	

Constants of Hyperboloid.

$2c = 1.68$ in.

$2\phi = 69° 00'$.

The positions of the points I and I', on the pelvis and femur were carefully ascertained by the same method as that used in the case of the Llama, and the following measurements and calculations made:—

"*Master Magrath*"—*Adductor secundus* (β).

$a = 1.20$ in.	$a' = 4.14$ in.
$a_1 = 0.60$,,	$a'_1 = 5.72$,,
$a_2 = 0.00$,,	$a'_2 = 7.30$,,

$$\frac{a}{p} = 0.249 \qquad \frac{a'}{p} = 0.860$$

$$\frac{a_1}{p_1} = 0.100 \qquad \frac{a'_1}{p_1} = 0.936$$

$$\frac{a_2}{p_2} = 0.000 \qquad \frac{a'_2}{p_2} = 0.890$$

$$\Sigma\left(\frac{a}{p}\right) = 0.349 \qquad \Sigma\left(\frac{a'}{p}\right) = 2.686$$

$$\frac{aa'}{p} = 1.031 \qquad \frac{1}{p} = 0.208$$

$$\frac{a_1 a'_1}{p_1} = 0.561 \qquad \frac{1}{p_1} = 0.163$$

$$\frac{a_2 a'_2}{p_2} = 0.000 \qquad \frac{1}{p_2} = 0.122$$

$$\Sigma\left(\frac{aa'}{p}\right) = 1.592 \qquad \Sigma\left(\frac{1}{p}\right) = 0.493$$

From these data we find

$$mX = 0.197$$

$$mX' = 1.520$$

$$K = 0.326;$$

and finally, equation (92) becomes

$$-0.479\,\lambda^4 + 2.395\,\lambda^3 + 2.128\,\lambda + 0.135 = 0.$$

This equation, as before, has a small real root, viz.,

$$\lambda = -0.063,$$

which leads to the construction (Fig. 90).

On measuring the lines, I found

$$SK = p = 1.65 \text{ in.}$$
$$SK' = p' = 0\ 80\ ,,$$
$$SI = \rho = 1.86\ ,,$$

From which we find

$$p = 1.65 \text{ in.}$$
$$\sin 2\phi = \frac{p'}{\sqrt{\rho^2 - p^2}}$$
$$2\phi = 68^\circ\ 28'.$$

These values differ from the constants of the hyperboloid—

$2c$, by 0.03 of an inch;
2ϕ, by $0^\circ\ 28'$.

(3.) *Adductor secundus* (β)—*Woman.*—The next example of skew muscles that I shall give is the case of a woman, on whom the following measurements were made :—

Woman—Adductor secundus (β).

Lengths of Fibres.	Lengths of Origin and Insertion.
$p = 5.67$ in.	$a = 2.55$ in.
$p_1 = 8.00$,,	$a' = 7.27$,,
$p_2 = 11.57$,,	

Constants of Hyperboloid.

$$2c = 3.60$$
$$2\phi = 70^\circ.$$

The positions of the points I and I' were determined, as in the case of the Llama, by taking the fibres in pairs, and

using the mean of all three. In this way I obtained (93) the following equation:—

$$14.54\ a' = 5.10\ a + 42.38$$
$$a' = 0.35\ a + 2.91.$$

Substituting this value of a' in the equation,

$$p^2 = a^2 + a'^2 - 2aa' \cos 2\phi + 4c^2,$$

we find

$$1.12\ a^2 + 2.03\ a - 10.71 = 0\ ;$$

and finally,

$$a = \begin{cases} +2.32 \\ -4.13 \end{cases} \qquad a' = \begin{cases} +3.72 \\ +1.47 \end{cases}$$

Taking the first pair of values, which correspond to the problem in hand, we find—

Woman—Adductor secundus (β).

$a = +2.32$ in.	$a' = 3.72$ in.
$a_1 = +1.04$,,	$a'_1 = 7.36$,,
$a_2 = -0.24$,,	$a'_2 = 11.00$,,
$\frac{a}{p} = +0.409$	$\frac{a'}{p} = +0.656$
$\frac{a_1}{p_1} = +0.130$	$\frac{a'_1}{p_1} = +0.920$
$\frac{a_2}{p_2} = -0.021$	$\frac{a'_2}{p_2} = +0.951$
$\Sigma\left(\frac{a}{p}\right) = +0.518$	$\Sigma\left(\frac{a'}{p}\right) = +2.527$

$$\frac{aa'}{p} = +\ 1.522 \qquad \frac{1}{p} = 0.176$$

$$\frac{a_1a'_1}{p_1} = +\ 0\ 957 \qquad \frac{1}{p_1} = 0.125$$

$$\frac{a_2a'_2}{p_2} = -\ 0.228 \qquad \frac{1}{p_2} = 0.086$$

$$\Sigma\left(\frac{aa'}{p}\right) = +\ 2.251 \qquad \Sigma\left(\frac{1}{p}\right) = 0.387$$

From these data we find

$$K = +\ 0\ 103$$
$$mX = +\ 0.297$$
$$mX' = +\ 1.448$$

and, finally, the equation of condition (92) becomes

$$-\ 0.153\ \lambda^4 + 5.288\ \lambda^3 + 3.710\ \lambda + 0.040 = 0.$$

The root of this equation, which gives the axis of maximum work, is

$$\lambda = -\ 0.011.$$

(4.) *Adductor secundus* (β)—*African Leopard.*—The following measurements of the skew adductor muscle were made upon a fine male Leopard, from the west coast of Africa:—

African Leopard.—Adductor secundus (β).

Lengths of Fibres.	Lengths of Origin and Insertion.
p = 2.42 in.	a = 2.76 in.
p_1 = 3.27 ,,	a' = 5.40 ,,
p_2 = 6.42 ,,	

Constants of Hyperboloid.

$$2c = 2.05$$

$$2\phi = 60^\circ\ 30'$$

The points I and I' were found by combining the fibres in pairs, the mean result giving the following relation between a and a':—

$$13.56\ a' = 10.90\ a - 16.31$$

$$a' = 0.804\ a - 1.203.$$

From which I obtained, as before,

$$0.84\ a^2 - 0.73\ a - 0.20 = 0$$

$$a = \begin{cases} + 1.09 \\ - 0.22 \end{cases} \qquad a' = \begin{cases} - 0.34 \\ - 1.38 \end{cases}$$

Taking the first pair of values, as before, I find—

African Leopard—Adductor secundus (β).

$a = + 1.09$ in.	$a' = - 0.34$ in.
$a_1 = - 0.29$ „	$a'_1 = + 2.36$ „
$a_2 = - 1.67$ „	$a'_2 = + 5.06$ „
$\frac{a}{p} = + 0.450$	$\frac{a'}{p} = - 0.145$
$\frac{a_1}{p_1} = - 0.090$	$\frac{a'_1}{p_1} = + 0.722$
$\frac{a_2}{p_2} = - 0.260$	$\frac{a'_2}{p_2} = + 0.788$
$\Sigma\left(\frac{a}{p}\right) = + 0.100$	$\Sigma\left(\frac{a'}{p}\right) = + 1.363$

$$\frac{aa'}{p} = -\,0.153 \qquad \frac{1}{p} = 0.413$$

$$\frac{a_1a'_1}{p_1} = -\,0.209 \qquad \frac{1}{p_1} = 0.306$$

$$\frac{a_2a'_2}{p_2} = -\,1.316 \qquad \frac{1}{p_2} = 0.155$$

$$\Sigma\left(\frac{aa'}{p}\right) = = -\,1.678 \qquad \Sigma\left(\frac{1}{p}\right) = 0.874$$

From these data we find,

$$K = -\,1.024$$

$$mX = +\,0.050$$

$$mX' = +\,0.681\,;$$

and, finally, the equation of condition (92) becomes

$$+\,1.372\,\lambda^4 + 1.343\,\lambda^3 + 1.091\,\lambda - 0.077 = 0.$$

The root of this equation, corresponding to the axis of maximum work, is

$$\lambda = +\,0.07.$$

In the preceding cases, and in others which I have calculated, too numerous to quote, we find that the generator of the hyperboloid, which constitutes the axis of maximum work, corresponds very closely with the value,

$$\lambda = 0,$$

the geometrical construction of which is shown in Fig. 90.

It is not difficult to show that, in this case, the entire skew muscle may be mechanically replaced by a single fibre, supposed to be endowed with the united force of all the fibres ;

and by constructing this fibre, we may obtain a clear and direct view of the action of the skew muscle, and, as I believe, discover the reason for the invention of such muscles, and, at the same time, test directly and satisfactorily the truth or falsehood of the Postulate (p. 238).

Making $\lambda = 0$, in equation (100), which gives the work done by the skew muscle for any generator, we find for the work done by the generator $I'X$ (Fig. 90), the following expression :—

$$\Sigma\left(\frac{ff'}{p}\right) = \frac{4cmX'}{\sqrt{m^2+1}} = 2c \sin 2\phi \; \Sigma\left(\frac{a'}{p}\right); \qquad (104)$$

because $mX' = \sin \phi \; \Sigma\left(\frac{a'}{p}\right)$.

Let us now assume

$$n\frac{L'}{P} = \Sigma\left(\frac{a'}{p}\right); \qquad (105)$$

where n denotes the number of fibres, P the length of the equivalent fibre, and L' its distance from the point I'. Hence we find

$$\Sigma\left(\frac{ff'}{p}\right) = n\,\frac{2c \times L' \sin 2\phi}{P}; \qquad (106)$$

which represents the work done by a single fibre, n times as strong as any single fibre, attached to the bone $A'B'$ at the point O', corresponding to L', and having a length, P.

The only assumption in the foregoing is (105), that

$$\frac{L'}{P} = \frac{1}{n}\Sigma\left(\frac{a'}{p}\right),$$

which means that the ratio of L' to P is given. Hence, there are an infinite number of single fibres, any one of which

would be the mechanical equivalent of the skew muscle, provided it be so placed between the bones as to make

$$\frac{L'}{P} = \frac{1}{n} \Sigma\left(\frac{a'}{p}\right).$$

I am, therefore, at liberty to make a second assumption, for the purpose of fixing the other end of the fibre, and so render the Problem of drawing an equivalent fibre definite.

I choose a point O, on the bone AB, determined on the same principle as the point O', viz. :—

$$\frac{L}{P} = \frac{1}{n} \Sigma\left(\frac{a}{p}\right); \qquad (107)$$

and if from this point I draw a fibre to $A'B'$, satisfying the condition (105), the fibre joining O and O', and having the united force of all the fibres, will be mechanically equivalent to the entire skew muscle.

But the single equivalent fibre OO', so found, will not satisfy the Postulate (p. 238), unless its extremities lie in a plane perpendicular to the axis of rotation, and at equal distances from that axis (*vide* p. 324); from which it readily follows that, if the Postulate be true, the points O and O' must lie at equal distances from the socket of the hip joint.

Let us proceed to enquire whether this be so or not.

Determination of the single Fibre equivalent to a Skew Muscle.

1. *Llama—Adductor secundus* (β).—In the case of the Llama, we have, for four fibres,

$$\Sigma\left(\frac{a}{p}\right) = +\,0.786 \qquad \Sigma\left(\frac{a'}{p}\right) = +\,4.007.$$

Hence, equations (105) and (107) give

$$\frac{L'}{P} = + 0.196, \qquad \frac{L}{P} = + 1.002 ;$$

but

$$P^2 = L^2 + L'^2 - 2LL' \cos 2\phi + 4c^2,$$

from which we find, by eliminating L and L', and using the constants of the hyperboloid,

$$2c = 2.38, \qquad 2\phi = 49^\circ\ 15',$$

$$P^2 = \frac{5.6644}{0.213}, \qquad P = 5.157 \text{ in.},$$

and

$$L = + 1.010 \text{ in.}, \qquad L' = + 5.167 \text{ in.}$$

On laying down the points O and O', just found, upon the pelvis and femur, and measuring their distances from the centre of the socket of the hip joint, I found—

Distance from O to centre of socket =	2.88 in.
Distance from O' to centre of socket =	3.14 ,,
Difference, . .	0.26 in.

The points O and O' both lie within the origin and insertion ; but it is to be noticed that the point O' is brought down to the near end of the insertion on the femur, within 0.56 in. of its extremity. We may also remark that the *equivalent fibre*, OO', does not coincide in direction with any of the real fibres of the muscle, none of which intersect each other in space.

2. "*Master Magrath*"—*Adductor secundus* (β).—In the case of the celebrated Irish Greyhound we have

$$2c = 1.68 \text{ in.}, \qquad 2\phi = 69^\circ\ 00',$$

$$\Sigma\left(\frac{a}{p}\right) = +\ 0.349, \quad \Sigma\left(\frac{a'}{p}\right) = +\ 2.686,$$

corresponding to three fibres: from these data we find, as before,

$$\frac{L}{P} = 0.116, \qquad \frac{L'}{P} = 0.895,$$

$$P^2 = \frac{2.8224}{0.26}, \qquad L = 0.38 \text{ in.},$$

$$P = 3.29 \text{ in.}, \qquad L' = 2.95 \text{ in.}$$

On laying down these points upon the pelvis and femur, I found—

Distance of O from centre of socket =	1.95 in.
Distance of O' from centre of socket =	2.28 ,,
Difference, . .	0.23 in.

Here the point O lies inside the origin of the muscle, but the point O' lies completely outside the insertion of the muscle, at a distance of 1.19 in. nearer to the hip joint than the upper extremity of the line of insertion;—and the *equivalent fibre* OO' is shorter than any real fibre of the skew muscle.

3. *Woman—Adductor secundus* (β).—In this case we have

$$2c = 3.60 \text{ in.}, \qquad 2\phi = 70^\circ.00',$$

$$\Sigma\left(\frac{a}{p}\right) = 0.518, \qquad \Sigma\left(\frac{a'}{p}\right) = +\ 2.527,$$

corresponding to three fibres: from these data, we find, as before,

$$\frac{L}{P} = +0.173, \qquad \frac{L'}{P} = +0.842,$$

$$P^2 = \frac{12.96}{0.361}, \qquad P = 5.99 \text{ in.},$$

$$L = +1.04 \text{ in.}, \qquad L' = +5.04 \text{ in.}$$

On laying down these points on the pelvis and femur, I found—

Distance from O to centre of socket	= 3.84 in.
Distance from O' to centre of socket	= 3.64 ,,
Difference, . . .	0.20 in.

The points O and O' are both inside the origin and insertion.

4. *African Leopard—Adductor secundus* (β).—In this case we have,

$$2c = 2.05 \text{ in.}, \qquad 2\phi = 60^\circ\ 30',$$

$$\Sigma\left(\frac{a}{p}\right) = +0.100, \qquad \Sigma\left(\frac{a'}{p}\right) = +1.363$$

corresponding to three fibres: from these data we find, as before,

$$\frac{L}{P} = +0.033, \qquad \frac{L'}{P} = +0.454,$$

$$P^2 = \frac{4.2025}{0.808}, \qquad P = 2.28 \text{ in.},$$

$$L = +0.07 \text{ in.}, \qquad L' = +1.03 \text{ in.}$$

On marking off the points O and O' on the bones, I found—

Distance of O from centre of socket = 1.82 in.
Distance of O' from centre of socket = 2.32 ,,

Difference, . . . 0.50 in.

The points O and O' are both inside the origin and insertion, but the point O' is only 0.41 in. inside the posterior end of the insertion, showing, as in all the former cases, that the *equivalent fibre* is inserted very high up on the femur.

5. *African Leopard—Adductor secundus* (a).—I shall select, as my last example of skew muscles, the ischiadic sheet of the second adductor of the African Leopard. In this case, the shortest fibres are the most posterior fibres on the ischium, and are inserted highest up on the femur: this difference of arrangement will require a change of sign in some of the quantities used in my calculation, which will be readily understood from the formulæ employed.

African Leopard—Adductor secundus (a).

Lengths of Fibres.	Lengths of Origin and Insertion.
p = 3.55 in.	a = 1.66 in.
p_1 = 4.24 ,,	a' = 5.40 ,,
p_2 = 6.42 ,,	

Constants of Hyperboloid.

$2c$ = 2.26 in.

$2\phi = 74^\circ\ 40'$.

The positions of the points I and I' were found by combining the lengths of the fibres, in pairs, as in the following equation:—

$$p_2^2 - p^2 = 2\{a - a' \cos 2\phi\}\ a + 2\{a' - a \cos 2\phi\}\ a' + \{a^2 + a'^2 - 2aa' \cos 2\phi\}.$$

The resulting relation between a and a', which is the mean of the three different combinations, is

$$3.04\ a + 9.92\ a' + 2.43 = 0.$$

From this relation we find, as before,

$$1.25\ a^2 - 0.02\ a - 7.43 = 0,$$

$$a = \begin{cases} + 2.44 \\ - 2.44 \end{cases} \qquad a' = \begin{cases} - 0.99 \\ + 0.50 \end{cases}$$

Adopting the second pair of values, which correspond with the question in hand, we find:—

African Leopard—Adductor secundus (a).

$a = - 2.44$ in.	$a' = + 0.50$ in.
$a_1 = - 1.61$,,	$a_1' = + 3.20$,,
$a_2 = - 0.78$,,	$a_2' = + 5.90$,,
$\frac{a}{p} = - 0.687$	$\frac{a'}{p} = + 0.141$
$\frac{a_1}{p_1} = - 0.380$	$\frac{a_1'}{p_1} = + 0.755$
$\frac{a_2}{p_2} = - 0.122$	$\frac{a_2'}{p_2} = + 0.919$
$\Sigma\left(\frac{a}{p}\right) = - 1.189$	$\Sigma\left(\frac{a'}{p}\right) = + 1.815$

Hence, as there are three fibres employed, we find

$$\frac{L}{P} = - 0.396, \qquad \frac{L'}{P} = + 0.605,$$

$$P^2 = \frac{5.1076}{0.35}, \qquad P = 3.82 \text{ in.},$$

$$L = - 1.51 \text{ in.}, \qquad L' = + 2.31 \text{ in.}$$

On marking off the points O and O' on the pelvis and femur, I found—

Distance of O from centre of socket =	2.63 in.
Distance of O' from centre of socket =	2.58 ,,
Difference, . .	0.05 in.

The preceding facts demonstrate in a remarkable manner the truth of the Postulate, p. 238, as applied to the case of skew muscles. The postulate requires the distances of the points O and O' from any point situated on the common intersection of the two planes containing the bones AB and $A'B'$ to be equal; and, from the conditions of the joint, the centre of the socket is supposed to lie on that intersection. Bringing together all the results, we find

Proof of the Postulate for Skew Muscles.

Muscle.	Distance from O to Socket.	Distance from O' to Socket.	Difference.	Angle.
Adductor secundus (β).				
1. Llama, ...	2.88 in.	3.14 in.	0.26 in.	47° 28′
2. "Master Magrath," .	1.95 ,,	2.28 ,,	0.23 ,,	49 28
3. Woman, ...	3.84 ,,	3.64 ,,	0.20 ,,	43 29
4. Leopard, . . .	1.82 ,,	2.32 ,,	0.50 ,,	51 53
Adductor secundus (α),				
5. Leopard, ...	2.63 ,,	2.58 ,,	0.05 ,,	44 27
			Mean, .	**47° 21′**

The last column of this Table shows the angle made by the fibre OO' with the plane containing the pelvic origin AB. This angle, according to the Postulate, ought to be 45°; its mean value is 47° 21′, which is a very remarkable approximation in observations of this kind.

Let P denote the length of the equivalent fibre joining O and O' situated in the two rectangular planes. In its position of maximum effect, the perpendicular let fall upon the line OO' from the intersection of the planes will be a maximum, and will measure the work done; we thus have

$$\text{Maximum Work} = \frac{P}{2}.$$

For any other position of the fibre, making an angle θ with one of the planes, it is easy to show that the perpendicular, or

$$\text{Work done} = P \sin\theta \cos\theta = \frac{P}{2} \sin 2\theta.$$

Hence we have

$$\frac{\text{Work done}}{\text{Maximum Work}} = \sin 2\theta. \qquad (108)$$

We can thus calculate a Table showing the percentage of the maximum theoretical work possible, which is actually done by the observed muscle:—

Ratio of Observed Work done by Skew Muscles to the Maximum Possible Work.

Muscle.	Work done.
Adductor secundus (β).	
1. Llama, ...	99.63 per cent.
2. "Master Magrath," . .	98.78 ,,
3. Woman, ...	99.86 ,,
4. Leopard, ...	97.13 ,,
Adductor secundus (α).	
5. Leopard, ...	99.98 ,,
Mean, . . .	**99.076** per cent.

The results obtained from the calculation of the equivalent fibre *OO′*, furnish us with more than a proof of the Postulate; they give us, as I believe, an inner insight into the reason for the invention and use of skew muscles. In order to illustrate my meaning, I shall place, in the form of a Table, the distances of the points of insertion on the femur, of the longest and shortest fibres of the muscle, together with the distances of the point of insertion of the (imagined) equivalent fibre *OO′*.

Position of Equivalent Fibre.

	Distance from Socket to upper end of insertion.	Distance from Socket to lower end of insertion.	Distance from Socket to insertion of *OO′*.
Adductor secundus (β).			
1. Llama, ...	2.52 in.	8.53 in.	3.14 in.
2. "Master Magrath," .	3.10 ,,	6.16 ,,	2.28 ,,
3. Woman, ...	3.32 ,,	13.10 ,,	3.64 ,,
4. Leopard, ...	1.47 ,,	6.70 ,,	2.32 ,,
Adductor secundus (α).			
5. Leopard, ...	1.47 ,,	6.70 ,,	2.58 ,,

This Table shows clearly the tendency of the skew form of muscle to bring the resultant line of force higher up on the femur, and nearer to the hip joint; but it may be still better seen from the next Table, which gives the actual transfer on the femur from the point of bisection of the line of insertion along the *linea aspera*.

Transfer of Force caused by Skew Muscle.

Muscle.	Distance from O' to centre of insertion.	Length of Femur.	Proportionate Transfer.
Adductor secundus (β).			
1. Llama, ...	2.34 in.	12.81 in.	18.3 per cent.
2. "Master Magrath," .	2.77 „	8.57 „	32.3 „
3. Woman, ...	2.32 „	16.50 „	14.1 „
4. Leopard, ...	1.33 „	8.80 „	15.1 „
Adductor secundus (α).			
5. Leopard, ...	0.89 „	8.80 „	10.1 „

10. **Theory of the Hip and Shoulder Joints**—*Hip Joint.*—We are now in a condition to attempt the solution of a most interesting problem, which may be thus stated:—Being given all the muscles that surround the hip joint, let it be proposed to ascertain the position of the socket of the joint which will enable all the muscles to perform the maximum amount of work. The following classification of the hip joint muscles may be employed:—

Muscles of Abduction.

1. Agitator caudæ.
2. Glutæus primus, ... (Gl. maximus.)
3. Tensor vaginæ femoris.
4. Glutæus secundus (α), . . (Pyriformis.)
5. Glutæus secundus (β), . . (Gl. medius.)
6. Glutæus tertius, ... (Gl. minimus.)
7. Glutæus quartus.
8. Glutæus quintus.
9. Obturator internus.

Muscles of Flexion.

1. Rectus femoris.
2. Sartorius.
3. Iliacus.
4. Psoas.

Muscles of Extension.

1. Biceps femoris.
2. Semitendinosus.
3. Semimembranosus.
4. Gracilis.
5. Quadratus femoris.

Muscles of Adduction.

1. Adductor primus, . . . } (Ad. magnus.)
2. Adductor secundus (α), . } (Ad. magnus.)
3. Adductor secundus (β), . } (Ad. magnus.)
4. Adductor tertius, . . . (Ad. brevis.)
5. Adductor quartus, . . (Ad. longus.)
6. Adductor quintus, . . (Pectinæus.)
7. Obturator externus.

These muscles are not all present together in any animal, but the greater number of them are to be found in all animals, and their constancy of occurrence and similarity of action have justly attracted the attention of anatomists, and several theories have been proposed to explain their similarity, among which the most remarkable is the theory which accounts for this similarity by the hypothesis of the descent of the several animals from a supposed common ancestor.

I shall choose the Lion as the animal, in the case of which I shall attempt to find the position of the socket of the hip

joint which shall produce the maximum effect of all the muscles.

Muscles used in Abduction.—The whole group of muscles (1 to 8) whose action is abduction of the femur, either wholly or in part, may be regarded as examples of muscles of the same class as the great pectoral in the bird's wing; that is, in the position of maximum extension of the fibres, the origin and insertion may be regarded as lying in the same plane, which plane contains also the centre of the socket of the hip joint. Hence the position of this centre, for maximum work, must lie somewhere on the axis of maximum instability.

1. *Agitator caudæ.*—This muscle takes its origin, for a length of 3.14 in. from the 1st, 2nd, and part of the 3rd caudal vertebræ, and is inserted by means of a long tendon into the outer side of the patella. Its form is shown in Fig. 91, in which AB is the origin and $A'B'$ the insertion. S is the actual socket of the hip joint, and ST the axis of rotation. The muscle is so nearly triangular in shape that we may assume as a close approximation (pp. 271, 289), that the axis LM, which bisects the intercept of OX between the bones, is the axis of maximum work. On measuring the space between LM and ST, I found—

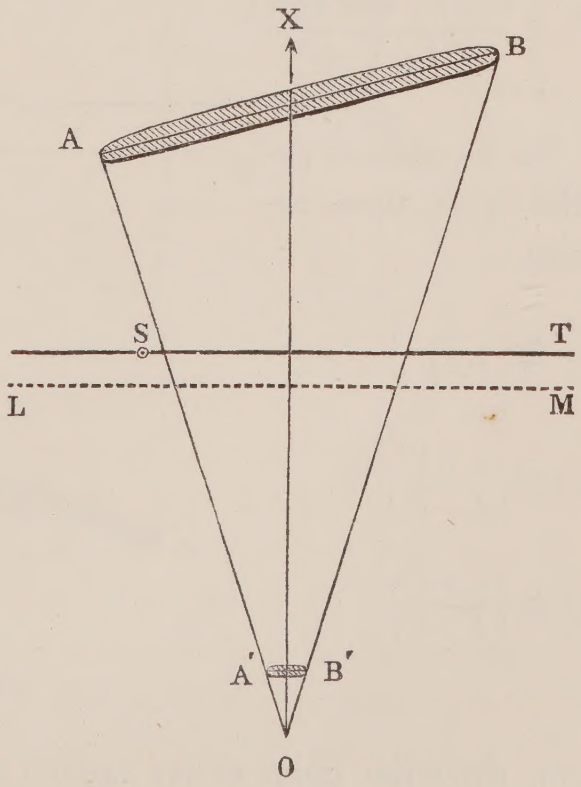


Fig. 91.

Distance between calculated and observed axes = 0.23 in.

2. *Glutæus primus.*—This muscle takes its origin from the sacral vertebræ, and is inserted into the outer edge of the femur, below the great trochanter, for a length of 2.90 inches. Its form is shown in Fig. 92, and its fibres are parallel to each other.

The coefficients L, M, N, p. 292, are—

$$L = \int_{-\theta}^{+\theta} \frac{d\theta}{l - l'}$$

$$M = \int_{-\theta}^{+\theta} \frac{(l + l') \cos \theta d\theta}{l - l'}$$

$$N = \int_{-\theta}^{+\theta} \frac{(l + l') \cos^2 \theta d\theta}{l - l'}$$

In the case of parallel fibres, these become,

$$L = \Sigma\left(\frac{1}{l - l'}\right)$$

$$M = \Sigma\left(\frac{l + l'}{l - l'}\right)$$

$$N = \Sigma\left(\frac{l + l'}{l - l'}\right)$$

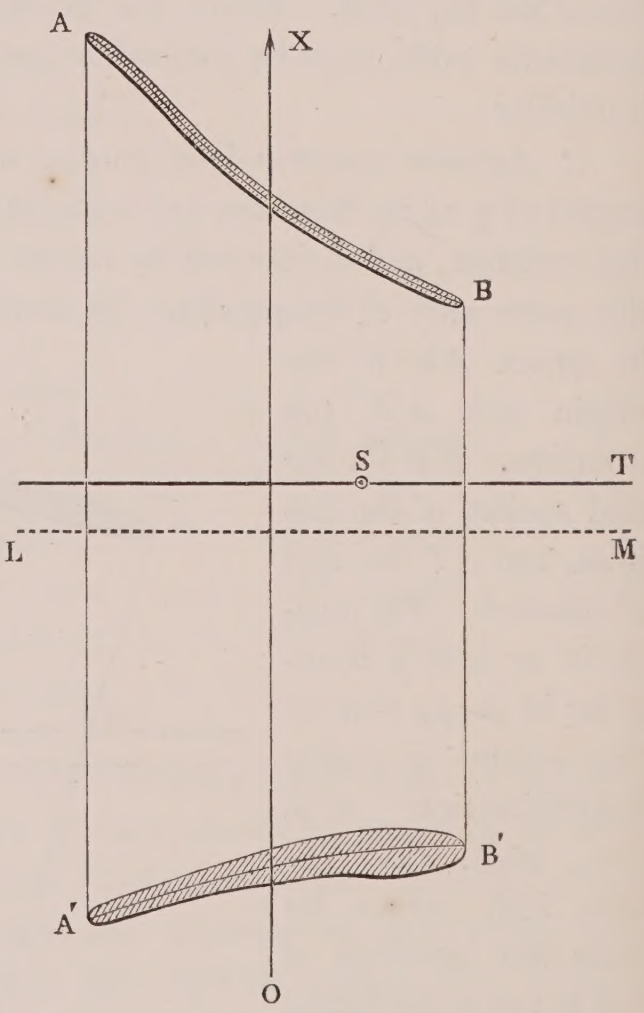


Fig. 92.

And, since the point O has moved off to infinity, $l + l'$ may be regarded as equal to twice the distance of the point of

bisection of the middle fibre from the point O: let this distance be represented by K, and we have,

$$L = \Sigma\left(\frac{1}{l - l'}\right),$$

$$M = 2K\,\Sigma\left(\frac{1}{l - l'}\right),$$

$$N_1 = 2K\,\Sigma\left(\frac{1}{l - l'}\right),$$

and the centre of the conic is found from the equation

$$2Lx + M = 0,$$

which becomes

$$2\,(x + K) = 0,$$

and shows that the axis of maximum work is the axis which bisects the middle fibre.

In Fig. 92, AB is the origin, and $A'B'$ the insertion of the muscle, and LM represents the calculated axis, and ST, passing through the socket S, the actual axis of rotation.

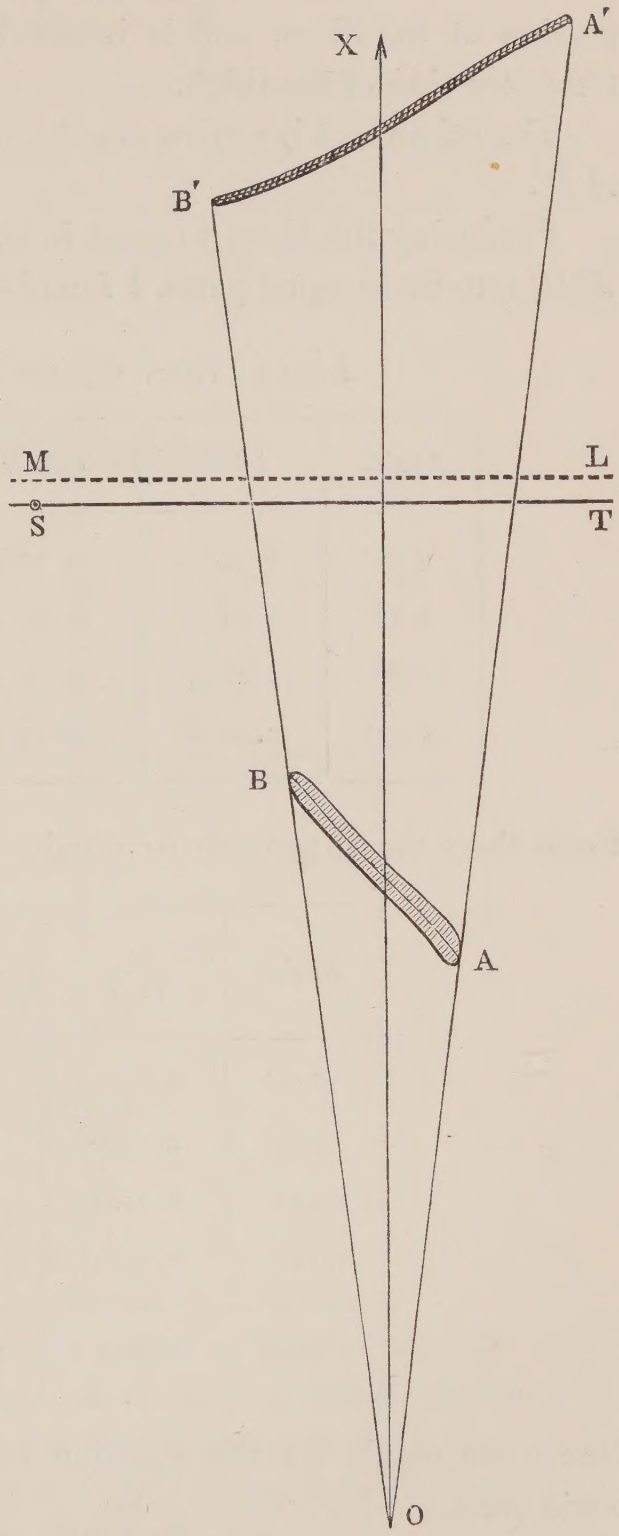


Fig. 93.

Distance between calculated and observed axes = 0.40 in.

3. *Tensor vaginæ femoris.*—This muscle takes origin from a length of 2.76 in. along the lower margin of the anterior portion of the ilium, and is inserted by a broad fascia into the outer side of the thigh.

The origin and insertion are shown in Fig. 93, at AB and $A'B'$.

Producing the fibres to meet in O, and dividing the angle AOB into three equal parts, I found—

Lion (*Tensor vaginæ femoris*).

Angle.	$l - l'$	$l \cos \theta$	$l' \cos \theta$
0° 00′	6.34 in.	14.10 in.	7.85 in.
5 15	7.75 „	14.80 „	7.06 „
10 30	9.20 „	15.60 „	6.70 „
15 45	10.40 „	16.25 „	5.91 „

From these measurements we obtain—

Angle.	$\frac{1}{l - l'}$	$\left(\frac{l + l'}{l - l'}\right) \cos \theta$
0° 00′	0.1577	3.4621
5.15	0.1290	2.8207
10.30	0.1087	2.4239
10.45	0.0961	2.1308
Sum,	**0·4915**	**10·8375**

Hence we obtain, by the equation to find the centre of the conic (66),

$$2Lx + M = 0$$

$$x = -\tfrac{1}{2}\frac{10.8375}{0.4915} = 11.02 \text{ in.}$$

In Fig. 93, the axis LM is drawn at this distance from the vertex O, and the axis ST is drawn parallel to it, passing through the socket S.

Distance between calculated and observed axes = 0.25 in.

4. *Glutæus secundus* (*a*).—This muscle (pyriformis) takes its origin from the under surface of the sacral vertebræ, for a length of 2.60 in., and is inserted into the inner surface of the top of the great trochanter. Its form is shown in Fig. 94,

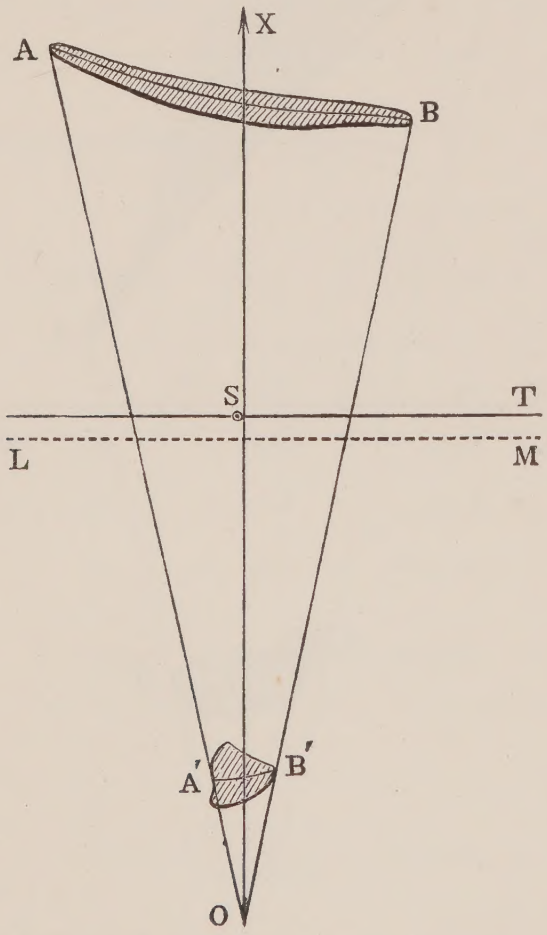


Fig. 94.

where AB and $A'B'$ are its origin and insertion. It is so nearly triangular in shape, that we may safely assume LM, which bisects the intercept of OX between the bones, to be the axis of maximum effect. ST, as usual, is the axis passing through the observed socket S.

Distance between calculated and observed axes = 0.16 in.

5. *Glutæus secundus* (β).—This muscle takes its origin from the upper two-thirds of the entire surface of the outer side of the ilium, and is inserted into the whole of the top and outer surface of the great trochanter. It is shown in

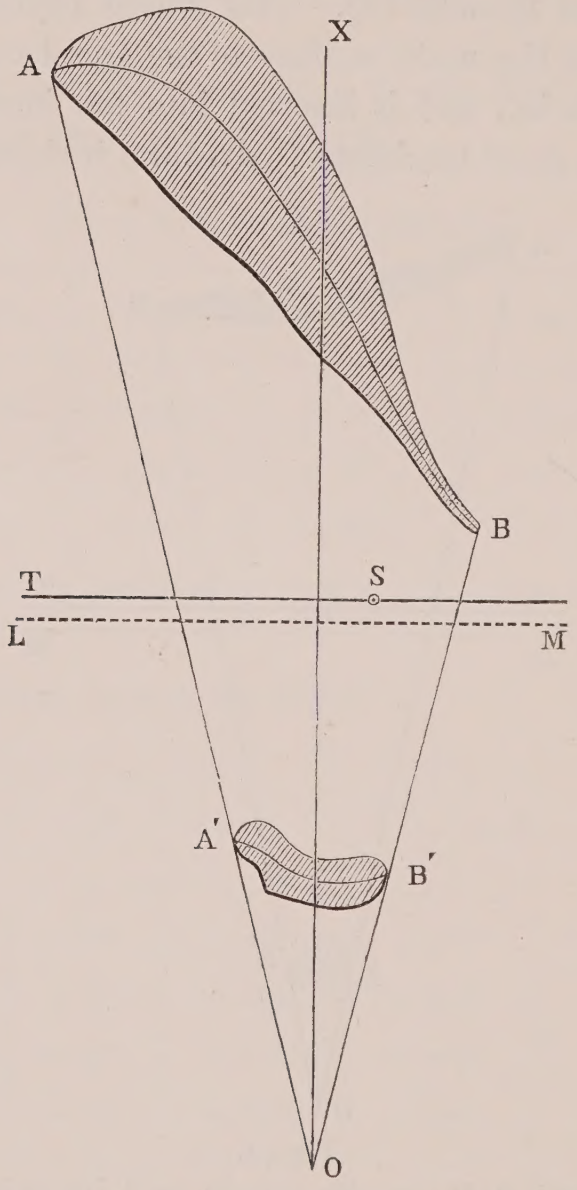


Fig. 95.

Fig. 95, where AB is the curved middle line of its origin, and $A'B'$ the curved middle line of its insertion. Dividing the angle AOB into three parts, we obtain—

Lion—Glutæus secundus (β)

Angle.	$l - l'$	$l \cos \theta$	$l' \cos \theta$
0°	3.91 in.	6.78 in.	3.09 in.
10	5.54 „	8 26 „	2.74 „
20	8.14 „	10.95 „	2.72 „
30	8.58 „	11.72 „	2.41 „

From these measurements we obtain—

Angle.	$\frac{1}{l - l'}$	$\left(\frac{l + l'}{l - l'}\right) \cos \theta$
0°	0.2558	2.5243
10	0.1805	1.9856
20	0.1228	1.6793
30	0.1015	1.5359
Sum,	**0·6606**	**7·7251**

and lastly,

$$x = -\tfrac{1}{2}\,\frac{7{\cdot}7251}{0.6606} = 5.85 \text{ in.}$$

The axis LM is drawn at this distance from the vertex O, and ST is drawn through the observed socket.

Distance between calculated and observed axes = 0.21 in.

6. *Glutæus tertius.*—This muscle takes its origin from the whole of the lower third of the outer surface of the ilium, and

is inserted into the top and outer side of the great trochanter, in front of the *Glutæus secundus* (β). Its form is shown in

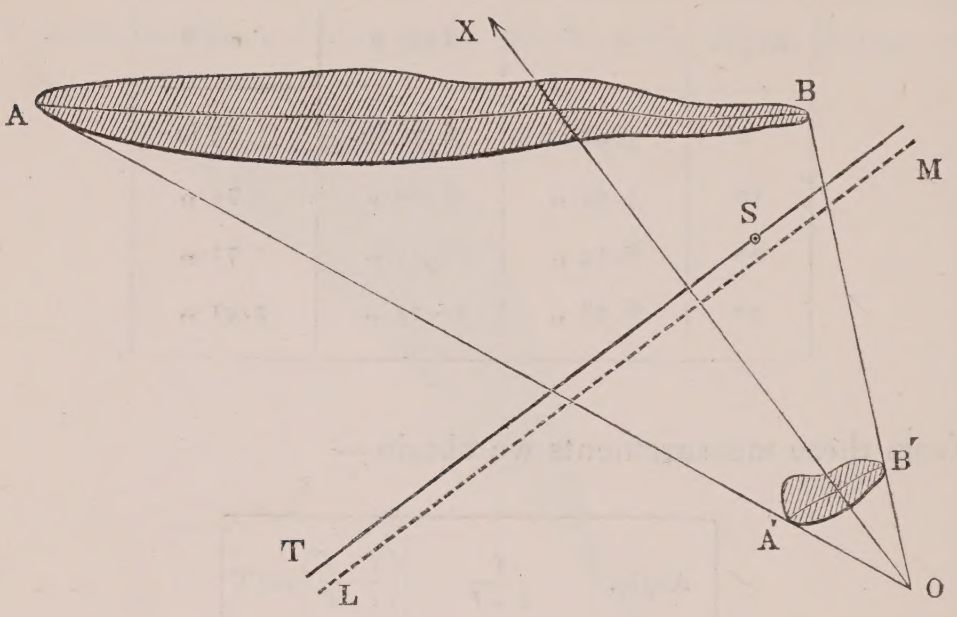


Fig. 96.

Fig. 96, where AB is the origin, and $A'B'$ the insertion of the muscle. By dividing the angle AOB into five equal parts, I found the following measurements :—

Lion—Glutæus tertius.

Angle.	$l - l'$	$l \cos \theta$	$l' \cos \theta$
0°	3.30 in.	4.02 in.	1.02 in.
10	3.50 ,,	4.47 ,,	1.08 ,,
20	4.04 ,,	5.15 ,,	1.12 ,,
30	4.68 ,,	5.81 ,,	1.15 ,,
40	5.77 ,,	6.74 ,,	1.20 ,,
50	7.75 ,,	8.30 ,,	1.20 ,,

From these measurements we obtain—

Angle.	$\frac{1}{l-l'}$	$\left(\frac{l+l'}{l-l'}\right)\cos\theta$
0°	0.3030	1.5273
10	0.2857	1.5857
20	0.2475	1.5520
30	0.2137	1.4872
40	0.1733	1.3761
50	0.1261	1.2258
Sum,	**1·3493**	**8·7541**

Hence we find

$$x = -\tfrac{1}{2}\frac{8.7541}{1.3493} = 3.24 \text{ in.}$$

The axis *LM*, in Fig. 96, is drawn at this distance from the vertex *O*, and the axis *ST*, is drawn parallel to it through the socket *S*.

Distance between calculated and observed axes = 0.18 in.

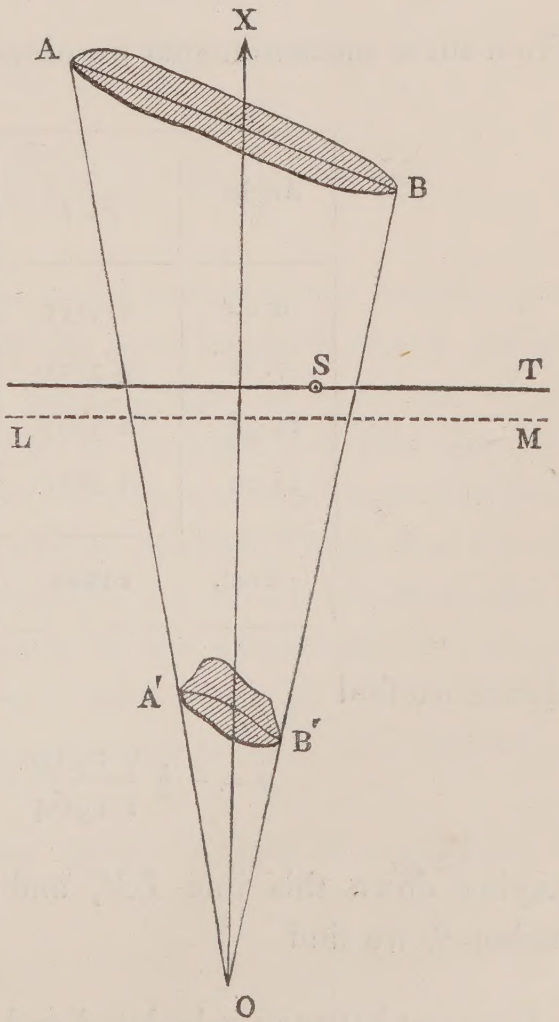


Fig. 97.

7. *Glutæus quartus.*—This muscle takes origin from a horizontal line on the ilium, 2 in. long, below the *Glutæus tertius*, and in front of the acetabulum, and is inserted on the inner front of the great trochanter, inside the insertion of *Glutæus*

tertius. Its origin AB, and insertion $A'B'$, are shown in Fig. 97. On dividing the angle AOB into three equal parts, I found the following results :—

Lion—Glutæus tertius.

Angle.	$l - l'$	$l \cos \theta$	$l' \cos \theta$
0°.00′	3.17 in.	4.47 in.	1.37 in.
7.20	3.20 ,,	4.70 ,,	1.50 ,,
14.40	3.33 ,,	4.95 ,,	1.62 ,,
22.00	3.73 ,,	5.21 ,,	1.65 ,,

From these measurements we obtain—

Angle.	$\frac{1}{l - l'}$	$\frac{l + l'}{l - l'} \cos \theta$
0°00′	0.3155	1.8423
7.20	0.3125	1.9375
14.40	0.3003	1.9730
22.00	0.2681	1.8391
Sum,	**1.1964**	**7.5919**

Hence we find

$$x = -\tfrac{1}{2} \frac{7.5919}{1.1964} = 3.17 \text{ in.}$$

Laying down this line LM, and drawing ST through the socket S, we find

Distance between calculated and observed axes = 0.19 in.

8. *Glutæus quintus.*—This muscle is not found in the Lion.

9. *Obturator internus.*—The tendon of this muscle, as in all animals, passes over a pulley formed by the posterior edge of the ischium, and is inserted into the upper portion of the pit of the great trochanter. If we measure the distances from the centre of the socket to the insertion of the muscle and to the centre of the tendon where it passes over the pulley, we find—

Distance from insertion to centre of socket = 2.40 in.
Distance from pulley to centre of socket = 2.30 ,,

From these measurements we readily find

$$\theta = 46^\circ.13$$

$$\sin 2\theta = \frac{\text{Work done}}{\text{Maximum Work}} = 99.91 \text{ per cent.}$$

We have now seen that in the case of all the muscles used in abduction, the centre of the acetabulum is nearly pierced by the axis of maximum work, proper to each one of the seven muscles; and it is to be remembered that the head of the femur does not fit the acetabulum exactly, so that there is a play permitted to the axis of rotation, which does not necessarily pass accurately through the centre of the socket in every motion. The diameter of the acetabulum is 1.42 in., and hence the rotation may take place, in some instances, round a tangent to the acetabulum sphere, at a distance of 0.71 in. from the centre.

Collecting together the seven muscles of abduction, we find:—

Abduction Muscles of Lion's Hip—Difference between Theory and Observation.

Muscle.	Weight of Muscle.	Distance between calculated and observed axes.
1. Agitator caudæ, ...	6.50 oz. av.	0.23 in.
2. Glutæus primus, ...	3.50 ,,	0.40 ,,
3. Tensor vaginæ femoris, . .	6.50 ,,	0.25 ,,
4. Glutæus secundus (α), . .	2.50 ,,	0.16 ,,
5. Glutæus secundus (β), . .	14.50 ,,	0.21 ,,
6. Glutæus tertius, ...	2.50 ,,	0.18 ,,
7. Glutæus quartus, . . .	0.53 ,,	0.19 ,,
8. Obturator internus, . . .	4.00 ,,	0.10 ,,
	40.53 oz. av.	**0.215** in.

The mean difference between calculation and observation amounts to only 0.215 of an inch, being less than one-third of the total range (0.71 in.) permitted to the axis of rotation by the arrangements of the joint.

The foregoing mean error is calculated as the arithmetical mean of the error in each muscle, or, in other words, all the muscles are treated as of equal importance. The work actually done by each muscle is, however, proportional to its weight, and we may calculate the mean error, taking into account the relative importance of each muscle, as measured by its work actually done. In order to find this mean, we multiply the weight of each muscle by its error, and divide the sum of all the products by the total weight of all the muscles. We thus find—

Abduction Muscles of Lion's Hip—Difference between Theory and Observation, taking into account the relative importance of each muscle.

Muscle.	Weight.	Weight × Error.
1. Agitator caudæ, ...	6.50 oz. av.	1.3950
2. Glutæus primus, ...	3.50 ,,	1.4000
3. Tensor vaginæ femoris, . .	6.50 ,,	1.6250
4. Glutæus secundus (α), . .	2.50 ,,	0.4000
5. Glutæus secundus (β), . .	14.50 ,,	3.0450
6. Glutæus tertius.	2.50 ,,	0.4500
7. Glutæus quartus, . . .	0.53 ,,	0.1007
8. Obturator internus, . . .	4.00 ,,	0.4000
Total, . .	**40.53** oz. av.	**8.8157**

Dividing this last sum by the total weight of the muscles, we find the mean error to be 0.217 of an inch, which agrees with the former result. This agreement shows that, in the plan of the Hip joint, the perfect action of each muscle, small and great, is equally provided for; just as in a well-ordered republic, the happiness of the humblest citizen is of as much importance to the State as the happiness of the citizen of highest rank.

Before leaving the muscles used in abduction, it may be useful to record the actual directions of the axes round which they cause the femur to rotate.

We may take the longest axis of the pelvis joining the tuber ischii with the anterior inferior spine of the ilium, as our reference line, measuring the angles made with this line towards the ischium, positive when above this line, and negative when below it.

On laying down the several axes, I found the following :—

Abduction Muscles of Lion's Hip—Position of Rotation axes.

Muscle.	Angle made with long axis of pelvis.
1. Agitator caudæ, ...	− 15° 45′
2. Obturator internus, . . .	0 00
3. Glutæus secundus (α), . .	+ 12 00
4. Glutæus primus, ...	+ 27 30
5. Glutæus quartus, . . .	+ 39 20
6. Glutæus tertius, ...	+ 45 05
7. Tensor vaginæ femoris, . .	+ 48 15
8. Glutæus secundus (β), . .	+ 54 00

Muscles used in Adduction.—In discussing the muscles used in abduction, we have gone half way round the hip joint; in the muscles used in adduction, we shall complete the remaining half of the circle, in the reverse order, as shown in the Table of muscles (p. 362).

1. *Adductor primus.*—This muscle takes its origin from a surface 2.61 inches in length, extending nearly from the *tuber ischii* to the posterior end of the *symphysis pubis*; and it is inserted for a length of 4.40 inches into the far end of the *linea aspera*. It forms a skew muscle, although not so markedly as the other adductors; and we shall investigate the conditions of its maximum work, by the method laid down for skew muscles.

Lion—Adductor primus.

Lengths of extreme Fibres.	Lengths of Origin and Insertion.
p = 8.00 in.	a = 2.61 in.
p_1 = 11.60 ,,	a' = 4.40 ,,

Constants of Hyperboloid.

$2c$ = 4.30 in.

$2\phi = 90°$.

It will be unnecessary to investigate the biquadratic (92), for the constant $2\phi = 90^\circ$ gives us $m = 1$, and therefore $\lambda = 0$, is a root of the equation, as may be seen on inspection.

To determine the points I and I', we have

$$p^2 = a^2 + a'^2 + 4c^2,$$
$$p_1^2 = (a + a)^2 + (a' + a')^2 + 4c^2,$$

from which we obtain

$$p_1^2 - p^2 = 2aa + 2a'a' + a^2 + a'^2;$$

or, introducing the numerical values,

$$5.22\ a + 8.80\ a' - 44.39 = 0,$$
$$a' = -\ 0.6\ a + \ 5.04.$$

Introducing this expression into the first equation, we find

$$1.36\ a^2 - 6.05\ a - 9.12 = 0,$$

and finally,

$$a = \begin{cases} + 5.64 \\ - 1.15 \end{cases} \qquad a' = \begin{cases} + 1.66 \\ + 5.73 \end{cases}$$

An inspection of the bones shows that the last pair of values are those to be employed, and thus we find

$a = -\ 1.15$ in.	$a' = +\ 5.73$ in.
$a_1 = +\ 1.46$,,	$a_1' = +\ 10.13$,,
$\frac{a}{p} = -\ 0.144$	$\frac{a'}{p} = +\ 0.716$
$\frac{a_1}{p_1} = +\ 0.126$	$\frac{a_1'}{p_1} = +\ 0.808$
$\Sigma\left(\frac{a}{p}\right) = -\ 0.018$	$\Sigma\left(\frac{a'}{p}\right) = +\ 1.524$

From these results we can calculate the equivalent fibre OO', as follows:—

$$\frac{L}{P} = -0.009, \qquad \frac{L'}{P} = +0.762,$$

$$P^2 = L^2 + L'^2 + 4c^2,$$

$$P^2 = \frac{18.49}{0.419}, \qquad P = 6.64 \text{ in.},$$

$$L = -0.06 \text{ in.} \qquad L' = +5.06 \text{ in.}$$

Laying down the points O and O' on the bones, I found—

Distance from O to centre of socket = 5.16 in.
Distance from O' to centre of socket = 6.40 ,,

Difference, . . . 1.24 in.

These measurements correspond to an angle of 51° 7′ made by the fibre OO' with the plane passing through the bone AB, or by equation (108).

$$\frac{\text{Work done}}{\text{Maximum work}} = \sin(2\theta) = 97.7 \text{ per cent.}$$

2. *Adductor secundus* (α).—This muscle takes its origin along a surface 2.68 inches long, from the *tuber ischii* to the posterior end of the *symphysis pubis*, and is inserted in continuation of the insertion of *adductor primus*, higher up on the linea aspera, for a length of 3.45 inches. It is more skew than *adductor primus*, but less so than *adductor secundus* (β). The following measurements were made upon this muscle:—

Lion—Adductor secundus (α).

Lengths of equidistant Fibres.	Lengths of Origin and Insertion.
$p = 5.34$ in.	$a = 2.68$ in.
$p_1 = 6.40$,,	$a' = 3.45$,,
$p_2 = 7.60$,,	

Constants of Hyperboloid.

$2c = 4.06$ in.

$2\phi = 91^\circ\ 15'$.

The angle between the bones is so nearly 90° that we may assume, as before, that $\lambda = 0$ is a root of equation (92).

To determine I and I', we have

$$p_2^2 - p^2 = 2aa + 2a'a' + (a^2 + a'^2),$$

which gives us

$$5.36\ a + 6.90\ a' - 10.17 = 0,$$
$$a' = -\ 0.78\ a + 1.47.$$

Substituting this value of a' in the expression for p^2, we obtain

$$1.61\ a^2 - 2.29\ a - 9.87 = 0,$$

and finally,

$$a = \begin{cases} +\ 5.30 \text{ in.} \\ -\ 1.87\ ,, \end{cases} \qquad a' = \begin{cases} -\ 1.10 \text{ in.} \\ +\ 2.93\ ,, \end{cases}$$

On inspection of the bones it is evident that the latter pair of roots are those required in the problem. Hence we find—

$a = -\ 1.87$ in.	$a' = +\ 2.93$ in.
$a_1 = -\ 0.53$,,	$a_1' = +\ 4.66$,,
$a_2 = +\ 0.81$,,	$a_2' = +\ 6.39$,,
$\frac{a}{p} = -\ 0.350$	$\frac{a'}{p} = +\ 0.549$
$\frac{a_1}{p_1} = -\ 0.083$	$\frac{a_1'}{p_1} = +\ 0.728$
$\frac{a_2}{p_2} = +\ 0.107$	$\frac{a_2'}{p_2} = +\ 0.841$
$\Sigma\left(\frac{a}{p}\right) = -\ 0.326$	$\Sigma\left(\frac{a'}{p}\right) = +\ 2.118$

In order to find O and O', we have

$$\frac{L}{P} = -0.109, \qquad \frac{L'}{P} = +0.706,$$

$$P^2 = L^2 + L'^2 + 4c^2,$$

$$P^2 = \frac{16.4836}{0.4897}, \qquad P = 5.802 \text{ in.}$$

$$L = -0.632 \text{ in.}, \qquad L' = +4.096 \text{ in.}$$

On laying down the points O and O', thus found, on the bones, I obtained—

Distance of O from centre of socket = 4.57 in.
Distance of O' from centre of socket = 5.31 ,,

Difference, . . . 0.74 in.

$$\theta = 49^\circ\ 17'.$$

$$\frac{\text{Work done}}{\text{Maximum work}} = \sin 2\theta = 98.88 \text{ per cent.}$$

3. *Adductor secundus* (β).—This muscle takes its origin for a length of 3.72 inches along the *symphysis pubis*, and is inserted, side by side, with *Adductor secundus* (α), into 3.45 inches of the upper portion of the *linea aspera*.

Lion—Adductor secundus (β).

Lengths of Equidistant Fibres.	Lengths of Origin and Insertion.
p = 5.86 in.	a = 3.72 in.
p_1 = 8.03 ,,	a' = 3.45 ,,
p_2 = 10.16 ,,	

Constants of Hyperboloid.

$2c$ = 3.20 in.
$2\phi = 49^\circ\ 00'$.

In order to calculate the positions of the points I and I', we have (taking into account the precautions as to sign already mentioned), *vide* equation (93),

$$p_2^2 - p^2 = -2(a + a' \cos 2\phi)\, a + 2(a' + a \cos \phi)\, a', \\ + (a^2 + a'^2 + 2aa' \cos 2\phi).$$

From this we find

$$11.78\, a' = 11.96\, a + 26.30,$$
$$a' = a + 2.23.$$

Substituting this value in the equation

$$p^2 = a^2 + a'^2 - 2aa' \cos 2\phi + 4c^2,$$

we find

$$0.69\, a^2 + 1.53\, a - 19.13 = 0,$$

and finally,

$$a = \begin{cases} +4.27 \\ -6.49 \end{cases} \qquad a' = \begin{cases} +6.50 \\ -4.26 \end{cases}$$

The first pair of values are those required by our present problem. Using them, we obtain—

Lion—Adductor secundus (β).

$a = +4.27$	$a' = +6.50$
$a_1 = +2.41$	$a_1' = +8.23$
$a_2 = +0.55$	$a_2' = +9.96$
$\frac{a}{p} = +0.728$	$\frac{a'}{p} = +1.109$
$\frac{a_1}{p_1} = +0.300$	$\frac{p'_1}{p_1} = +1.025$
$\frac{a_2}{p_2} = +0.054$	$\frac{a_2'}{p_2} = +0.980$
$\Sigma\left(\frac{a}{p}\right) = +1.082$	$\Sigma\left(\frac{a'}{p}\right) = +3.114$

$$\frac{1}{p} = 0.171 \qquad \frac{aa'}{p} = +4.736$$

$$\frac{1}{p_1} = 0.124 \qquad \frac{a_1a_1'}{p_1} = +2.470$$

$$\frac{1}{p_2} = 0.098 \qquad \frac{a_2a_2'}{p_2} = +0.539$$

$$\Sigma\left(\frac{1}{p}\right) = 0.393 \qquad \Sigma\left(\frac{aa'}{p}\right) = +7.745$$

Hence we obtain, using equations (96),

$$K = +0.535,$$
$$mX = +0.449,$$
$$mX' = +1.292.$$

If we now employ these values to calculate the coefficients of (92), we find for the biquadratic which determines the position of the generator of maximum work,

$$-0.647\,\lambda^4 + 5.456\,\lambda^3 + 4.495\,\lambda + 0.405 = 0.$$

One of the real roots of this equation is

$$\lambda = -0.09.$$

Hence the construction (Fig. 90) applies, and we can find the positions of O and O', and the length of the equivalent fibre, as follows:—

$$\frac{L}{P} = +0.361, \qquad \frac{L'}{P} = +1.038.$$

Using these values in the equation,

$$P^2 = L^2 + L'^2 - 2LL' \cos 2\phi + 4c^2,$$

we find

$$P^2 = \frac{10.24}{0.282} \qquad P = 6.026 \text{ in.}$$

$$L = +2.175 \text{ in.} \qquad L' = +6.255 \text{ in.}$$

Laying down these points, as before, on the bones, I found—

Distance of O from centre of socket = 3.63 in.
Distance of O' from centre of socket = 4.46 „

Difference, . . . 0.83 in.

$$\theta = 50^\circ 51.$$

$$\frac{\text{Work done}}{\text{Maximum work}} = \sin 2\theta = 97.92 \text{ per cent.}$$

4. *Adductor tertius* (*brevis*).—This muscle is wanting in the Lion.

5. *Adductor quartus* (*longus*). } These muscles take
6. *Adductor quintus* (*pectinæus*). } origin, side by side, from the anterior margin of the pubes, for a length of

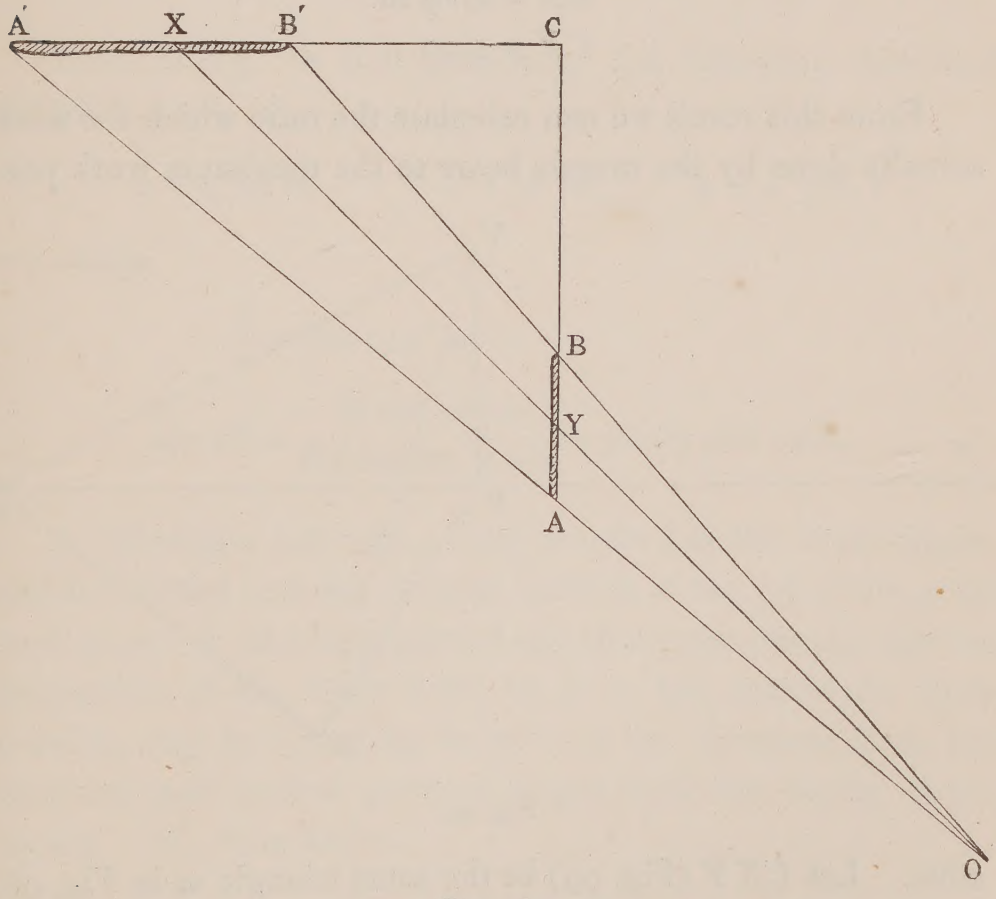


Fig. 98.

1.70 in., and are inserted in like manner into a line 2.95 in. in length, lying beside the insertion of *Adductor secundus*,

and to its inner side. When the thigh is abducted, these muscles form plane quadrilaterals, and I have already shown (p. 328) that they are nearly inscribable in circles, which is the geometrical condition necessary to produce the maximum work.

In Fig. 98 I have drawn these muscles to scale, from the measurements given in p. 328. The pelvic origin is AB, and the femoral insertion is $A'B'$. If the quadrilateral $ABB'A'$ be inscribable in a circle, then if $A'A$ and $B'B$ be produced to meet in O, and OYX be drawn bisecting the angle AOB, the triangle CXY should be isosceles, and the side CX equal to the side CY. I found the following measurements, on the scale of nature:—

$$CX = 4.29 \text{ in}$$
$$CY = 4.30 \text{ ,,}$$

From this result we can calculate the ratio which the work actually done by the muscle bears to the maximum work pos-

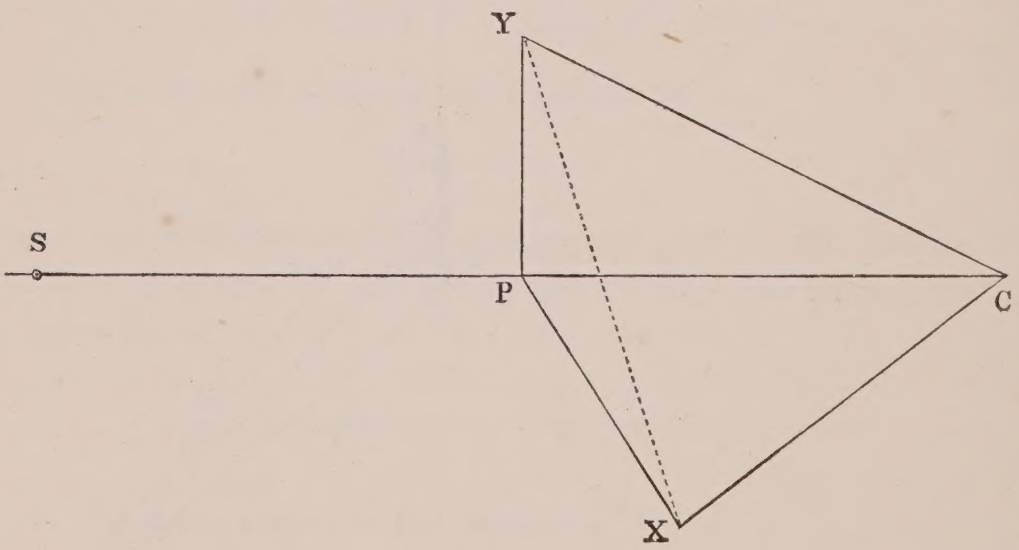


Fig. 99.

sible. Let CXY (Fig. 99) be the same triangle as in Fig. 98, and let SC be the axis of rotation passing through the socket S. Draw through XY the plane YPX perpendicular

(or nearly so) to the axis SC, and let us use the following notation :—

$$CY = b, \qquad CX = a,$$
$$XCY = \phi, \qquad YP = y, \qquad XP = x.$$

It is easy to see that we have the following relations :—

$$y^2 - x^2 = b^2 - a^2,$$
$$y^2 + x^2 = b^2 + a^2 - 2ab \cos \phi,$$

from which we obtain

$$\frac{y}{x} = \sqrt{\frac{b\,(b - a \cos \phi)}{a\,(a - b \cos \phi)}}$$

$$\frac{y}{x} = \tan \theta, \qquad \frac{\text{Work done}}{\text{Maximum work}} = \sin 2\theta.$$

Substituting, in the foregoing, the following observed values,

$$a = 4.29 \text{ in.}, \qquad b = 4.30 \text{ in.}, \qquad \phi = 9^\circ\, 30',$$

we obtain

$$\frac{y}{x} = \tan (49^\circ\, 45'),$$

$$\sin 2\theta = \frac{\text{Work done}}{\text{Maximum work}} = 98.63 \text{ per cent.}$$

7. *Obturator externus.*—This muscle has the usual origin, and is inserted into the deepest portion of the pit of the great trochanter; it may be regarded as a triangular muscle, and the proportion of the work done by it to the maximum work possible may be found by measuring the distances from the insertion and central point of origin from the centre of the socket. We thus find—

Distance from insertion to centre of socket = 2.33 in.
From centre of *Obturator foramen* to centre of socket = 2.27 ,,

Hence we find

$$\theta = 45^\circ\ 45',$$

$$\sin 2\theta = \frac{\text{Work done}}{\text{Maximum work}} = 99.96 \text{ per cent.}$$

Collecting the preceding results into one Table, we find—

Adduction Muscles of Lion's Hip—Difference between Theory and Observation.

Muscle.	Work done.	Difference between work done and maximum work.
1. Adductor primus, . . .	97.70 per cent.	2.30 per cent.
2. Adductor secundus (α), . .	98.88 ,,	1.12 ,,
3. Adductor secundus (β), . .	97.92 ,,	2.08 ,,
4. Adductor tertius, . . .		
5. Adductor quartus, . . .	98.63 ,,	1.37 ,,
6. Adductor quintus, . . .	98.63 ,,	1.37 ,,
7. Obturator externus, . . .	99.96 ,,	0.04 ,,
Mean, . . .	**98·62** per cent.	**1·38** per cent.

Nothing can be more complete than the proof afforded by the foregoing Table, that the adduction muscles, like the abduction muscles, are all placed in reference to the socket of the hip joint, in such a manner as to produce the maximum amount of work.

It is unnecessary to discuss the case of the flexion and extension muscles, for, in p. 258, *et seq.*, I have fully proved that they are placed (not merely in relation to the hip joint, but also in relation to the knee and heel) so as to produce the maximum of work possible.

It would be tedious to make similar calculations for the muscles of other animals; and it is sufficient to state that I have completed many such calculations, for the hip, shoulder,

and other joints of many animals, and that I have never met with any exception to the following Propositions, which may be regarded as summing up my results :—

1°. *Each muscle is constructed in relation to its joint, in such a manner as to perform one kind of work, only ; and it performs that work to maximum advantage.*

2°. *The number of muscles employed is determined by the number of distinct actions required from the limb.*

3°. *The shape and form of the bones employed are the necessary consequence of the shape and power of the muscles in action.*

4°. *The smallest muscle in the combination is as carefully adapted to its conditions of maximum work as the largest muscle.*

From these Propositions, supposed to be extended to the action of every muscle and joint, it appears to me to follow as necessary consequences :—

1°. *That a foreseeing Mind planned the type of the limb, and of its actions.*

2°. *The idea of the limb and of its necessary actions being given ; the number, shape, and arrangement of the necessary muscles can be calculated and predicted with as much certainty as an astronomer can predict an eclipse.*

3°. *That the shape and arrangement of the bones follow of necessity, from the necessary arrangement of the muscles.*

4°. *That any alteration, however slight, in any part of the combination of bones, muscles, and joints, would entail a loss of work, and lead to a less perfect mechanism.*

5°. *Hence, the permanence and stability of each species (so far as relates to bones, muscles, and joints) is absolutely secured, on the principles so admirably laid down by Mr. Darwin.*

6. *The profound study of the mechanism of joints lends no support to the postulate, that the similarities found to exist in the*

bones, muscles, and joints of animals may be explained by common descent from a supposed common ancestor.

Shoulder Joint.—The shoulder girdle differs from the pelvic, in being moveable, so that the centre of the socket round which the arm revolves is not fixed in one definite position, but may be fixed for the moment by the action of its own muscles, or it may be itself in motion during the motion of the arm. Notwithstanding its greater complexity of action, the same principles may be applied to the shoulder joint, that were used in the discussion of the hip joint; and they lead us to the same general conclusions, as to the absolute perfection of the mechanism of the shoulder joint in all animals.

If we compare together the muscles of the shoulder and hip, setting aside those of the shoulder which are peculiar to it, from the mobility of the shoulder girdle, we find the following Table of comparative muscles, corresponding to each other in action :—

Shoulder Joint.	*Hip Joint.*
Muscles of Abduction.	Muscles of Abduction.
1. Trapezius inferior, ...	Agitator caudæ.
2. Trapezius superior, } . . .	Glutæus primus.
3. Deltoideus, }	
4. Teres major, ...	Tensor vaginæ femoris.
5. Supraspinatus, ...	Glutæus secundus.
6. Infraspinatus, ...	Glutæus tertius.
7. Teres minor, ...	Glutæus quartus.
8. Infraspinatus secundus, . .	Glutæus quintus.
9. Subclavius, ...	Obturator internus.
Muscles of Flexion.	Muscles of Flexion.
1. Triceps longus, ...	Rectus femoris.
2. Tricipiti accessorius, . . .	Sartorius.
3. Subscapularis, ...	Iliacus.
4. Latissimus dorsi, ...	Psoas magnus.

Muscles of Extension.	Muscles of Extension.
1. Biceps coracoradialis, . . .	Biceps femoris.
2. Biceps glenoideus, . . .	Semitendinosus.
3. Brachiæus, ...	Semimembranosus.
4. Coracobrachialis capsularis, .	Quadratus femoris.

Muscles of Adduction.	Muscles of Adduction.
1. Coracobrachialis, ...	Adductor primus.
2. Pectoralis major, ...	Adductor secundus. Adductor tertius. Adductor quartus. Adductor quintus.
3. Pectoralis minor, ...	Obturator externus.
4. Pectoralis quartus, ...	Gracilis.

The similarity in the muscles of the shoulder and hip is a necessary consequence of the *idea* which is common to both joints; viz., a single long bone revolving round a centre, and acted on by muscles terminating in that bone, and taking their origin from a girdle, either absolutely or relatively at rest. There are some twenty or twenty-five actions necessary in such a system, and, therefore, a muscle is "told off" to perform to maximum advantage each proper action. In the order of nature, the following method is observed in all the mechanisms of the body, of every animal:—

1°. A contriving Mind.
2°. The Idea of the mechanism and its actions.
3°. A perfect muscle to perform each action.
4°. Bones of form, magnitude, and position, which can be predicted from the condition of maximum work imposed on each muscle employed.

Here we obtain a solid and permanent foundation for the Science of Comparative Osteology, founded on Geometry and

Mechanics, combined with the Postulate of maximum work; and starting from this foundation, in the inverse order to tha employed in the case of the Lion's hip, we might proceed to re-clothe a given skeleton with its proper muscles, and calculate with accuracy the amount of the forces employed in every action of the body of the living animal.

11. **Muscular Types.**—I now propose to develop, by means of a few examples, the theory of Muscular Types, of various animals, based upon the preceding proofs of the individual importance of each muscle; and shall choose as my first example the well-marked Type of the *Felidæ*.

(*a.*) *Muscular Type of the Felidæ.*—In discussing the theory of muscular types, I shall confine my attention, chiefly, to the hip and shoulder muscles. Let the weight of each muscle be taken, as measuring the work done by it, and let the total weight of all the muscles acting on the joint be represented by 100, so that the weight of each muscle is given as a percentage; we may thus construct Tables for the several animals in which all the figures are comparable. In order to avoid unnecessary columns of figures, I do not reproduce the actual weight of each muscle, but give the total actual weight of all, so that any reader can calculate back from the Table the original weight of any muscle. The animals selected for the present purpose are—

1. Adult African Lion (magnificent specimen).
2. Adult African Lioness (not related by blood to Lion).
3. Bengal Tiger (magnificent specimen).
4. Indian Leopard (male).
5. African Leopard (male).

Muscular Type of Larger Felidæ—Hip Joint.

Muscle.	Lion.	Lioness.	Tiger.	Mean.
Abductors.	Per cent.	Per cent.	Per cent.	Per cent.
1. Agitator caudæ,	3·57	3·12	4·10	3·60
2. Glutæus primus,	1·93	2·07	1·60	1·87
3. Tensor vaginæ femoris,	3·57	4·39	3·76	3·91
4. Glutæus secundus (α),	7·97	7·21	7·49	7·56
5. Glutæus secundus (β),	1·37	1·19	0·83	1·13
6. Glutæus tertius,	1·37	1·24	0·98	1·19
7. Glutæus quartus,	0·29	0·16	0·30	0·25
8. Obturator internus,	2·20	2·24	1·41	1·95
Flexors.				
1. Rectus femoris,	6·05	5·56	5·62	5·74
2. Sartorius,	6·45	5·39	7·90	6·58
3. Psoadiliacus,	5·91	5·68	4·75	5·45
Extensors.				
1. Biceps femoris,	15·25	17·50	17·21	16·65
2. Bicipiti accessorius,	——	0·45	0·47	0·30
3. Semitendinosus,	5·50	5·27	4·94	5·24
4. Semimembranosus,	9·18	8·72	8·47	8·79
5. Gracilis,	4·94	4·78	4·85	4·86
6. Quadratus femoris,	0·86	0·71	0·79	0·78
Adductors.				
1. Adductor primus, 2. Adductor secundus,	20·24	21·02	7·88 / 14·07	21·07
3. Adductor quartus, 4. Adductor quintus,	1·92	1·72	1·42	1·69
5. Obturator externus,	1·43	1·58	1·16	1·39
	100	**100**	**100**	**100**
	Oz. av.	Oz. av.	Oz. av.	
Total weights,	**181·96**	**143·86**	**276·09**	

The total weights of the muscles in the Tiger and Lion show the great superiority of the Tiger, and this superiority is further confirmed by the weights of the muscles of the shoulder joint, which will be given presently. In fact, if we take the Tiger as our standard, and compare with him the

Lion and Lioness, it is easy, from the weights of muscles given in the Tables, to calculate the following:—

Comparative Strength of Tiger and Lion.

	Tiger.	Lion.	Lioness.
Hip joint, ...	100	65.90	52.11
Shoulder joint, . . .	100	69.93	51.55

From this Table it appears that the strength of the Lion is about two-thirds of that of the Tiger, and that the strength of the Lioness is about one-half that of the Tiger. From the greater development of the fore-quarter of the Lion, as compared with that of the Lioness, we find that the shoulder muscles of the Lion bear a somewhat greater proportion to those of the Tiger, than the hip muscles do; while in the Lioness the proportions of the muscles of the shoulder and hip continue to have the same relation to the corresponding muscles of the Tiger.

In the time of the Emperor Titus, Tigers and Lions were forced to fight in the Amphitheatre at Rome, and it is recorded by Martial that the Tiger was able to kill the Lion, a fact which has been confirmed by accidental contests between these animals occurring in modern menageries.

"Lambere securi dextram consueta magistri
Tigris ab Hyrcano gloria rara jugo
Sæva ferum rabidâ laceravit dente leonem:
Res nova, non ullis cognita temporibus.
Ausa est tale nihil, sylvis dum vixit in altis,
Postquam inter nos est, plus feritatis habet."*

* Martial. *De Spectaculis Ep. X.*

Muscular Type of Lesser Felidæ—Hip Joint.

Muscle.	Indian Leopard.	African Leopard.	Mean.
Abductors.	Per cent.	Per cent.	Per cent.
1. Agitator caudæ, ...	4.44	3.57	4.01
2. Glutæus primus, ...	1.82	1.36	1.59
3. Tensor vaginæ femoris, . .	3.93	3.38	3.66
4. Glutæus secundus (α), . .	5.89	6.62	6.25
5. Glutæus secundus (β), . .	0.99	0.80	0.89
6. Glutæus tertius, ...	1.31	1.09	1.20
7. Glutæus quartus, ...	0.20	0.31	0.25
8. Obturator internus, . . .	2.01	1.72	1.87
Flexors.			
1. Rectus femoris, ...	6.69	6.20	6.45
2. Sartorius, ...	6.82	6.44	6.63
3. Psoadiliacus, ...	5.64	5.84	5.74
Extensors.			
1. Biceps femoris, ...	16.57	17.56	17.07
2. Bicipiti accessorius, . . .	0.30	0.35	0.32
3. Semitendinosus, ...	5.62	6.04	5.83
4. Semimembranosus, . . .	9.19	10.32	9.76
5. Gracilis, ...	4.14	4.08	4.11
6. Quadratus femoris, . . .	0.63	0.80	0.71
Adductors.			
1. Adductor primus, . . .	7.67	7.61	7.64
2. Adductor secundus, . . .	13.83	13.38	13.61
3. Adductor quartus, } . . .	1.14	1.39	1.26
4. Adductor quintus, }			
5. Obturator externus, . . .	1.17	1.14	1.15
	100	**100**	**100**
	Oz. av.	Oz. av.	
Total weights, . .	**58·33**	**48·33**	

These results show a general close agreement with those obtained from the larger Felidæ; and, if we compare the muscles in groups, according to their general action, we obtain the following comparative Table :—

Muscular Type of Felidæ—Hip Joint.

	Abductors.	Flexors.	Extensors.	Adductors.
Lion and Tiger, .	21.46	17.77	36.62	24.15
Leopard, . . .	19.72	18.82	37.80	23.66

We shall now proceed to examine the muscular type of the shoulder joint in the Felidæ, using the same animals as before:—

Muscular Type of Larger Felidæ—Shoulder Joint.

Muscle.	Lion.	Lioness.	Tiger.	Mean.
Scapular.	Per cent.	Per cent.	Per cent.	Per cent.
1. Trapezius clavicularis, . .	3.85	3.74	4.53	4.04
2. Cleidomastoideus, ...	1.80	1.20	1.15	1.38
3. Trapezius inferior, ...	1.48	1.64	1.84	1.65
4. Trapezius superior, . . .	1.30	1.47	2.06	1.61
5. Omo-Atlanticus, ...	1.27	1.30	1.35	1.31
6. Serratus magnus, ...	8.81	8.77	8.95	8.84
7. Rhomboideus, ...	3.50	4.08	4.73	4.10
Abductors.				
1. Deltoideus, ...	4.85	4.72	5.25	4.94
2. Teres major, ...	4.03	3.83	3.55	3.80
3. Supraspinatus, ...	6.16	7.22	6.92	6.77
4. Infraspinatus, ...	5.28	5.32	5.19	5.26
5. Teres minor, ...	0.23	0.25	0.26	0.25
Flexors.				
1. Triceps longus, ...	12.51	12.15	9.10	11.25
2. Tricipiti accessorius, . . .	0.80	0.86	0.87	0.84
3. Subscapularis, ...	6.25	6.36	5.48	6.03
4. Latissimus dorsi, ...	10.83	8.23	11.79	10.29
Extensors.				
1. Biceps humeri, ...	3.39	3.86	3.68	3.64
2. Brachiæus, ...	1.71	1.49	1.68	1.63
Adductors.				
1. Coracobrachialis, ...	0.12	0.07	0.06	0.08
2. Pectoralis major, ...	17.81	19.06	21.56	22.29
3. Pectoralis quartus, ...	4.04	4.38		
	100	**100**	**100**	**100**
	Oz. av.	Oz. av.	Oz. av.	
Total weights, . .	**235·67**	**173·72**	**337·00**	

Muscular Type of Lesser Felidæ—Shoulder Joint.

Muscle.	Indian Leopard.	African Leopard.	Mean.
Scapular.	Per cent.	Per cent.	Per cent.
1. Trapezius clavicularis, . .	3.91	3.83	3.87
2. Cleidomastoideus, ...	1.13	1.95	1.54
3. Trapezius inferior, ...	1.74	1.34	1.54
4. Trapezius superior, . . .	1.46	1.46	1.46
5. Omo-Atlanticus, ...	1.09	1.03	1.06
6. Serratus magnus, ...	9.11	9.22	9.17
7. Rhomboideus, ...	5.43	4.22	4.82
Abductors.			
1. Deltoideus, ...	5.57	5.39	5.48
2. Teres major, ...	3.66	3.87	3.76
3. Supraspinatus, ...	7.05	7.34	7.20
4. Infraspinatus, ...	5.78	4.89	5.33
5. Teres minor, ...	0.35	0.27	0.31
Flexors.			
1. Triceps longus, ...	10.03	9.40	9.72
2. Tricipiti accessorius, . . .	0.92	0.87	0.89
3. Subscapularis, ...	6.61	6.10	6.35
4. Latissimus dorsi, ...	11.58	14.43	13.01
Extensors.			
1. Biceps humeri, ...	3.96	3.34	3.65
2. Brachiæus, ...	1.68	1.70	1.69
Adductors.			
1. Coracobrachialis, ...	0.11	0.03	0.07
2. Pectoralis major, } . . .	18.83	19.32	19.08
3. Pectoralis quartus, }			
	100	**100**	**100**
	Oz. av.	Oz. av.	
Total weights, . .	**72·50**	**58·93**	

If we compare the muscles of the shoulder joint in groups, according to their general action, we obtain the following results:—

Muscular Type of Felidæ—Shoulder Joint.

	Scapular.	Abductors.	Flexors.	Extensors.	Adductors.
Lion and Tiger, .	22.93	21.02	28.41	5.27	22.37
Leopard, . . .	23.46	22.08	29.97	5.34	19.15

(*b.*) *Muscular Type of the Canidæ.*—The muscular type of the *Canidæ* is quite distinct from that of the Felidæ, and is not so constant in the members of the entire family, which differ from each other more considerably than the members of the Felidæ do.

The Canidæ differ from the Felidæ in the hip and shoulder joints in the following particulars :—

First, negatively—

The Canidæ do not possess the following muscles :—

1°. *Agitator caudæ*,
2°. *Glutæus quartus*, } Hip.
3°. *Pectoralis quartus*—Shoulder.

Secondly, positively—

The Canidæ possess always a double *Sartorius*, and occasionally a double *Tensor vaginæ femoris.*

I shall take the Greyhound as the type of the family, and give the percentages of this animal in detail, adding a summary of the percentages of groups of muscles taken from other members of the family.

In the following Table, I give "Master Magrath's" muscles in one column, and place beside it the average muscles found from other Greyhounds :—

Muscular Type of Greyhound—Hip Joint.

Muscle.	"Master Magrath."	Average Greyhound.
	Percentage.	Percentage.
Abductors.		
1. Glutæus primus, ...	2.03	1.90
2. Tensor vaginæ femoris, . .	4.98	4.70
3. { Glutæus secundus (α), / Glutæus secundus (β), } .	10.13	9.07
4. Glutæus tertius, ...	0.91	0.86
5. Obturator internus, . . .	1.40	1.53
Flexors.		
1. Rectus femoris, ...	4.69	5.71
2. Sartorius (double), . . .	4.26	5.42
3. Psoadiliacus, ...	5.18	5.12
Extensors.		
1. Biceps femoris, ...	21.23	18.79
2. Bicipiti accessorius, . . .	0.31	0.30
3. Semitendinosus, ...	5.63	5.22
4. Semimembranosus, . . .	6.91	7.86
5. Gracilis, ...	5.81	5.32
6. Quadratus femoris, . . .	0.68	0.52
Adductors.		
1. Adductor primus, ...	7.82	9.42
2. Adductor secundus (α), / 3. Adductor secundus (β), .	15.33	15.50
4. Adductor quartus, ...	0.85	0.84
5. Adductor quintus, ...	0.60	0.75
6. Obturator externus, . . .	1.25	1.17
	100	**100**
	Oz. av.	Oz. av.
Total weights, . .	**51·31**	**40·43**

It will be seen from this Table that "Master Magrath," although a slightly built dog, possessed an amount of muscular force greatly in excess of ordinary Greyhounds; and a similar conclusion follows from the examination of the muscles of the shoulder joint. From an examination of the dog during

life, I had expressed the opinion that his chief muscular peculiarity consisted in the development of the shoulder muscles—a conclusion which was fully borne out by the *post mortem* examination. If we regard "Master Magrath" as the unit of comparison, we find the percentage of the average Greyhound to be—

In Hip Joint, ... 78.04 per cent.
In Shoulder Joint, . . 75.56 „

These figures show the muscular superiority of "Master Magrath," and the greater development of the muscles of his shoulder.

In the following Table, I have collected into groups the muscular percentages of various Canidæ, for the purpose of comparison :—

Muscular Type of Canidæ—Hip Joint.

Animal.	Abductors.	Flexors.	Extensors.	Adductors.	Total weight.
	Percentage.	Percentage.	Percentage.	Percentage.	Oz. av.
1. "Master Magrath,"	19.45	14.13	40.57	25.85	51.81
2. Average Greyhound,	18.06	16.25	38.01	27.68	40.43
3. Irish Terrier, . .	21.94	15.83	36.41	25.82	14.72
4. Australian Dingo, .	19.37	15.87	42.39	22.37	24.63
5. Pyrenean Mastiff, .	15.82	15.51	41.90	26.77	67.90
6. European Wolf, .	21.45	17.31	34.14	27.10	26.57
7. Indian Jackall, . .	17.43	17.33	39.36	25.88	9.12
8. Bengal Fox, . . .	17.73	15.60	41.14	25.53	2.82

This Table shows the great muscularity of the Greyhounds, for the only dog that excels them is the Mastiff, which is a much larger and heavier animal. It may be worth adding that "Master Magrath's" running weight was 54 lbs.

The muscular type of the shoulder joint in the Canidæ is shown in the following Tables :—

Muscular Type of Greyhound—Shoulder Joint.

Muscle.	"Master Magrath."	Average Grey-hound.
	Per-centage.	Per-centage.
Scapular.		
1. Trapezius clavicularis, . .	1.93	1.48
2. Cleidomastoideus, ...	1.65	1 27
3. Trapezius inferior, ...	1.38	2.40
4. Trapezius superior, . . .	1.17	
5. Omo-Atlanticus, ...	2.02	1.98
6. Serratus magnus, ...	9.28	9.38
7. Rhomboideus, ...	4.24	3.86
Abductors.		
1. Deltoideus, ...	4.22	4.40
2. Teres major, ...	2.29	2.46
3. Supraspinatus, ...	7.35	7.71
4. Infraspinatus, ...	5.15	5.51
5. Teres minor, ...	0.22	0.30
Flexors.		
1. Triceps longus, ...	18.70	22.67
2. Tricipiti accessorius, . . .	0.38	
3. Subscapularis, ...	4.16	4.40
4. Latissimus dorsi, ...	11.40	9.29
Extensors.		
1. Biceps humeri, ...	1.84	2.40
2. Brachiæus, ...	1.34	1.40
Adductors.		
1. Coracobrachialis, ...	0.21	0.23
2. Pectoralis major, ...	21.07	18.86
	100	**100**
	Oz. av.	Oz. av.
Total weights, . .	**63·45**	**47·94**

In the following Table, I have placed the percentages of groups of muscles, of various Canidæ, arranged in the same manner as in the corresponding Table for the Felidæ :—

Muscular Type of Canidæ—Shoulder Joint.

Animal.	Scapular.	Abductors.	Flexors.	Extensors.	Adductors.	Total weight.
	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Oz. av.
1. "Master Magrath,"	21.67	19.23	34.64	3.18	21.28	63.45
2. Average Greyhound,	20.37	20.38	36.36	3.80	19.09	47.94
3. Irish Terrier, . .	26.25	20.35	35.55	3.54	14.31	21.18
4. Australian Dingo, .	24.41	19.81	35.46	3.85	16.47	37.40
5. Pyrenean Mastiff, .	26.56	18.91	30.51	4.12	19.90	80.01
6. European Wolf, .	25.11	25.38	29.54	4.77	15.20	35.65
7. Indian Jackall, . .	22.63	24.13	32.49	4.32	16.43	10.65
8. Bengal Fox, . .	27.30	22.09	30.06	3.99	16.56	3.26

(*c.*) *Muscular Types of Man, and of the Quadrumans.*—I shall now proceed to the comparison of the muscular types of Man and the Quadrumans—a subject full of interest to comparative anatomists.

The comparison includes the following animals:—

1. Man (mean of three subjects).
2. Gorilla.*
3. Chimpanzee (mean of two).
4. Macacus Rhesus (mean of two).
5. Macacus cynomolgus.
6. Cynocephalus Hamadryas (Hepi, or Lion Baboon).
7. Papio Maimon (Mandrill).
8. Ateles ater (mean of two). (Spider Monkey.)

I shall commence with the hip joint:—

* I am indebted to Mr. T. J. Moore, Director of the Liverpool Museum, for the opportunity of dissecting a young female Gorilla, which was brought home in spirits by Captain J. B. Walker, F. R. G. S., from Africa.

Muscular Types of Man and the Quadrumans—Hip Joint.

Muscle.	Man.	Gorilla.	Chimpanzee.	Hamadryas.	Cynomolgus.	Mandrill.	Rhesus.	Ateles.
Abductors.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.
1. Glutæus primus, . .	18.28	13.29	11.67	9.33	5.38	3.72	6.78	4.96
2. Tensor vaginæ femoris,	2.33	2.10						
3. Glutæus secundus (α),	10.89	13.98	14.58	13.03	13.80	13.85	11.60	9.79
4. Glutæus secundus (β),	1.43							
5. Glutæus tertius, . .	3.52	1.75	4.17	1.93	1.93	2.06	1.71	0.89
6. Glutæus quartus, . .	—	—	—	—	—	—	—	1.27
7. Obturator internus, .	3.22	1.92	2.08	2.13	1.79	3.14	2.15	3.93
Flexors.								
1. Rectus femoris, . .	4.94	3.50	3.75	5.55	5.79	5.15	6.19	6.49
2. Sartorius, ...	4.13	2.62	2.50	0.98	2.34	1.37	2.48	3.93
3. Iliacus, ...	4.25	7.00	11.67	11.47	10.49	8.47	8.99	11.18
4. Psoas, ...	4.89							
Extensors.								
1. Biceps femoris, . .	5.50	7.86	5.84	17.21	16.56	17.34	13.36	8.13
2. Semitendinosus, . .	3.60	5.95	4.58	6.04	4.97	6.24	5.49	4.70
3. Semimembranosus, .	4.27	2.45	3.33	3.62	3.58	4.45	5.13	5.08
4. Gracilis, ...	2.27	6.46	5.83	6.24	5.65	5.32	7.24	5.59
5. Quadratus femoris, .	1.04	1.40	1.25	1.48	0.69	1.37	1.49	2.29
Adductors.								
1. Adductor primus, . .		9.97						
2. Adductor secundus (α)	16.58		19.18			23.00	23.69	
3. Adductor secundus (β)		14.33		17.59	24.14			26.55
4. Adductor tertius, . .	2.39		2.91					
5. Adductor quartus, .	3.82	2.45	3.33	—	—	0.45	1.15	
6. Adductor quintus, .	1.35	0.87	0.83	0.65	0.96	1.49	0.71	1.78
7. Obturator externus, .	1.30	2.10	2.50	2.75	1.93	2.58	1.93	3.44
	100	**100**	**100**	**100**	**100**	**100**	**100**	**100**
	Oz. av.	Oz. av.	Oz. av.	Oz. av.	Oz. av.	Oz. av.	Oz. av.	Oz. av.
Total weights, . .	**93·17**	**2·86**	**2·40**	**24·34**	**7·25**	**17·48**	**18·12**	**7·87**

The peculiarities of the Hip joint muscles in Man are the following:—

1°. The enormous size of the *Glutæus primus*, and its complete separation from the *Tensor vaginæ femoris;* in all the Quadrumana, these two muscles form a continuous thin sheet.

2°. The complete separation of the *iliacus* and *psoas magnus*.

3°. The separation of *Glutæus secundus* (*a*) and (*β*).

4°. The absence of *Glutæus quartus*.

5°. The perfect differentiation of the *adductors*.

The broad differences in type will be best seen from the following Table, in which the muscles are arranged in groups as before:—

Muscular Types of Man and the Quadrumans—Hip Joint.

Animal.	Abductors.	Flexors.	Extensors.	Adductors.	Total weight.
	Per cent.	Per cent.	Per cent.	Per cent.	Oz. av.
1. Man, ...	39.67	18.21	16.68	25.44	93.17
2. Gorilla, . . .	33 04	13.12	24.12	29.72	2.86
3. Chimpanzee, . .	32.50	17.92	20.83	28.75	2.40
4. Hamadryas, . .	26.42	18.00	34.59	20.99	24.34
5. Cynomolgus, . .	22.90	18.62	31.45	27.03	7.25
6. Mandrill, . . .	22.77	14.99	34.72	27.52	17.48
7. Rhesus, . . .	22.24	17.66	32.62	27.48	18.12
8. Ateles, ...	20.84	21.60	25.79	31.77	7.87

In this Table, it will be observed that the flexors and adductors are much alike, and that man's superiority consists in the increase of the abductors at the expense of the extensors. This is mainly due to the developement of the *Glutæus primus*, which is essential to man's erect posture; and to the diminution of the *biceps femoris*, which is essentially a brute or quadrupedal muscle.

Taking together the *Glutæus primus* and *tensor vaginæ femoris*, we obtain the following instructive comparison:—

Comparison of Man and Quadrumans—Glutæus primus and Tensor vaginæ femoris.

	Percentage of Hip Joint Muscles.	Relative Proportions.	Successive Differences.
1. Man, ...	20.61	100	—
2. Gorilla, . . .	15.39	74.6	25.4
3. Chimpanzee, . .	11.67	56.6	43.4
4. Hamadryas, . .	9.33	45.3	11.3
5. Rhesus, . . .	6.78	33.0	12.3
6. Cynomolgus, . .	5.38	26.1	6.9
7. Ateles, ...	4.96	24.1	2.0
8. Mandrill, . . .	3.72	18.1	6.0

This Table shows that the difference between Man and the Gorilla is greater than the differences between the Quadrumans compared with each other.

The remarkable differences between Man and the Quadrumans, found in the muscles of the Hip joint, are further illustrated by an examination of the muscles of the foot. These muscles may be divided into three distinct groups, viz. :—

1°. Walking muscles.
2°. Grasping muscles.
3°. Extending muscles.

So far as the presence or absence of individual muscles is concerned, the peculiarities of Man's foot consist in—

1°. The presence of the *peronæus tertius*.
2°. The absence of *peronæus quinti*.
3°. The absence of *extensor internodii hallucis*.

The following Tables enable us to compare the relative development of the various muscles, or groups of muscles :—

Muscular Types of Man and Quadrumans—Foot.

	Man.	Gorilla.	Chimpanzee.	Hamadryas.
Walking Muscles.				
1. Gastrocnemius,	20.35	13.68	16.32	22.52
2. Solæus,	31.80	14.62	16.32	17.00
3. Plantaris,	1.48	—	0.48	4.20
4. Tibialis posticus,	8.60	8.49	9.18	4.20
Grasping Muscles.				
1. Flexor digitorum longus,	2.28	6.60	8.16	5.15
2. Flexor hallucis longus,	4.82	17.94	15.30	10.88
3. Flexor digitorum brevis,	1.48	4.24	0.58	4.58
4. Fl. dig. long. accessorius,	0.51	—		
Extending Muscles.				
1. Tibialis anticus,	9.94	10.85	10.20	14.51
2. Extensor digitorum longus,	3.60	5.19	6.12	4.96
3. Extensor hallucis longus,	3.63	2.83	3.06	1.71
4. Extensor secundus internodii hallucis,	—	—	2.04	—
5. Peronæus longus,	5.53	7.54	8.16	6.10
6. Peronæus brevis,	3.38	8.02	4.08	4.19
7. Peronæus tertius,	2.60	—	—	—
8. Peronæus quinti,	—	—	—	Small.
	100	**100**	**100**	**100**
	Oz. av.	Oz. av.	Oz. av.	Oz. av.
Total weight,	**31·10**	**1·06**	**0·98**	**5·24**

Taking the sums of the muscles in groups, we find—

Muscular Types of Man and Quadrumans—Foot.

	Walking.	Grasping.	Extending.	Total weight.
	Per cent.	Per cent.	Per cent.	Oz. av.
1. Man,	**62·23**	**9·09**	28.68	31.10
2. Gorilla,	36.79	28.78	34.43	1.06
3. Chimpanzee,	42.30	24.04	33.66	0.98
4. Hamadryas,	47.92	20.61	31.47	5.24
5. Cynomolgus,	40.27	23.53	36.20	2.21
6. Mandrill,	40.59	24.54	34.87	5.42
7. Rhesus,	44.65	23.43	31.92	5.42
8. Ateles,	54.96	20.21	24.83	2.82

The following facts are evident from an inspection of this Table:—

1°. The great developement of the walking muscles in Man.

2°. The great deficiency of the grasping (or hand-like) action in the foot of Man.

We may also remark that in both these respects the New World Monkeys come nearer to Man than either the Gorilla or Chimpanzee.*

The deficiency of grasping action in the foot of Man, stated as a physical constant characteristic of him, fully justifies the title of Quadrumanous applied by Cuvier to the Apes, and denied by him to Man.

In the next Table, I have resolved the walking muscles into their component parts:—

Walking Muscles of Man and Quadrumans.

	Gastro-cnemido-Solæus.	Tibialis posticus.	Plantaris.
1. Man, ...	52.15	8.60	1.48
2. Gorilla, . . .	28.30	8.49	—
3. Chimpanzee, . .	32.64	9 18	0.48
4. Hamadryas, . .	39.52	4.20	4.20
5. Cynomolgus, .	34.17	2.71	3.39
6. Mandrill, . . .	33.76	3.14	3.69
7. Rhesus, . . .	36.72	3.14	4.79
8. Ateles, ...	48.57	6.39	—

If we now examine the muscles of the Shoulder joint and Hand, we shall find the same profound differences between Man and the Quadrumans as in the Hip joint and Foot.

In the Shoulder muscles, Man is characterised by the presence of—

1°. *Levator anguli scapulæ;*

2°. Distinct *Pectoralis major* and *minor;*

* Professor Huxley, some years ago, directed my attention to the remarkable power of standing and walking erect, possessed by the Spider Monkey; an observation fully confirmed on mechanical grounds by the foregoing Table.

and by the absence of—

1°. Occipital slip of *Rhomboideus*.
2°. *Omo-Atlanticus*.
3°. *Tricipiti accessorius*.
4°. *Pectoralis quartus*.

The following Tables show the relative developement of the various muscles and groups of muscles :—

Muscular Types of Man and Quadrumans—Shoulder Joint.

	Man.	Gorilla.	Chimpanzee.	Hamadryas.
Scapular Muscles.	Per cent.	Per cent.	Per cent.	Per cent.
1. Trapezius,	9.33	8.73	9.98	7.68
2. Cleidomastoides,	2.49	0.60	4.50	3.37
3. Rhomboideus,	3.07	3.02	3.75	3.29
4. Serratus magnus,	7.64	11.30	13.71	8.46
5. Levator anguli scapulæ,	1.96	—	—	—
6. Omo-Atlanticus,	—	0.60	0.99	1.70
7. Subclavius,	0.19	0.15	0.24	0.54
Abductors.				
1. Deltoideus,	15.32	14.02	8.98	7.45
2. Teres major,	4.28	4.21	3.49	2.75
3. Supraspinatus,	2.40	1.96	2.49	4.54
4. Infraspinatus,	4.56	3.91	3.99	6.04
5. Teres minor,	1.54	1.20	1.00	0.77
Flexors.				
1. Triceps longus,	8.95	4.52	3.99	9.04
2. Tricipiti accessorius,	—	1.51	0.74	1.63
3. Subscapularis,	5.04	6.92	5.99	7.25
4. Latissimus dorsi,	9.35	12.50	12.22	10.15
Extensors.				
1. Biceps humeri,	4.34	7.68	5.99	7.52
2. Brachiæus,	5.46	5.12	3.74	3.21
Adductors.				
1. Coracobrachialis,	1.46	1.20	1.52	0.77
2. Pectoralis major,	10.45	9.34	11.73	8.95
3. Pectoralis minor,	2.17	1.51		3.61
4. Pectoralis quartus,	—	—	0.96	1.28
	100	**100**	**100**	**100**
	Oz. av.	Oz. av.	Oz. av.	Oz. av.
Total weight,	**77·11**	**3·32**	**4·01**	**25·80**

Taking the sums of the muscles in groups, I find the fol lowing Table :—

Muscular Types of Man and Quadrumans—Shoulder Joint.

	Scapular.	Abductors.	Flexors.	Extensors.	Adductors.	Total weight.
	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Oz. av.
1. Man, . . .	24.68	28.10	23.34	9.80	14.08	77.11
2. Gorilla, . .	24.40	25.30	25.45	12.80	12.05	3.32
3. Chimpanzee,	33.17	19.95	22.94	9.73	14.21	4.01
4. Hamadryas, .	25.04	21.55	28.07	10.73	14.61	25.80
5. Cynomolgus,	19.95	16.94	31.42	12.15	19.54	7.32
6. Mandrill, . .	18.96	20.87	30.60	14.07	15.50	16.77
7. Rhesus, . .	20.86	19.96	31.46	11.42	16.50	15.58
8. Ateles, . .	18.94	21.93	31.86	14.86	12.41	7.34

On examining these Tables, we find in Man and the Gorilla a remarkable developement of the abductor muscles, due mainly to the great size of the *Deltoid* muscle, which corresponds to the *Glutæus maximus* in the hip. This is evident from the following Table :—

Comparison of the Deltoid Muscle in Man and the Quadrumans.

	Percentage of Shoulder Muscles.	Relative Proportion.	Successive Differences.
1. Man, ...	15.32	100.00	—
2. Gorilla, . . .	14.02	91.51	8.49
3. Ateles, ...	10.34	67.49	32.51
4. Chimpanzee, . .	9.00	58.75	8.74
5. Hamadryas, . .	7.45	48.63	10.12
6. Mandrill, . . .	7.15	46.67	1.96
7. Rhesus, . . .	6.42	41.91	4.76
8. Cynomolgus, . .	5.61	36.62	5.29

This Table shows that the Gorilla and Spider Monkey approach nearer to Man, with regard to relative size of the Deltoid muscle, than the other Quadrumans do.

In examining the hand of Man and the Quadrumans, the following well-known differences are found :—

1°. Man posseses a distinct *flexor pollicis longus*,* which does not occur in any Quadruman.

2°. Man possesses two distinct extensors of the phalanges of the thumb; while the Quadrumans possess either one or none.

3°. In Man the *Indicator* belongs to the second finger only; while in the Quadrumans it belongs to the second and third fingers jointly.

4°. In Man the *Auricularis* belongs to the fifth finger only; the Chimpanzee and Ateles resemble Man in this respect; and in the other Quadrumans, the *Auricularis* belongs to the fourth and fifth fingers jointly.

The following Tables show the relative developement of the various muscles and groups of muscles :—

Muscular Types of Man and Quadrumans—Hand.

	Man.	Gorilla.	Chimpanzee.	Hamadryas.
Walking Muscles.	Per cent.	Per cent.	Per cent.	Per cent.
1. Flexor carpi radialis, . . .	5 40	7.17	7.77	8.24
2. Flexor carpi ulnaris, . . .	8.06	9.73	6.89	16.49
3. Palmaris longus, ...	2.58	0.75	1.72	3.59
Grasping Muscles.				
1. Flexor digitorum sublimis, .	17.71	18.92	14.65	12.02
2. Flexor digitorum profundus,	21.53	33.25	34.49	28.13
3. Flexor pollicis longus, . .	4.24			
Extending Muscles.				
1. Extensor carpi radialis (α), .	9.65	2.56	6.03	5.01
2. Extensor carpi radialis (β), .	6.48	5.12	5.17	6.08
3. Extensor carpi ulnaris, . .	5.65	4.60	3.45	5.55
4. Extensor digitorum longus, .	8.31	8.71	9.49	6.81
5. Auricularis, ...	1.41	2.04	1.72	1.62
6. Extensor assis metacarpi pollicis, ...	4.24	4.60	4.31	4.48
7. Extensor primi internodii pollicis, ...	0.67	—	—	—
8. Extensor secundi internodii pollicis, ...	2.41	1.53	2 59	0.90
9. Indicator, ...	1.66	1.02	1.72	1.08
	100	**100**	**100**	**100**
	Oz. av.	Oz. av.	Oz. av.	Oz. av.
Total weight, . .	**12·03**	**0·98**	**1·16**	**5·58**

* For a remarkable exception to this rule, *vide* pp. 122–3.

Taking the sums of the muscles in groups, I find the following results :—

Muscular Types of Man and the Quadrumans—Hand.

	Walking.	Grasping.	Extending.	Total weight.
	Per cent.	Per cent.	Per cent.	Oz. av.
1. Man, ...	16.04	43.48	40·48	12.03
2. Gorilla, . . .	17.65	52.17	30.18	0.98
3. Chimpanzee, . .	16.38	49.14	34.48	1.16
4. Hamadryas, . .	28.32	40.15	31.53	5.58
5. Cynomolgus, . .	15.54	53.38	31.08	1.48
6. Mandrill, . . .	19.15	50.00	30.85	4.44
7. Rhesus, . . .	22.77	46.68	30.55	3.47
8. Ateles, ...	16.24	54.82	28.94	1.97

From this Table it appears that the chief muscular peculiarity of Man's hand consists in the great development of the extensor muscles; and on analysing these muscles it will be found that it is the extensors of the wrist that are principally developed.

Wrist Extensors of Man and the Quadrumans.

	Extensor carpi radialis (α).	Extensor carpi radialis (β).	Extensor carpi ulnaris.	Total.
	Per cent.	Per cent.	Per cent.	Per cent.
1. Man, ...	9.65	6.48	5.65	21.78
2. Gorilla, . . .	2.56	5.12	4.60	12.28
3. Chimpanzee, . .	6.03	5.17	3.45	14.65
4. Hamadryas, . .	5.01	6.08	5.55	16.64
5. Cynomolgus, . .	3.80	6.33	5.40	15.53
6. Mandrill, . . .	6.30	5.63	4.95	16.88
7. Rhesus, . . .	4.61	7.49	5.76	17.86
8. Ateles, ...	3.55	5.58	4.06	13.19

The remarkable developement of the extensor muscles of the wrist in Man is probably connected with the corresponding developement of the Deltoid muscle in the shoulder, and the combination of both is essential to characteristic human action—such, for example, as stone-throwing and oratorical gestures.

It is interesting to observe that the Gorilla approaches

Man in the shoulder muscles more nearly than the Chimpanzee does, while he deviates farther from the hand of Man than does the Chimpanzee.

If we take the walking muscles of the hand and foot as our standard of comparison, and inquire the proportion which the grasping muscles bear to them, we shall obtain the following instructive Tables, which show, in a remarkable way, the profound difference between a hand and a foot, even in Quadrumanous animals :—

Comparison of Walking and Grasping Muscles in the Foot *of Man and the Quadrumans.*

	Walking Muscles.	Grasping Muscles.
1. Man, ...	100	14.6
2. Gorilla, . . .	100	78.2
3. Chimpanzee, . .	100	56.8
4. Hamadryas, . .	100	43.1
5. Cynomolgus, . .	100	58.4
6. Mandrill, . . .	100	60.5
7. Rhesus, . . .	100	52.4
8. Ateles, ...	100	36.8

In the foot, the grasping power is less than the walking power.

Comparison of Walking and Grasping Muscles in the Hand *of Man and the Quadrumans.*

	Walking Muscles.	Grasping Muscles.
1. Man, ...	100	271.0
2. Gorilla, . . .	100	295.6
3. Chimpanzee, . .	100	300.0
4. Hamadryas, . .	100	141.7
5. Cynomolgus, . .	100	343.5
6. Mandrill, . . .	100	261.1
7. Rhesus, . . .	100	205.0
8. Ateles, ...	100	337.5

In the hand, the grasping power is greater than the walking power.

These Tables show—

1°. The deficiency of grasping power in feet as compared with hands.

2° The enormous developement of the walking power in the foot of Man.

These results possess considerable interest when taken in conjunction with the results previously obtained by me, in discussing the differences in the flexor tendons of the hand and foot. It may be worth while to note, in connexion with the exceptional position of the Llama, p. 134 (which proves that the hand of this animal is a true foot), that the proportions of the walking and grasping muscles completely confirms this view.

	Walking Power.	Grasping Power.
Llama (Hand), . . .	100	90.3

Taking, as before, the walking muscles of the hand as our standard of comparison, we find the following proportions for the extensors of the wrist:—

Comparison of Walking and Extending Muscles in the Wrist *of Man and the Quadrumans.*

	Walking Muscles.	Extensors of Wrist.
1. Man, ...	100	135.7
2. Gorilla, . . .	100	171.0
3. Chimpanzee, . .	100	89.5
4. Hamadryas, . .	100	58.8
5. Cynomolgus, . .	100	100.0
6. Mandrill, . . .	100	88.2
7. Rhesus, . . .	100	78.5
8. Ateles, ...	100	81.2

The next example of muscular type that I shall give, is the remarkable case of the hind limbs of the *Struthionidæ*.

I have had an opportunity of examining the following specimens of this interesting group of Birds :—

1°. Three Ostriches.
2°. Four Emus.
3°. Two Rheas.
4°. One Cassowary.

In the Hip joint of the Struthionidæ, the following peculiarities are observable :—

1°. The *Agitator caudæ* and *Tensor femoris vaginæ* form an outer sheet of powerful muscles, which overlies the glutæal muscles.

2°. The *Glutæus quartus* muscle is always present.

3°. The *Iliacus* muscle is also present.

4°. The *Adductor tertius* (*brevis*) is invariably absent.

5°. The *Rectus femoris* is present in the Ostrich and Rhea, and has relation in these, as in other birds, to the *Solæus* muscle, and to the perforate flexors of the most important toe. This muscle is totally wanting in the Emu and Cassowary.

6°. The *Biceps femoris* acquires an extraordinary developement, intimately related to the unusual running power of this remarkable group of birds.

The following Table shows the muscular type of the Ostrich :—

Muscular Type of Ostrich—Hip Joint.

	Male.	Female.	Female.	Mean.
Abductors.	Per cent.	Per cent.	Per cent.	Per cent.
1. Agitator caudæ, ...	23.76	15.64	12 17	24.04
2. Tensor vaginæ femoris, . .		10.05	10.52	
3. Glutæus primus, ...	*x*	*x*	*x*	*x*
4. Glutæus secundus, ...	6.70	6.71	6.77	6.73
5. Glutæus tertius, ...	0.51	0.72	0.56	0.60
6. Glutæus quartus, ...	0.89	1.15	0.98	1.01
7. Obturator internus, . . .	9.04	8.56	8.76	8.79
Flexors.				
1. Rectus femoris, ...	3.68	3.84	3.02	3.51
2. Sartorius, ...	9.15	8.63	8.86	8.88
3. Iliacus, ...	0.89	0.98	1.13	1.00
Extensors.				
1. Biceps femoris, ...	20.98	21.18	22.71	21.62
2. Semitendinosus, ...	7.12	5.30	8.19	6.87
3. Semitendinoso-accessorius, .	0.51	0.45	0.52	0.49
4. Semimembranosus, . . .	2.28	2.68	2.53	2.50
Adductors.				
1. Adductor primus, ...	8.65	6.76	8.68	8.03
2. Adductor secundus, . . .				
3. Adductor quartus, ...	2.41	3.84	2.43	2.89
4. Adductor quintus, ...	2.03	2.14	1.29	1.82
5. Obturator externus, . . .	1.40	1.37	0.88	1.22
	100	**100**	**100**	**100**
	Oz. av.	Oz. av.	Oz. av.	
Total weight, . .	**196·75**	**133·96**	**284·51**	

In the next Table, I show the corresponding muscular type of the Emu :—

Muscular Type of Emu—Hip Joint.

	Male.	Male.	Male.	Young Female.	Mean.
Abductors.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.
1. Agitator caudæ, . . . } 2. Tensor vaginæ femoris, }	23.68	25.97	24.02	23.75	24.36
3. Glutæus primus, . .	1.99	1.25	1.28	1.47	1.50
4. Glutæus secundus, . .	10.15	8.58	9.85 }		9.88
5. Glutæus tertius, . .	0.40	0.29	0.37 }	13.32	0.34
6. Glutæus quartus, . .	2.19	1.67	1.85 }		1.95
7. Obturator internus, .	5.27	4.85	5.56	5.25	5.23
Flexors.					
1. Rectus femoris, . . .	None.	None.	None.	None.	None.
2. Sartorius, ...	12.34	13.00	12.81	9.96	11.91
3. Iliacus, ...	*x*	*x*	*x*	0.15	0.15
Extensors.					
1. Biceps femoris, . . .	18.71	17.15	17.90	20.51	18.57
2. Semitendinosus, . .	8.76	9.97	8.27	4.47	7.87
3. Semitendinoso-accessorius, ... }	1.19	1.10	1.41	0.81	1.13
4. Semimembranosus, . .	1.99	2.49	1.59	2.07	2.03
Adductors.					
1. Adductor primus, . . } 2. Adductor secundus, . }	6.77	7.18	7.28	7.11	7.08
3. Adductor quartus, . .	1.99	2.63	3.77	2.37	2.69
4. Adductor quintus, . .	1.79	1.52	1.41	2.01	1.68
5. Obturator externus, .	2.78	2.35	2.63	6.75	3.63
	100	**100**	**100**	**100**	**100**
	Oz. av.	Oz. av.	Oz. av.	Oz. av.	
Total weight, . .	**50·25**	**72·30**	**95·62**	**33·34**	

The following Table shows the muscular type of the Rhea :—

Muscular Type of Rhea—Hip Joint.

	Male.	Female.	Mean.
Abductors.	Per cent.	Per cent.	Per cent.
1. Agitator caudæ, ... } 2. Tensor femoris vaginæ, . . }	23.12	25.54	24.33
3. Glutæus primus, ...	0.89	1.20	1.04
4. Glutæus secundus, ...	9.61	9.02	9.32
5. Glutæus tertius, ...	0.96	0.59	0.77
6. Glutæus quartus, ...	0.89	0.90	0.89
7. Obturator internus, . . .	8.65	8.09	8.38
Flexors.			
1. Rectus femoris, ...	0.46	0.47	0.46
2. Sartorius, ...	10.76	8.35	9.56
3. Iliacus, ...	0.31	0.23	0.27
Extensors.			
1. Biceps femoris, ...	14.60	19.48	17.04
2. Semitendinosus, ...	8.34	11.00	9.92
3. Semitendinoso-accessorius, .	1.92	1.06	1.49
4. Semimembranosus, . . .	2.73	2.07	2.40
Adductors.			
1. Adductor primus, ... } 2. Adductor secundus, . . . }	10.00	8.14	9.07
3. Adductor quartus, ...	3.84	2.26	3.05
4. Adductor quintus, ...	0.46	0.28	0.37
5. Obturator externus, . . .	1.96	1.32	1.64
	100	**100**	**100**
Total weight, . .	Oz. av. **26·02**	Oz. av. **47·73**	

The muscular types of the Cassowary and of the other Struthionidæ are given in the following Table:—

Muscular Types of Struthionidæ—Hip Joint.

	Ostrich.	Emu.	Rhea.	Cassowary.
Abductors.	Per cent.	Per cent.	Per cent.	Per cent.
1. Agitator caudæ, ... }	24.04	24.36	24.33	25.08
2. Tensor vaginæ femoris, . . }				
3. Glutæus primus, ...	*x*	1.50	1.04	1.49
4. Glutæus secundus, ...	6.73	9.88	9.32	10.90
5. Glutæus tertius, ...	0.60	0.34	0.77	0.32
6. Glutæus quartus, ...	1.01	1.95	0.89	2.53
7. Obturator internus, . . .	8.79	5.23	8.38	5.44
Flexors.				
1. Rectus femoris, ...	3.51	None.	0.46	None.
2. Sartorius, ...	8.88	11.91	9.56	15.82
3. Iliacus, ...	1.00	0.15	0.27	0.08
Extensors.				
1. Biceps femoris, ...	21.62	18.57	17.04	15.40
2. Semitendinosus, ...	6.87	7.87	9.92	7.70
3. Semitendinoso accessorius, .	0.49	1.13	1.49	1.36
4. Semimembranosus, . . .	2.50	2.03	2.40	0.64
Adductors.				
1. Adductor primus, ... }	8.03	7.08	9.07	4.77
2. Adductor secundus, . . . }				
3. Adductor quartus, ...	2.89	2.69	3.05	5.58
4. Adductor quintus, ...	1.82	1.68	0.37	2.66
5. Obturator externus, . . .	1.22	3.63	1.64	0.23
	100	**100**	**100**	**100**
				Oz. av.
Total weight, . . .				**53·63**

If we combine these results in groups as before, we find the following results :—

Muscular Type of Struthionidæ—Hip Joint.

	Abductors.	Flexors.	Extensors.	Adductors.
	Per cent.	Per cent.	Per cent.	Per cent.
1. Ostrich, . . .	41.17	13.39	31.48	13.96
2. Emu, ...	43.26	12.06	29.60	15.08
3. Rhea, ...	44.73	10.29	30.85	14.13
4. Cassowary, . .	45.76	15.90	25.10	13.24

If we compare this Table with the preceding Tables, we shall see that the peculiarities of the Struthionidæ consist in the development of the abductor muscles (which give the power of standing erect) and of the extensor muscles (which measure the power of running). This will be better seen by the following comparison of the most important individual muscles, on which these actions mainly depend :—

Comparative View of the Standing and Running Powers of the Struthionidæ and other Animals.

	Ostrich.	Emu.	Rhea.	Cassowary.	Man.	"Master Magrath."
Standing.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.
1. Agitator caudæ, Tensor vaginæ femoris, Glutæus primus, . .	24.04	25.86	25.37	26.57	20.61	7.01
Running.						
2. Biceps femoris, . . .	21.62	18.57	17.04	15.40	5.50	21.23

In this Table, the contrast between Man and the Dog is well brought out; and it is shown that the Struthionidæ excel in both the respective peculiarities of Man and the Dog;

The last example that I shall give of the theory of muscular types, is the remarkable case of the wings of birds, which, for many reasons, is to be considered in detail.

The development of the great pectoral in birds has, of course, been noticed by every anatomist; but the theory of muscular types enables us to present it to the eye in a manner that admits of immediate comparison with the corresponding muscle in other animals; and to demonstrate the impossibility of flight, in other animals not endowed with the peculiar muscular type of birds.

The birds from which I have made my measurements are the following:—

1. Albatross (two examples, measuring, respectively, from tip to tip of extended wings, 11 ft. 0 in. and 9 ft. 3 in.).
2. Common Swan.
3. Dorking Cock.
4. Aylesbury Duck (two examples).
5. Gannet.
6. Curassow.
7. Jabiru (two examples).
8. Stork (two examples).
9. Common Heron.
10. Squacco Heron.
11. Flamingo.
12. White-Headed Eagle (two examples).
13. Grebe.
14. African Parrot.
15. Peregrine Falcon.
16. White Crane.
17. New Zealand Weka Rail (two examples).

Muscular Type of Birds' Wings.

	I. Albatross (mean of two).	II. Common Swan.	III. Dorking Cock.	IV. Aylesbury Duck (mean of two).	V. Gannet.	VI. Curassow.
Scapular.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.
1. Trapezius, . .	3.64	1.45	2.20	2.28	2.05	1.84
2. Rhomboideus, .	1.02	0.36	1.59		1.10	
3. Serratus magnus,	3.04	0.83	0.61	1.39	2.36	0.74
Abductors.						
1. Deltoideus, . .	5.07	6.52	2.73	3.75	3.94	4.67
2. Pectoralis avium,	5.11	6.84	19.94	11.28	4.88	15.23
4. Teres major, . .	1.43	0.83	0.53	0.49	1.73	0.49
5. Spinatus, . . .	2.03	4.86	5.15	4.73	2.52	5.90
Flexors.						
1. Triceps longus, .	4.14	5.02	2.43	4.48	4.89	3.31
2. Subscapularis, .	4.00	1.55	2.27	2.37	3.30	1.84
3. Latissimus dorsi,	2.76	1.92	0.68	0.98	3.46	1.35
Extensors.						
1. Biceps humeri, .	1.84	3.57	2.35	2.61	0.63	3.94
2. Brachiæus, . .	0.80	1.24	0.76	0.90	0.63	0.24
Adductors.						
1. Coraco-brachialis,	0.69	0.46	0.76	0.90	0.95	2.83
2. Pectoralis major,	62.67	62.90	35.42	61.96	64.42	55.04
3. Pectoralis minor.	1.76	1.65	2.58	1.88	3.14	2.58
	100	**100**	**100**	**100**	**100**	**100**
	Oz. av.	Oz. av.	Oz. av.	Oz. av.	Oz. av.	Oz. av.
Total weight, .	**14·22**	**19·32**	**13·19**	**6·13**	**6·35**	**4·07**

Muscular Type of Birds' Wings (continued).

	VII. Jabiru (mean of two).	VIII. Stork (mean of two).	IX. Common Heron.	X. Squacco Heron.	XI. Flamingo.	XII. White-headed Eagle (mean of two).
Scapular.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.
1. Trapezius, . .	1.16	1.32	1.64	1.68	0.81	1.45
2. Rhomboideus, .	0.82	0.62	0.97	1.01	0.81	0.85
3. Serratus magnus,	1.14	0.59	2.70	1.17	0.81	1.54
Abductors.						
1. Deltoideus, . .	5.81	4.58	3.38	3.02	7.50	8.45
2. Pectoralis avium,	8.93	8.23	7.45	7.89	8.42	6.09
3. Teres major, . .	0.75	0.68	0.29	0.33	0.61	1.19
4. Spinatus, . . .	3.70	4.20	3.28	3.02	4.05	4.75
Flexors.						
1. Triceps longus, .	5.61	4.56	3.67	2.51	5.58	4.83
2. Subscapularis, .	1.72	3.12	2.12	1.68	1.83	1.79
3. Latissimus dorsi,	1.17	0.93	0.97	0.50	1.12	0.96
Extensors.						
1. Biceps humeri, .	2.66	2.31	2.99	2.02	2.84	4.55
2. Brachiæus, . .	0.43	0.79	0.39	0.50	0.71	0.85
Adductors.						
1. Coraco-brachialis,	2.20	1.39	1.16	0.50	1 32	1.31
2. Pectoralis major,	62.06	65.18	67.54	72.83	62.47	59.82
3. Pectoralis minor,	1.84	1.50	1.45	1.34	1.12	1.57
	100	**100**	**100**	**100**	**100**	**100**
	Oz. av.	Oz. av.	Oz. av.	Oz. av.	Oz. av.	Oz. av.
Total weight, .	**21·45**	**12·89**	**5·18**	**2·98**	**4·93**	**8·89**

The first fact that strikes us, on examining the foregoing Tables, is the extraordinary development of the great pectoral muscle, which in many cases absorbs two-thirds of the entire muscular force of the shoulder joint. This muscle is the great depressor of the wing, and I have already shown what extraordinary care is bestowed upon the placing of the socket

of the joint in such a position as to give the muscle the maximum amount of advantage in performing its work. The muscles that lift the wing are the *Deltoid* and *Pectoralis avium*, which latter corresponds with the *Subclavius* in Mammals.

Combining together the Depressor and Levator muscles, I have found the following interesting results :—

Depressor and Levator Muscles in the Wings of Birds.

	Depressors. Pect. major and Pect. minor.	Levators. Deltoid and Pect. avium.	Ratio.
1. Albatross, ...	64.43	10.18	15.80 per cent.
2. Swan, ...	64.55	13.36	20.70 ,,
3. Dorking Fowl, ...	58.00	22.67	39.09 ,,
4. Aylesbury Duck, . . .	63.84	15.03	23.54 ,,
5. Gannet, ...	67.56	8.82	13.06 ,,
6. Curassow, ...	57.62	19.90	34.54 ,,
7. Jabiru, ...	63.90	14.74	23.07 ,,
8. Stork, ...	66.68	12.81	19 21 ,,
9. Common Heron . .	68.99	10.83	15.70 ,,
10. Squacco Heron , . .	74.17	10.91	14.71 ,,
11. Flamingo, ...	63.59	15.92	25.04 ,,
12. White-headed Eagle, .	61.39	14.54	23.68 ,,
13. Grebe, ...	75.82	11 26	14.84 ,,
14. African Parrot, ...	69.14	15.96	23.08 ,,
15. Falcon, ...	59.27	17.22	29.05 ,,
16. White Crane, ...	54 90	20.95	38.16 ,,
17. Weka Rail, ...	38.28	20.00	52.25 ,,

The birds in which the great pectoral becomes least are the Dorking Fowl and Curassow, which are notoriously heavy fliers, and the Weka Rail, which uses its wings principally in propelling itself through the long grass. The birds of greatest *soaring* power are those in which the levator muscles bear the least proportion to the depressor muscles, viz., the Gannet, Grebe, Heron, and Albatross.

If we arrange the muscles into groups, as before, we find the following Table :—

Muscular Type of Wings of Birds.

	Scapular.	Abductors.	Flexors.	Extensors.	Adductors.	Total weight.
	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Oz. av.
1. Albatross, ...	7.70	13.64	10.90	2.63	65.13	14.22
2. Common Swan, ...	2.64	19.05	8.49	4.81	65.01	19.32
3. Dorking Cock, ...	4.40	28.35	5.38	3.11	58.76	13.19
4. Aylesbury Duck, . . .	3.67	20.25	7.83	3.51	64.74	6.13
5. Gannet, ...	5.51	13.07	11.65	1.26	68.51	6.35
6. Curassow, ...	2.58	26.29	6.50	4.18	60.45	4.07
7. Jabiru, ...	3.12	19.20	8.50	3.09	66.09	21.45
8. Stork, ...	2.53	17.68	8.62	3.10	68.07	12.89
9. CommonHeron, . . .	5.31	14.40	6.76	3.38	70.15	5.18
10. Squacco Heron, . . .	3.86	14.26	4.69	2.52	74.67	2.98
11. Flamingo, ...	2.43	20.58	8.53	3.55	64.91	4.93
12. White-headed Eagle, .	3.84	20.47	7.58	5.41	62.70	8.89
13. Grebe, ...	2.05	13.93	6 97	1.23	75.82	2.44
14. African Parrot, ...	5.22	18.73	5.22	2.63	68.20	0.94
15. Falcon, ...	4.30	22.18	9.27	3.97	60.28	1.51
16. White Crane, ...	7.16	23.88	7.69	5.30	55.97	1.89
17. Weka Rail, ...	10 43	29.56	13.04	6.95	40.02	0.58

If we compare the adductor muscles of any of the birds (ranging from 40 to 76 per cent.) with the adductor muscles of the Felidæ, Canidæ, Man, and the Quadrumans, already given, we shall see at a glance how peculiar is the type of the muscular arrangements of the wing of the Bird, and how hopeless it would be to expect that any of the other types could attempt to fly, even if aided by mechanical contrivances to increase the surface exposed to the air. And yet the muscular type of the wing of the Bird, differing profoundly as it does from the corresponding type of the fore-limbs of the other animals, is composed of groups of muscles bearing the same names, and fulfilling similar functions.

The skilful artizan can produce from the same number of wheels and pinions either a clock or a roasting jack, fulfilling the very different functions of marking time, and of roasting meat. An ignorant but intelligent savage, who was shown the interior of these machines, would come to the conclusion

that they were very like each other—simply because he would consider only their superficial resemblances, and would be unable to appreciate the purposes which the machines were respectively intended to fulfil. In like manner, anatomists, from observation of apparent resemblances in the structure of organs, such as the brain (of the specific action of whose parts little or nothing is known), have sometimes, rashly, inferred a greater degree of affinity between various animals than there is any logical ground for admitting. If we confine our attention to the arrangement of muscles and bones, the objects and uses of which are perfectly known and understood, we may readily perceive that, under a superficial appearance of similarity of parts, there really exists a profound difference of function and intention as to the purposes to which these organs are devoted. In the present state of anatomical science, it seems safer to collect facts and physical constants, characteristic of each group of organs in various animals, than to indulge in premature speculations as to possible modes of production of such organs. In the language of the schoolmen, the Final Cause of the wing of a bird is to fly in air, and the Final Cause of the fore-limb of an Otter or Seal is to swim in water; and we can show that the intention of the organ is, in each case, perfectly fulfilled; but it appears unscientific to speculate as to the possible derivation of one of these structures from the other, by means of an hypothetical common ancestor.

I shall now give the muscular type of the wings of two large Bats, *Pteropus edulis* and *Epomophorus Whitii*, for the purpose of comparing it with the type of the wings of birds :—

Muscular Type of Wings of Bats.

	Pteropus edulis.	Epomophorus Whitii.
Scapular.	Per cent.	Per cent.
1. Trapezius, ...	4.36	5.00
2. Rhomboideus, ...	1.80	2.14
3. Serratus magnus, ...	11.80	11.43
4. Levator anguli scapulæ, . .	—	2.14
5. Omo-Atlanticus, ...	0.80	1.43
Abductors.		
1. Deltoideus—		
(α.) Clavicular, ...	2.99	2.14
(β.) Acromial, ...	4.58	3.57
(γ.) Scapular, ...	2.00	2.14
2. Occipito-pollicalis, ...	2.39	—
3. Subclavius, ...	1.00	1.43
4. Teres major, ...	1.80	1.44
5. Supraspinatus, ...	1.60	1.43
6. Infraspinatus, ...	1.60	2.14
7. Teres minor, ...	0.20	—
Flexors.		
1. Triceps longus, ...	4.20	4.30
2. Subscapularis, ...	6.18	6.42
3. Latissimus dorsi, ...	2.60	2.85
Extensors.		
1. Biceps humeri, ...	11.18	8.57
2. Brachiæus, ...	0.60	0.71
Adductors.		
1. Coraco-brachialis, ...	1.00	2.86
2. Pectoralis major, ...	36.12	35.01
3. Pectoralis quartus, ...	1.20	2.85
	100	**100**
	Oz. av.	Oz. av.
Total weight, . .	**2·51**	**0·70**

This Table suggests many interesting reflections, arising from the great number of muscles employed, as compared with those of birds; but we must pass on to other subjects, noting merely the bird-like power of the adductor muscles.

In Swimming Animals we find, as might be expected, a type of muscular structure of the fore-limb, which in many respects approaches to that of the Flying Animals; for the act of swimming resembles that of flying in water instead of air. I have selected the following animals to illustrate the swimming type of the fore-limb :—

1. The Green Turtle.
2. The Hawk's Bill Turtle.
3. The Logger Head Turtle.
4. The Alligator ($6\frac{1}{2}$ feet in length).
5. The Otter.
6. The Seal.

Muscular Type of Swimming Animals.

	Green Turtle.	Hawk's-bill Turtle.	Logger-Head Turtle.	Alligator.	Otter.	Seal.
Scapular.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.
1. Trapezius, ...	1.12	1.62	0.57	1.60	14.88	6.61
2. Rhomboideus, . . .				2.18	3.77	5.19
3. Serratus magnus, . .	2.28	4.86	7.44	20.12	11.98	11.80
4. Omo-Atlanticus, . . .	—	—	0.77	—	2.31	2.67
5. Cleidomastoideus, . .	—	—	—	—	1.67	7.30
6. Levator anguli scapulæ,	—	—	—	—	1.72	—
Abductors.						
1. Deltoideus, ...	5.71	5.81	5.86	7.92	3.50	3.28
2. Pectoralis avium, . .	4.62	5.57	7.67	4.37	—	—
3. Teres major, ...	2.88	3.04	0.52	0.87	1.35	1.29
4. Supraspinatus, . . .	3.09	7.39	4.55	1.36	4.35	4.87
5. Infraspinatus, . . .					1.80	1.23
6. Teres minor, ...	—	—	—	—	Trace.	0.08
Flexors.						
1. Triceps longus, . . .	0.27	0.26	0.30	8.11	7.80	4.42
2. Subscapularis, . . .	10.39	1.72	14.29	4.08	5.31	15.41
3. Latissimus dorsi, . . .	6.26	5.17	5.47	4.61	3.68	2.19
4. Tricipiti accessorius, .	—	—	—	—	1.91	1.64
Extensors.						
1. Biceps humeri, . . .	4.26	10.03	7.07	2.23	1.64	0.95
2. Brachiæus, ...	1.41	3.34	2.07	2.48	1 30	0.73
Adductors.						
1. Coraco-humeralis, . .	16.00	8.79	8.33	2.77	—	—
2. Coraco-capsularis, . .	0.33	0.38	0.50			
3. Pectoralis major—						
(α.) ——.	26.64	27.22	18.13	37.30	20.64	25.60
(β.) ——.	2.22	3.07	2.70			
4. Pectoralis minor, . .	12.52	11.73	13.76			
5. Pectoralis quartus, . .	—	—	—	—	10.39	4.74
	100	**100**	**100**	**100**	**100**	**100**
	Oz. av.	Oz. av.	Oz. av.	Oz. av.	Oz. av.	Oz. av.
Total weight, . .	**105·13**	**24·69**	**162·66**	**20·59**	**18·84**	**24·72**

The relations among the types of these animals may be seen from the following Table, in which the muscles are arranged in groups, as before :—

Muscular Types of Flying and Swimming Animals.

	Scapular.	Abductor.	Flexor.	Extensor.	Adductor.
	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.
1. Pteropus edulis, . .	18.76	18.16	12.98	11.78	38.32
2. Epomophorus Whitii,	22.14	14.29	13.57	9.28	40.72
3. Green Turtle, . .	3.40	16.30	16.92	5.67	57.71
4. Hawk's-Bill Turtle,	6.48	21.81	7.15	13.37	51.19
5. Logger-Head Turtle,	8.78	18.60	20.06	9.14	43.42
6. Alligator, ...	23.90	14.52	16.80	4.71	40.07
7. Otter, ...	36.33	11.00	18.70	2.94	31.03
8. Seal, ...	33.57	10.75	23.66	1.68	30.34

The muscular types of these various animals are, of course, very diverse ; but they all agree in the development of the adductor muscles used in flying or swimming ; and no other animals, except birds, present an approach to this type.

I shall conclude this section on muscular types by a few observations on the "undeveloped" fore-limbs of the Struthionidæ, as I believe that a profound study of the muscular types of these limbs leads us up to the conclusion, that they could never have been intended for wings at all, but are prepared for uses quite distinct from flying.

As the question is one of some theoretical interest, I shall use all the materials at my disposal, and allow my readers to draw their own conclusions from a consideration of all the facts I am able to lay before them.

As it is a well known, although unexplained fact, that the Ostrich and Rhea form a natural group on the one hand, contrasted with the Emu and Cassowary on the other,* I shall discuss the muscular types of their wings separately.

* Among muscular peculiarities not noticed by previous anatomists, I may mention that in the Ostrich and Rhea, the *Obturator internus* muscle is single,

Muscular Type of Wings of Ostrich and Rhea.

	Ostrich, No. 1 (female).	Ostrich, No. 2 (female).	Rhea, No. 1 (male).	Rhea, No. 2 (female).	Rhea, No. 3 (male).
Scapular.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.
1. Trapezius, ...	4.14	3.27	3.50	4.75	2.71
2. Rhomboideus, . . .	2.06	0.54	0.87	1.65	1.93
3. Serratus magnus, . .	1.16	3.93	2.62	2.13	2.97
Abductors.					
1. Deltoideus, ...	33.10	28.21	15.72	15.64	16.91
2. Pectoralis avium, . .	3.53	3.27	16.16	17.77	15.23
3. Teres major, . . .	13.58	16.77	8.73	4.98	8.38
4. Spinatus, ...	3.28	4.47	3.49	7.11	9.94
Flexors.					
1. Triceps longus, . .	2.97	4.47	7.42	4.50	6.33
2. Subscapularis, . . .	12.37	10.78	9.61	10.90	3.09
3. Latissimus dorsi, . .	5.15	4.03	6.55	5.69	3.74
Extensors.					
1. Biceps humeri, . .	3.93	4.58	6.55	6.88	7.35
2. Brachiæus, ...	1.31	1.52		1.18	1.29
Adductors.					
1. Coraco-brachialis, . .	1.41	1.09	6.99	3.32	9.93
2. Pectoralis major, . .	11.46	13.07	11.79	13.50	10.20
3. Pectoralis minor, . .	0.55				
	100	**100**	**100**	**100**	**100**
	Oz. av.	Oz. av.	Oz. av.	Oz. av.	Oz. av.
Total weight, . .	**19·82**	**9·18**	**2·29**	**4·22**	**3·88**

It will be seen from this Table that the Ostrich and Rhea belong to distinct types, well marked, but agreeing with each

as in other birds, and possesses a single tendon only; whereas in the Emu and Cassowary the *Obturator internus* is double, viz.:—

(α.) —— with *puboischial* origin, and inserted an inch higher up than (β) on the femur, so that the tendons cross each other.

(β.) —— with *ilioischial* origin.

These muscles are quite distinct in their actions, and must have some unknown relations to peculiarities in the gait and habit of the Emu and Cassowary, which do not occur in the Ostrich and Rhea. This muscular peculiarity alone places the Emu and Cassowary far remote from all other birds.

other in a manner that shows a wide variation from the type of ordinary wings. This will appear better by arranging the muscles, as before, into groups.

Muscular Type of Wings of Ostrich and Rhea.

	Scapular.	Abductor.	Flexor.	Extensor.	Adductor.
	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.
Ostrich, No. 1, . . .	7.36	53.49	20.49	5.24	13.42
Ostrich, No. 2, . . .	7.74	52.72	19.28	6.10	14.16
Rhea, No. 1, . . .	6.99	44.10	23.58	6.55	18.78
Rhea, No. 2, . . .	8.53	45.50	21.09	8.06	16.82
Rhea, No. 3, . . .	7.61	50.46	13.16	8.64	20.13

On comparing this Table with that already given for common birds, we find the columns of abductor and adductor muscles precisely reversed: so that it may be asserted that the wings of the Ostrich and Rhea are constructed on a type the very opposite of that of the wings of common birds. The wings recorded in the preceding Table are utterly incapable of being used in flight, and it is extremely difficult to understand how they could possibly represent common wings altered from want of use, and so undeveloped.

This argument becomes stronger from an examination of the wings of the Emu and Cassowary, which are formed on a type quite different from that of the Ostrich and Rhea, and equally irreconcileable with the type of ordinary wings. This is shown in the following Table :—

Muscular Type of Wings of Emu and Cassowary.

	Emu, No. 1 (male).	Emu, No. 2 (male).	Cassowary, (male).
Scapular.	Per cent.	Per cent.	Per cent.
1. Trapezius, ...	27.56	26.41	25.74
2. Rhomboideus, ...	12.24	10.38	9.90
3. Serratus magnus, ...	14.28	8.49	14.85
Abductors.			
1. Deltoideus, ...	5.10	7.55	7.92
2. Pectoralis avium, ...	9.19	3.77	14.85
3. Teres major, ...	6.12	6.61	2.97
4. Spinatus, ...	1.02	0.94	—
Flexors.			
1. Triceps longus, ...	5.10	7.55	2.97
2. Subscapularis, ...	1.02	0.94	—
3. Latissimus dorsi, ...	7.15	7.55	10.89
Extensors.			
1. Biceps, 2. Brachiæus, ...	1.02	3.77	1.00
Adductors.			
1. Coraco-brachialis, ...	5.10	8.02	1.98
2. Pectoralis major, 3. Pectoralis minor, . . .	5.10	8.02	6.93
	100	**100**	**100**
	Oz. av.	Oz. av.	Oz. av.
Total weight, . .	**0.98**	**1·06**	**1·01**

On arranging the muscles in groups, as before, we find the following Table :—

Muscular Type of Wings of Emu and Cassowary.

	Scapular.	Abductor.	Flexor.	Extensor.	Adductor.
	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.
Emu, No. 1, . . .	54.08	21.43	13.27	1.02	10.20
Emu, No. 2, . . .	45.28	18.87	16.04	3.77	16.04
Cassowary, . . .	50.49	25.74	13.86	1.00	8.91

The agreement in muscular type here shown between the Emu and Cassowary is very interesting, and corroborates the evidence as to their similarity drawn from other sources; but the muscular type here given is totally different from that of the Ostrich and Rhea, and equally irreconcileable with that of ordinary birds.

From the foregoing facts, I think we are entitled to conclude that the fore limbs of the Struthionidæ were never intended to be used as wings, and that they are constructed for some other purposes, which differ essentially in the two groups, into which these remarkable birds may be divided.

12. **Principle of least Action applied to the Arrangement of the muscular Fibres of the Heart.**—At page 137 and following pages, I have shown the amount of work done by the heart; and I propose here to inquire how that work is done, and to show that it is done in conformity with the principle of "least action."

The heart is composed of innumerable muscular fibres arranged in what Borelli calls a *glomifile* manner, similar to a ball of twine. More correctly, we might compare the heart to two balls of twine, each having a cavity in its centre, and both surrounded by a third ball of twine enveloping the whole structure.

The muscular fibres of the heart are, however, not continuous, as this illustration supposes, but are independent of each other, the appearance of the whole being *glomifile*.

The law of muscular contraction which has to be fulfilled is the following: if l denote the length of a single fibre before contraction, and l' its length after contraction, whenever the muscular fibre is perfectly free, it is found that l' bears a constant ratio to l; or

$$l' = nl. \qquad (109)$$

If, from any cause, the fibre be prevented from contracting to its full extent represented by (109), there is a loss of work which might otherwise be done, and the principle of least action is violated. We may, therefore, state the principle of "least action," as applied to the heart, in the following manner:—

The arrangement of the fibres of the heart must be such as to allow each fibre to contract to the fullest extent required by the law of muscular contraction.

Before proceeding to prove that the actual arrangement of the fibres of the heart is such as will satisfy the foregoing Law, it will be worth while to show by an illustration how easy it would be to devise a heart that would not satisfy the Law, and would, therefore, be an imperfect contrivance.

The special work done by the heart is to contract itself upon an internal cavity containing fluid at a given pressure, and so to expel that fluid from the cavity. Let us imagine that it is proposed to do this work by a number of concentric spheres of muscular fibres placed over each other, the fibres of each sphere being quite independent of those of the other spheres. Let V denote the volume of the ventricles of the heart distended with blood at the commencement of the systole, which volume is reduced to zero at the close of the contraction; let r denote the radius of any sphere of muscular fibres at the commencement, and r' the radius of the same sphere at the end of the systole. It is obvious that r, r' are proportional to l, l', the lengths of the fibres of the sphere in question before and after the systole. Now, as V denotes the difference in volume of the sphere before and after its contraction, we have

$$V = \frac{4\pi}{3}(r^3 - r'^3),$$

$$r' = nr.$$

From these equations, it is easy to see, after a few reductions, that

$$n = \sqrt[3]{1 + \frac{3V}{4\pi r^3}}. \qquad (110)$$

This equation establishes a relation between n and r; that is, between the coefficient of muscular contraction and the radius of the muscular sphere, and shows that if n be taken for the fullest amount of contraction possible to the fibre, the value of r becomes fixed, and there is only one sphere of muscular fibres that does the whole of the work possible for it.

Solving equation (110) for r, we find

$$r = \sqrt[3]{\frac{3V}{4\pi\,(1 - n^3)}}. \qquad (111)$$

As an example of the foregoing we may take the human heart, in which the united volumes of the two ventricles is 10 cubic inches; and assuming that each fibre contracts through one-ninth of its length, we have

$$V = 10,$$
$$n = \frac{8}{9}.$$

Introducing these values into (111), we find

$$r = 2.002 \text{ in.}$$

Hence, in the case supposed, the muscular sphere whose radius is 2 inches, does its full work; the spheres of greater radius do less than their full work, and the spheres of lesser radius are called upon to do more than their full work. In fact, in order to make a heart of the structure supposed a perfect mechanism, it would be necessary to have a different law of muscular contraction for each sphere of fibres; the

amount of contraction n proper to each sphere being determined as a function of r by means of equation (110).

We are familiar, in the construction of large guns, with an instance that may be regarded as the converse of the supposed case of the heart just given. A large piece of ordnance cannot be made strong enough to resist the explosive force of a heavy charge of powder, merely by increasing the thickness of the gun; for according to principles similar to those employed above (using square roots instead of cube roots), it can be shown that the inner rings of the gun are called upon to do more than their share of the work of resistance, while the outer rings are called upon to do less than their share. Hence it follows that the inside portion of a very large gun might be torn up by the force of the explosion before the outside portion had suffered any strain at all. In order to prevent this, and to compel every portion of the gun to take its proper share of the work of resistance, large guns have been made in separate rings, the outermost rings being shrunk on while red hot, so that when cold the inner rings are in a state of compression, and the outer rings are in a state of tension. When the explosion now takes place, the work done by the outer rings is represented by the compression of the inner rings, and these latter take no share in the work of resistance until the force of the explosion has destroyed their state of compression caused by the resistance of the outer rings; and by this ingenious contrivance, an attempt is made to compel every particle of metal of which the gun is composed to bear its own individual fair share of the work to be done.

The celebrated Armstrong 600 pounder gun was formed in this manner of eight distinct rings, and when the charges of powder were gradually increased until the gun was burst, it was found that the 1st, 6th, and 8th rings were burst, the remaining five being apparently uninjured. If the principle

aimed at in the construction of the gun could have been completely fulfilled in practice, all the eight rings would have been ruptured simultaneously when the bursting charge of the gun was reached.

Having shown by the preceding example the conditions required by the principle of "Least Action" when applied to the heart, I shall now describe the actual arrangement of the fibres of the heart, and demonstrate that the law of least action is completely fulfilled by them. The heart has been compared by Borelli and Winslow to two muscles enclosed in a third, and the fibres composing these three muscles may be classified as follows :—

1°. *Common* fibres passing round both ventricles.

2°. *Proper* fibres passing round the left ventricle.

3°. *Proper* fibres passing round the right ventricle.

I shall commence by describing the course of the *common* fibres. The muscular fibres of the auricles and ventricles,

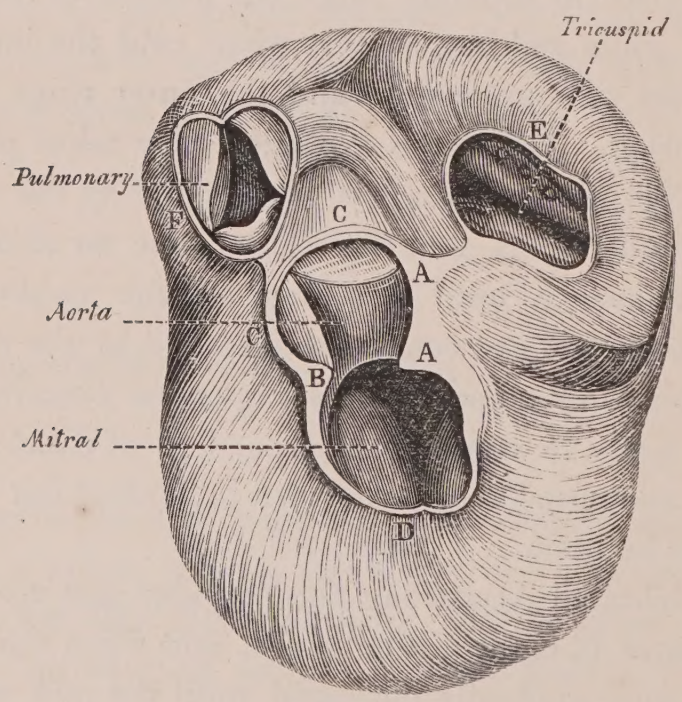


Fig. 100.

forming quite distinct systems, both take origin from the

common tendinous zone which surrounds all the orifices of the heart, with the exception of that of the pulmonary artery. This tendinous zone is shown in Fig. 100.*

In this figure (*AA*) is the great central fibro-cartilage, situated upon the septum with which it is incorporated, between the mitral and tricuspid orifices, and behind the right posterior sinus of the aorta.

The left fibro-cartilage (*B*) which is much smaller than the central fibro-cartilage, is seated to the left of the mitral orifice, and behind the left posterior sinus, with which it is closely attached.

At (*CC*) is shown the strong fibrous loop situated in front and to the left of the root of the aorta, stretching from the left to the central fibro-cartilage, with both of which it is intimately incorporated.

At *D* is shown the tendinous ring of the mitral orifice, stretching from the left to the central fibro-cartilage.

The tendinous ring of the tricuspid orifice is shown at *E*, attached at both its extremities to the central fibro-cartilage.

Lastly, at *F* is shown the insular fibrous ring that surrounds the orifice of the pulmonary artery; and is but slightly connected with the central mass of fibro-cartilaginous structure.

All the muscular fibres of the ventricles take origin from the tendinous zone, and having wound round the heart in the manner presently to be described, either return directly to the tendinous zone, or indirectly through the intervention of the *columnæ carneæ* and *papillary muscles*.

The common fibres of the heart arise from the outer side of the tendinous ring, and from the outer surface of the heart, as shown in Fig. 101,† which represents the posterior and anterior aspects of the dissected heart. The general

* Copied by permission from Dr. Sibson's Medical Anatomy.

† This figure is taken, by permission, from Dr. Sibson's Medical Anatomy.

appearance of the bundles of common fibres is well compared by Dr. Sibson to the overlapping capes of a coachman's great coat when twisted. As each layer proceeds from right to left, it twists upon itself and at length dips underneath the layers to which it was superficial at the beginning of its course.

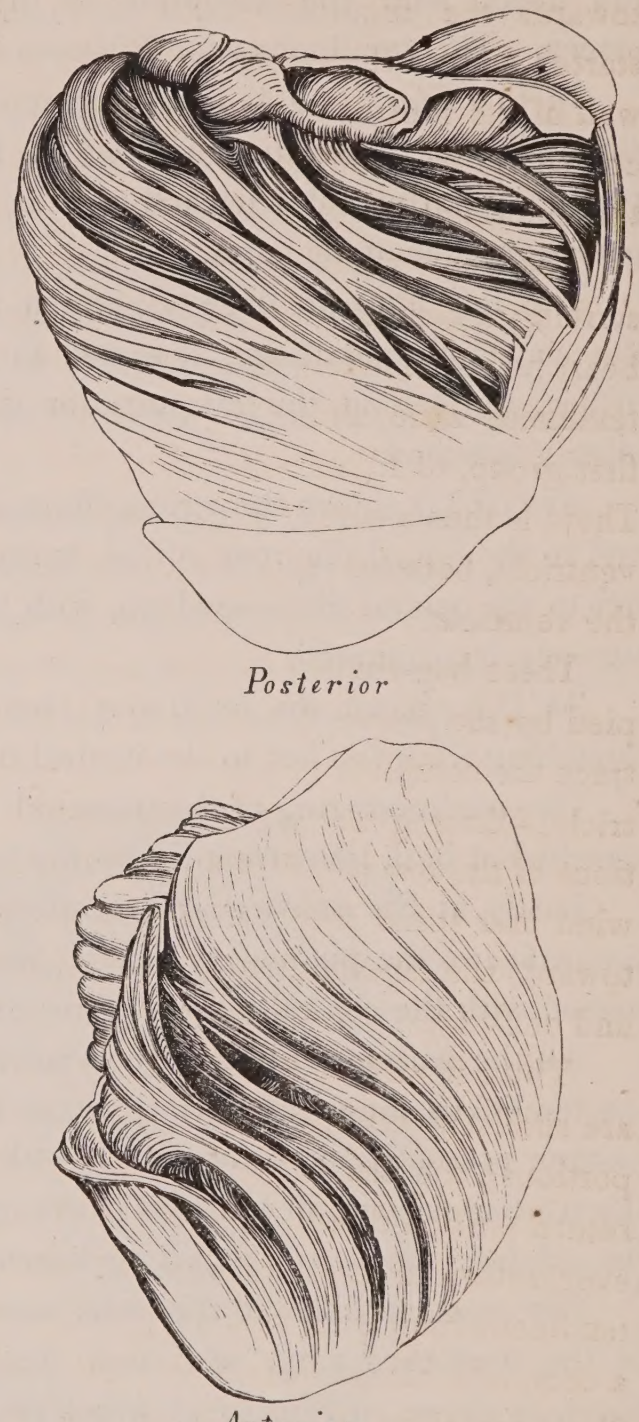


Fig. 101.

The following observations and measurements were made on the heart of the ox, several preparations of which were placed at my disposal by Dr. Sibson, and several others made by myself were examined while fresh after long continued boiling. On tracing the course of the superficial common fibres, it is found that they describe a complete circumference in a spiral line previous to entering deeper into the heart in the neighbourhood of the

vortex, from which point they return almost in a straight line towards the tendinous zone from which they originally started, but on the return path they now form the innermost wall of the ventricle; so that these fibres during their spiral course are the outermost, and during their direct course are the innermost fibres of the heart.

The fibres which lie inside the superficial fibres, after describing a complete spiral, plunge into the interior of the heart before reaching the vortex, and return back towards the tendinous zone almost in a straight line, lying outside the first group of fibres which line the cavity of the ventricle. There is thus a sort of bag-shaped cavity formed round each ventricle, between the outside of the heart and the walls of the ventricle.

These bag-shaped cavities are now to be supposed occupied by the *proper* fibres of each ventricle. I was able to trace the complete course of the *proper* fibres of the left ventricle, filling up the space between the spiral and direct portions of the *common* fibres, and I found that the proper fibres wind in a spiral line round the ventricle, and return back towards the tendinous zone, after describing a circumference and one-fifth.

Thus, it will be seen that the *common* and *proper* fibres are similarly arranged, and may be divided each into two portions; the first portion spiral and *outer*, and the second or return portion direct and *inner*. The common fibres, however, return to the same azimuth on the circumference of the tendinous ring, from which they started, after having described a complete circumference in their course; while the proper fibres return to an azimuth on the tendinous ring 72° in advance of the azimuth from which they started, after having described a complete circumference and one-fifth in their course.

It is easy to see that as the common fibres describe a

complete circumference, the transverse components of the force of contraction will equilibrate each other, and thus no motion of rotation is given to the heart by these fibres; on the other hand, the transverse components of the force of contraction of the proper fibres are unsymmetrical, and they will give a resultant twisting motion to the cavity of the ventricle, the effect of which will be, to wring out completely every drop of blood from the ventricle at the close of the systole.

Having ascertained the arrangement of the several varieties of fibres, I dissected them out carefully, and measured their lengths. On doing so, it became apparent that the common fibres were equal to each other in length, within the errors of observation; and also that the proper fibres were equal to each other in length, this length being distinct from the length of the common fibres. This result is caused by the peculiar shape of the heart,* which compels the fibres that enter the interior above the vortex to describe their spiral circumference on a more equatorial region of the heart than the apex; so that although the return path of these fibres is shorter than that of the superficial fibres that enter the vortex, their total lengths remain the same.

From the preceding statement, it appears that each fibre is independent of the others, and is at full liberty to contract to the utmost extent required by the law of muscular contraction, and thus the law of least action is fulfilled by the common fibres and proper fibres respectively, each fibre belonging to either group having the same length. But, in order that the two groups of fibres, working simultaneously, may produce the maximum of work required by the law of least action, it is necessary that a certain relation should exist between the

* Or, to speak more correctly, the shape of the heart is the necessary consequence of the law of least action, which requires the fibres of the same group to have the same length.

respective lengths of the two groups of fibres. This relation may be investigated as follows:—

Let L denote the length of the common fibres passing round both ventricles, and let the volumes of the left and right ventricle be denoted by λ and ρ.

Let l denote the length of the proper fibres that pass round the left ventricle only.

Let the fibres L and l, become L' and l' when contracted; then it is plain that we have

$$\frac{L^3 - L'^3}{l^3 - l'^3} = \frac{\lambda + \rho}{\lambda}. \qquad (112)$$

If n denote the coefficient of muscular contraction, and if it be supposed to be the same for both groups of fibres, then equation (112) becomes

$$\frac{L^3(1 - n^3)}{l^3(1 - n^3)} = \frac{\lambda + \rho}{\lambda};$$

or

$$\frac{L^3}{l^3} = \frac{\lambda + \rho}{\lambda}. \qquad (113)$$

As the left-hand side of this equation depends on measurements of the lengths of the two systems of fibres, while the right-hand side depends on measurements of the volumes of the two ventricles, it is plain that it affords us a test, within limits of errors of observation, of whether the principle of least action applies in all its force to the theory of the action of the heart.

As the lengths of the fibres and sizes of the hearts examined differed considerably, I selected a large and small heart for measurement, with the following results:—

	No. 1.	No. 2.
Length of common fibres,	13.25 in.	8.5 in.
Length of proper fibres,	10.75 ,,	6.5 ,,

Hence we find

$$\frac{L^3}{l^3}\,(\text{No. 1.}) = \left(\frac{13{\cdot}25}{10{\cdot}75}\right)^3 = 1.873$$

$$\frac{L^3}{l^3}\,(\text{No. 2.}) = \left(\frac{8{\cdot}5}{6{\cdot}5}\right)^3 = 2.236$$

Mean, . **2.055**

Brown, from observations made on the heart of the Ox, states that the right ventricle exceeds the left in the proportion of 110 to 100, which would make

$$\frac{\lambda + \rho}{\lambda} = 2.10.$$

The following observations have been made by anatomists on the relative volumes of the two ventricles in the human heart:—

Human Heart.	$\frac{\lambda+\rho}{\lambda}$.
1. Lower, ...	2.00
2. Sabatier, ...	2.00
3. Andral, ...	2.00
4. Gordon, ...	2.25
5. Cruveilhier, ...	2.00 (less than.)
6. Portal, ...	2.40
7. Sibson, ...	2.00
8. Quain, ...	2.00 (greater than.)
9. Lieberkuhn, ...	2.50
Mean, . . .	**2.128**

There are good physiological grounds for believing that, during life and action, the volumes of the two cavities are

equal in a healthy heart ; and that the slight preponderance in favour of the right ventricle found by anatomists depends on the lesser thickness of its walls, which causes it to distend more than the left ventricle, under the hydrostatical pressure to which it is subjected by the anatomical observer. If this be so, then the principle of least action applied to the heart leads to the remarkable prediction, that the lengths of the common fibres ought to bear to the lengths of the proper fibres the ratio of the cube root of 2 to unity,

$$\frac{L}{l} = \sqrt[3]{2}. \qquad (114)$$

Assuming this Law to be correct, we can calculate from the observed lengths of the common fibres, measured in our two hearts, what the length of the proper fibres ought to be, and so compare our observations with theory. I find the following results :—

Comparison of Observed and Calculated lengths of Muscular Fibres in the Heart of the Ox.

	Common Fibre.	Proper Fibre (observed).	Proper Fibre (calculated).	Difference.
No. 1,	13.25 in.	10.75 in.	10.52 in.	+ 0.23 in.
No. 2,	8.50 „	6.50 „	6.74 „	− 0.24 „

These differences are less than a quarter of an inch, and will be readily admitted by practical anatomists to be well inside the unavoidable errors of observation.

If there be any Naturalist who thinks it possible that a mechanism such as I have here described could grow up of itself, from chance combinations of fibres developed in a

long lapse of time, and without the aid of a Contriving Mind, I would ask him to study the remarkable words of Cicero:—

"Nihil est enim, quod ratione et numero moveri possit, sine consilio; in quo nihil est temerarium, nihil varium, nihil fortuitum."

13. **Applications of General Laws of Muscular Action.**—I shall conclude my observations on Animal Mechanics by laying down in this section a few General Laws, and showing the inferences that may be deduced from them. In drawing these inferences, I have the advantage of being able to make use of the experiments of Mr. W. Stanley Jevons,* of Manchester, and of Mr. Frank E. Nipher,† of Iowa University, U. S. A., in addition to my own experiments and observations; and I believe it will be admitted, from the results of the investigations I have been able to make, that a great future in the way of useful practical applications lies open to the Science of Animal Mechanics.

I have been led to the establishment of the three following Laws, the proofs of which I shall give in detail:—

LAW I.—*In comparing together different muscles, the work done in contracting is proportional to the weight of each.*

LAW II.—*In comparing the same muscle (or group of muscles) with itself, when contracting under different external conditions, the work done is always constant in a single contraction.*

LAW III.—*When the same muscle (or group of muscles) is kept in constant action until fatigue sets in, the total work done, multiplied by the rate of work, is constant.*

I shall now take these Laws in order, and show the evidence on which each is based.

* *Nature*, 30th June, 1870, page 158.

† *School Laboratory*, September and December, 1871, Iowa City, page 108.

LAW I. *Work proportional to weight.*—The germ of this Law is to be found in Borelli's great book, Vol. I., Props. cxxi., cxxii., cxxiii. (pp. 209–211).

I here give the statements of Borelli's Propositions.

PROPOSITION CXXI.

Si duo musculi ejusdem animalis fuerint æque crassi, scilicet compositi ex pari multitudine fibrarum, sed earum longitudines inæquales fuerint; suspendent quidem æqualia pondera : at potentiæ motivæ, et altitudines suspensionum eandem proportionem habebunt, quam longitudines musculorum.

PROPOSITION CXXII.

Si vero, altitudines musculorum fuerint æquales, et crassities eorum inæquales : potentiæ motivæ, et pondera suspensa, proportionalia erunt crassitiebus musculorum ; at pondera ad æquales altitudines ascendent.

PROPOSITION CXXIII.

At, si tam altitudines, quam crassities musculorum inæquales fuerint, pondera suspensa erunt, ut crassities musculorum ; altitudines elevationum eorum ut longitudines musculorum; at potentiæ compositam proportionem habebunt ex ratione crassitierum et longitudinum.

If in these Propositions we replace the phrase *potentia motiva* by its modern equivalent, *Work done*, we obtain the following statement :—

The Work done by the contraction of a muscle is proportional to its length and area of cross section jointly.

But it is evident, *cæteris paribus*, that the weight of a muscle is proportional to the product of its length and cross section ; so that Law I. may be fairly called Borelli's Law ; although he failed to see the important consequences that

flow from it. In my section on Muscular Types I have endeavoured to show what valuable applications may be made of this Law by the Comparative Anatomist, when combined with the Law of Least Action applied to muscular structure and arrangements.

Law II. *Constancy of Work done.*—The second Law may be described as the law of constancy of work done by a group of muscles, in a single contraction, performed under different external conditions.

The admirable experiments published by Mr. W. Stanley Jevons in *Nature* (30 June, 1870) enable me to demonstrate the truth of this Law in a satisfactory manner.

Mr. Jevons ascertained by numerous experiments the comparative distances to which various weights could be thrown by hand, on level ground. The weights employed ranged from ½ lb. up to 56 lbs., and were thrown as nearly as possible in a uniform manner, and at the most advantageous angle of projection. About 57 experiments at different times were made with each weight, or 456 experiments in all; and it was quite obvious that good average results were obtained, the correspondence of different sets being very satisfactory. The following results were obtained :—

Mr. Jevons' Experiments (Throwing Weights).

Weight in lbs.	Average Distance thrown in feet.	Calculated.	Difference.
56	1.84	1.93	+ 0.09
28	3.70	3.63	− 0.07
14	6.86	6.46	− 0.40
7	10.56	10.61	+ 0 05
4	14.61	14.65	+ 0.04
2	18.65	19.61	+ 0.96
1	23.05	23.61	+ 0.56
½	27.15	26.30	− 0.85

The calculated distances contained in the foregoing Table were obtained by Mr. Jevons, by assuming the following empirical Law :

$$x = \frac{p}{w + q}, \qquad (115)$$

where

x = distance thrown,

w = weight thrown,

p, q, unknown constants to be determined.

The experiments gave Mr. Jevons eight distinct equations by which to determine the two unknown quantities p and q ; and by means of the method of least squares, Mr. Jevons found their most probable values to be

$$p = 115{\cdot}7$$
$$q = 3{\cdot}9$$

Mr. Jevons correctly stated that the agreement between observation and calculation is so close as to prove the accuracy of the empirical law (115), but he professes his inability to explain it on mechanical principles. I subsequently succeeded in doing so, and published my results in *Nature*. My investigation is as follows :—

In throwing weights by the hand in the manner described by Mr. Jevons, the arm, after a little practice, instinctively pitches the weight at the angle (45°) corresponding to the maximum range (x), and as the maximum range is proportional to the square of the velocity of projection, it may be used to replace that quantity, in estimating the work done by the arm. The total work done by the arm is the same as if the weight used and the weight of the arm were concentrated at the centre of Gyration of the loaded arm, regarded as a compound pendulum.

Let us assume

w = weight held in hand,
a = weight of arm,
v = velocity of centre of Gyration.

By the law of *Constancy of Work*, we have

$$(w + a)\, v^2 = \text{constant.} \qquad (116)$$

Let

V = velocity of the hand,
k = radius of gyration,
a = length of the arm.

We now have the additional equations,

$$v = V\frac{k}{a}, \qquad (117)$$

and since the arm may be assumed to be approximately cylindrical, we have

$$\left(w + \frac{a}{3}\right) a^2 = (w + a)k^2,$$

or,

$$\frac{k^2}{a^2} = \frac{w + \frac{a}{3}}{w + a}. \qquad (118)$$

Substituting from (117) and (118) in equation (116), we find

$$V^2\left(w + \frac{a}{3}\right) = \text{Const.}:$$

and finally, writing x for V^2, as before explained, we have

$$x = \frac{p}{w + \frac{a}{3}}. \qquad (119)$$

This equation is identical with the empirical formula (115) employed by Mr. Jevons, and shows that his constant, q,

represents one-third of the weight of the moving arm expressed in pounds. The weight of Mr. Jevons' arm, including the scapula and its muscles actually in motion, is therefore 11.7 lbs.

The remarkable agreement between the observations and equations (115) and (119) which may be derived directly, as I have shown, from the Law of constancy of work done in a single contraction of a group of muscles, proves completely the accuracy of that Law.

Although the work done by the contraction of the muscles is constant, the *useful work done* is not constant, and it may be worth while to inquire how it varies, and whether it admits of a maximum, a circumstance which would have an important practical significance.

Using the same notation as before

$$\text{The useful work done} = wx = \eta.$$
$$\text{The weight thrown} \quad = w = \xi.$$

If ξ and η be used to designate the abscissa and ordinate of a curve, by constructing this curve of useful work done, we may ascertain whether its ordinates admit of a maximum or not. Multiplying Mr. Jevons' equation (115) by w, we find

$$wx = \frac{pw}{w + q}$$

or

$$\eta = \frac{p\xi}{\xi + q}. \qquad (120)$$

This represents an equilateral hyperbola referred to axes parallel to its asymptotes, and intersecting on the curve itself.

In Fig. (102) I have constructed from calculation the curve of useful effect, which is an equilateral hyperbola, having OX and OY for asymptotes.

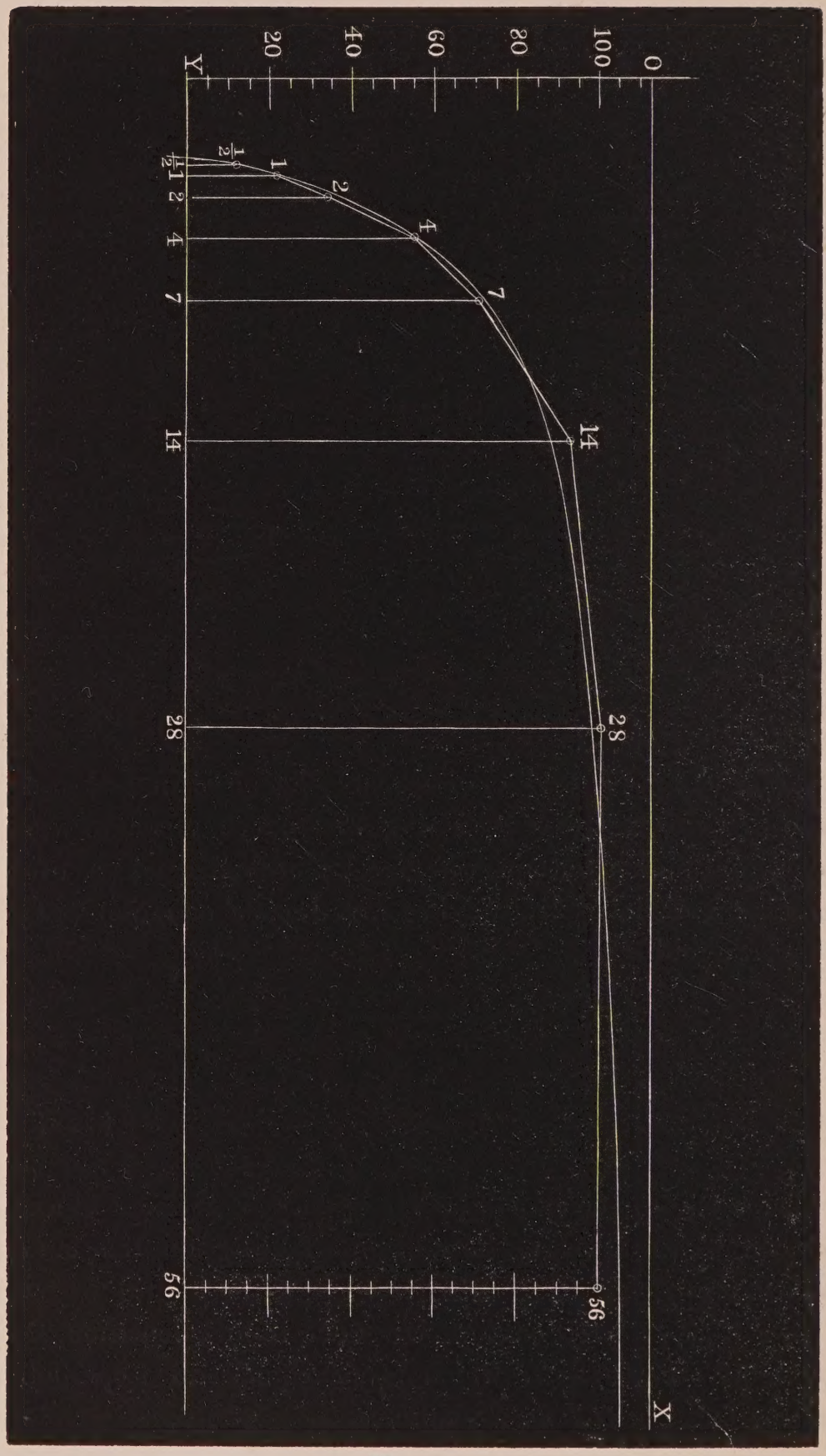


Fig. 102.

Fig. 102. Curve of useful work, $\eta = \dfrac{p\xi}{\xi + q}$ $\begin{matrix} p = 115{\cdot}7 \\ q = 3{\cdot}9 \end{matrix}$

and have shown the actual useful effects of each weight, by means of the centres of the small circles corresponding to each weight. This Figure conveys to the eye a complete idea of the accuracy of the Law of Constancy of Work, and of its agreement with observation.

The upper line in the Figure shows the horizontal asymptote, towards which the ordinate continually approaches, and both the curve itself and the observations show that the useful effect increases with the weight employed.

Equation (116) may be thus written,

$$wv^2 + av^2 = \text{constant};$$

in which wv^2 represents the *useful work*, and av^2 the work done by the arm; in this equation, a is a constant, being the weight of the arm, and v grows less as the weight thrown increases; hence the work expended on the arm is less as the weight is greater, and therefore the *useful work* must increase continually with the weight, and reaches a maximum when the weight is infinite.

In Figure 102, the useful work is represented by the ordinate of the hyperbola. and the work done on the arm is represented by the intercept of the ordinate between the curve and its asymptote, which diminishes continually as the weight (abscissa) increases.

The coordinates of the centre of the hyperbola (120) are

$$\alpha = -q,$$
$$\beta = p,$$

and the equation of the curve, referred to its asymptotes, is

$$\xi\eta + pq = 0. \qquad (121)$$

From the preceding data, we may calculate the maximum velocity which the muscles employed as in the experiments are capable of giving to the hand. For, since the work done on the arm is a maximum when $w = 0$, we find from equation (115) the maximum range of an infinitely small weight,

$$x = \frac{p}{q} = \frac{115 \cdot 7}{3 \cdot 9} = 29\tfrac{2}{3} \text{ feet.}$$

Now, it is well known that the velocity of projection corresponding to the maximum range on a horizontal plane is

$$V = \sqrt{gx}, \qquad (122)$$

from which we find

$$V = 30 \cdot 9 \text{ feet per second,}$$

showing that the muscles in question can produce on the hand a velocity nearly equal to that of a body which has fallen for one second.

LAW III. *The Law of Fatigue.*—The third Law of Muscular Action states that the *total work done* multiplied by the *rate of work* is constant;—and as soon as this product attains its constant value, fatigue sets in, and the muscles become incapable of giving out more work, until they have been allowed to rest. Hence, the Third Law may be appropriately termed the *Law of Fatigue.*

First Illustration of Law of Fatigue.—I shall take my first proof of this Law from Mr. Jevons' experiments.* Mr. Jevons raised and lowered various weights by a pulley and cord through the convenient range of the arm, continuing the motion with unrelaxed rapidity until the power of the muscles was entirely exhausted. From these experiments, he found the following results:—

* *Nature*, 30th June, 1870, p. 159.

Mr. Jevons' Experiments (lifting Weights with Pulley).

Weight lifted.	No. of times lifted.	Useful Work.
56 lbs.	5.7	319.2
42 „	11.9	499.8
28 „	23.0	644.0
21 „	37.6	789.6
14 „	111.0	1554.0

From the manner in which these experiments were made, it is easy to see that the weight of the arm does not enter into the result. Let

w = weight raised;

n = number of times it is raised, carried to point of fatigue of muscles;

h = height through which weight is raised;

t = time occupied by a single lift.

Hence we have

$$\text{Total work done} = whn,$$

$$\text{Rate of work} = \frac{wh}{t}.$$

The rate of work is found by dividing the work done in a given time by the time itself. Hence we find by the Law of Fatigue

$$\frac{w^2h^2n}{t} = \text{constant}; \qquad (123)$$

or, since h and t are constant,

$$w^2n = A. \qquad (124)$$

If we write

$$\xi = w,$$

$$\eta = n,$$

we have

$$\xi^2\eta = A, \qquad (125)$$

which represents a cubical hyperbola.

If we write

$$\xi = w,$$
$$\eta = wn \text{ (useful effect)},$$

we find

$$\xi\eta = A, \qquad (126)$$

which represents an equilateral hyperbola.

If we write $A = 19600$, in equation (124), we can calculate the following values of n, and compare them with the observed values:—

Mr. Jevons' Experiments (lifting Weights with Pulley).

w.	n (observed).	n (calculated).	Difference.
56	5.7	6.2	− 0.5
42	11.9	11.1	+ 0.8
28	23.0	25.0	− 2.0
21	37.6	44.3	− 6.7
14	111.0	100.0	+ 11.0

Cubical hyperbola, $w^2n = 19600$.

In Fig. 103, I have constructed the cubical hyperbola,

$$w^2n = 19600,$$

showing both its asymptotes, and marking with small circles the observations of Mr. Jevons; the agreement between calculation and observation is sufficiently close for thi sclass of experiments.

The relation between the useful effect and weight lifted is represented by an equilateral hyperbola (126), and the useful

effect diminishes as the weight increases, and becomes infinite when the weight is zero. There is, therefore, no true maximum of effect in these experiments.

Second Illustration of Law of Fatigue.—I shall take my second illustration of the Law of Fatigue from experiments similar to those described (pp. 24–44), from which I obtained

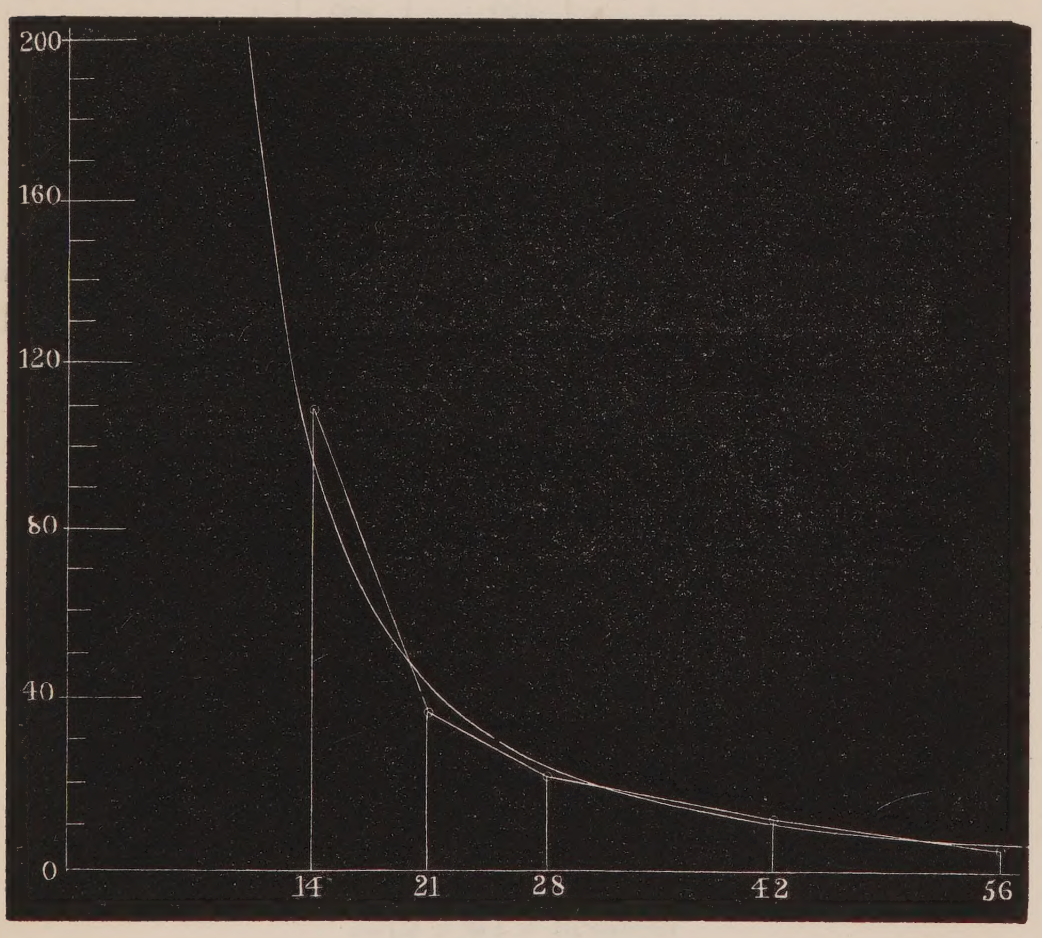


Fig. 103.

my first view of the Law. The experiments consist in holding out the arm, horizontally, either unloaded, or carrying weights in the palm of the hand, held upwards, and noting the time during which the arm could be held out horizontally. The following results are the means of many observations made on myself, and include those before described :—

Dr. Haughton's Experiments (holding Arms horizontal).

Weight.	Time.	Useful Effect.
0 lbs.	420 secs.	0
1 ,,	275 ,,	275
2 ,,	200 ,,	400
3 ,,	147 ,,	441
4·5 ,,	101 ,,	454·5
7 ,,	62 ,,	434
11 ,,	33 ,,	363
14 ,,	23 ,,	322

The useful effect is estimated by taking the product of the weight and the time during which it is held in the horizontal position.

Using the notation employed in p. 25 ; let

w = weight held in hand.

a = weight of arm.

t = time of holding arm horizontal, until fatigue sets in.

x = distance of centre of gravity of loaded or unloaded arm from centre of glenoid cavity.

ω = the unknown angular velocity.

The moment of the weights equilibrated by the action of the muscles is

$$\text{moment} = (w + a)\, x.$$

The total work done will be proportional to this moment multiplied by the time.

Hence

$$\text{Total work done} = \omega\,(w + a)\, xt,$$

$$\text{Rate of work} = \omega\,(w + a)\, x.$$

Hence, by the Law of Fatigue,

$$\omega^2\,(w + a)^2\, x^2 t = \text{const.} \qquad (127)$$

Now, we have

$$(w + a)\,x = wa + a\,\frac{a}{2},$$

where a is the length of the arm, supposed to be an uniform cylinder; therefore

$$(w + a)\,x = a\left(w + \frac{a}{2}\right),$$

substituting in equation (127), we obtain (assuming ω^2 to be an unknown constant)

$$\left(w + \frac{a}{2}\right)^2 t = A. \qquad (128)$$

This equation represents a cubical hyperbola, whose asymptotes are

$$t = 0. \qquad (129)$$

$$w + \frac{a}{2} = 0.$$

Solving equation (128) for t, we find

$$t = \frac{A}{\left(w + \frac{a}{2}\right)^2}. \qquad (130)$$

The useful effect is found by multiplying (130) by w, and is

$$\text{Useful effect} = wt = \frac{Aw}{\left(w + \frac{a}{2}\right)^2}. \qquad (131)$$

This equation represents a cuspidal cubic, whose shape I shall presently describe.

I find the following values of A and a to give me the best agreement between theory and observation, viz.:—

$$A = 7730.$$
$$a = 8.5 \text{ lbs.}$$

Using these values I find the following comparison of theory and observation:—

No. 1.—*Dr. Haughton's Experiments.*—(*Time of holding Arm horizontal*).

w.	*t* (observed).	*t* (calculated).	Difference.
0	420 secs.	427 secs.	− 7
1	275 ,,	280 ,,	− 5
2	200 ,,	198 ,,	+ 2
3	147 ,,	147 ,,	0
4.5	101 ,,	98 ,,	+ 3
7	62 ,,	61.4 ,,	+ 0 6
11	33 ,,	31.6 ,,	+ 1.4
14	23 ,,	23.3 ,,	− 0 3

No. 2.—*Dr. Haughton's Experiments.*—(*Useful Effect*).—(*Holding Arm horizontal*).

w.	Useful Effect observed.	Useful Effect calculated.	Difference.
0	0	0	0
1	275	280	− 5
2	400	395.5	+ 4.5
3	441	441	0
4.5	454.5	454	+ 0 5
7	434	430	+ 4
11	363	366	− 3
14	322	326	− 4

The cubical hyperbola (130) is shown, drawn to scale in Fig. (104); its asymptotes (129) being OX and OY; and the small circles as before denoting the actual observations.

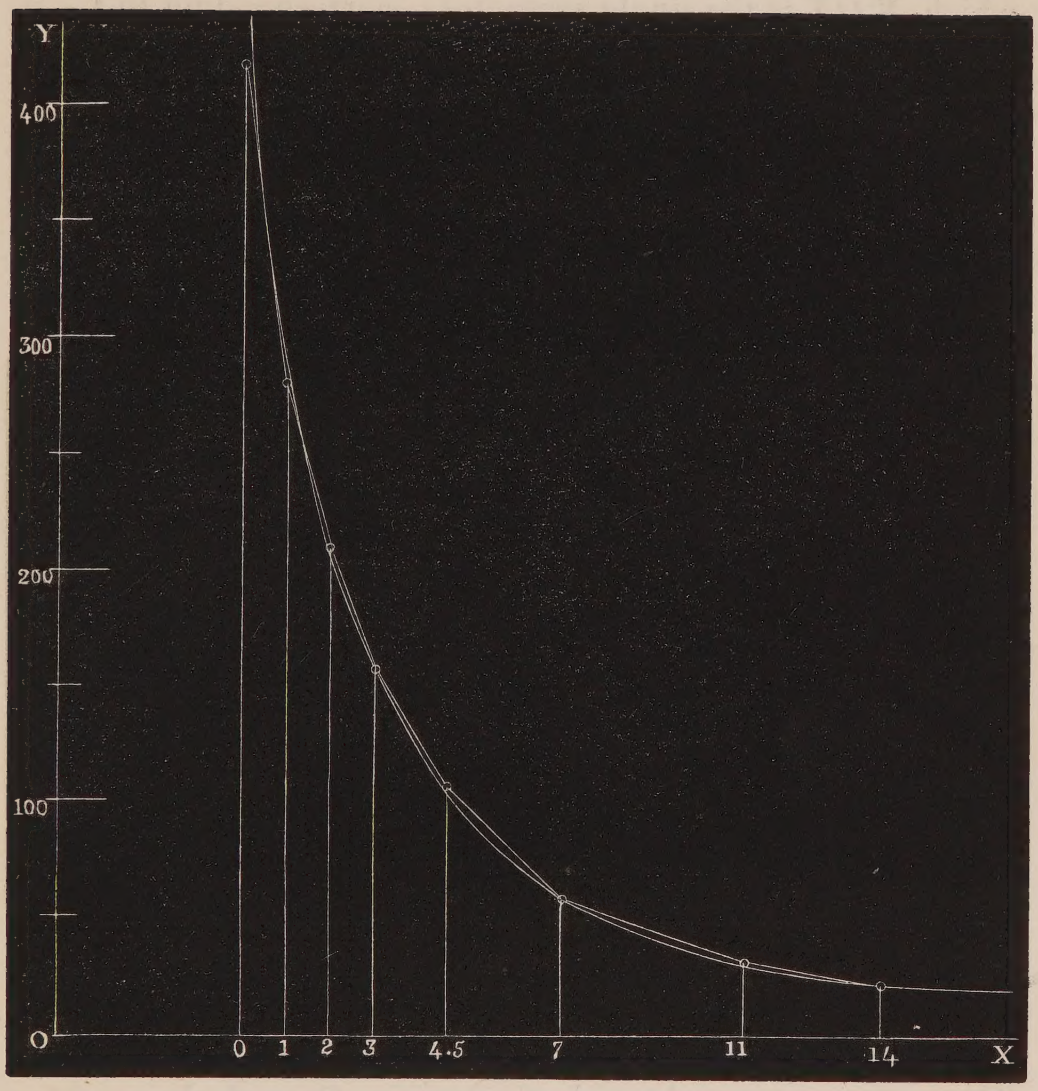


Fig. 104.

Fig. 104.—*Dr. Haughton's Experiments—(Arms horizontal).*

$$\left\{ \begin{array}{c} \text{Cubical Hyperbola } t = \dfrac{A}{\left(w + \dfrac{a}{2}\right)^2}. \\ \\ A = 7730. \quad a = 8.5 \text{ lbs.} \end{array} \right\}$$

The general form of the cuspidal cubic (131) whose ordinates represent the useful effect, is shown in Fig. (105), in

which $X'OX$ is the single asymptote, corresponding to $t = 0$, and ApA' is the double asymptote corresponding to

$$w + \frac{a}{2} = 0,$$

and having a cusp at negative infinity.

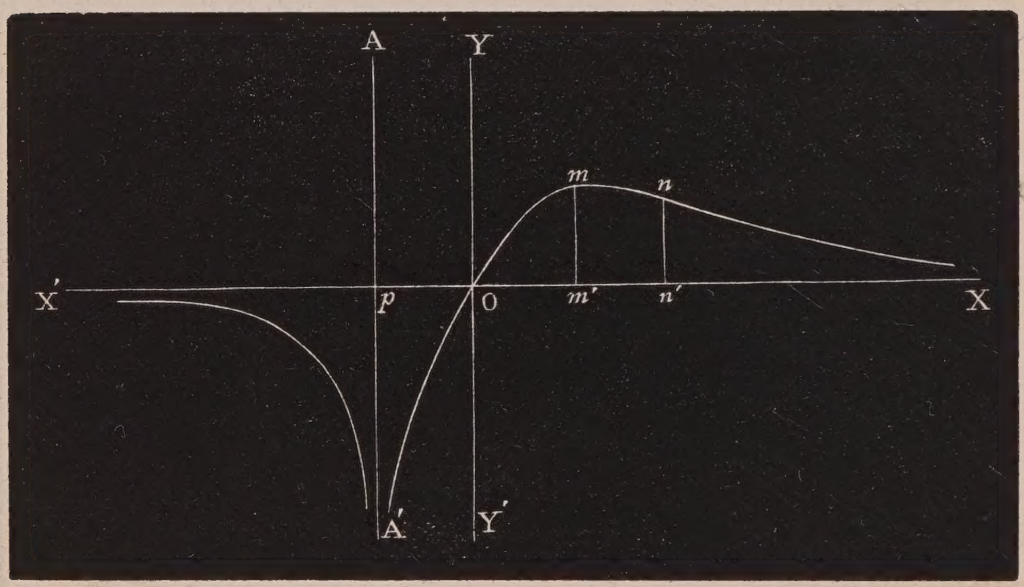


Fig. 105.

There is an hyperbolic branch lying between pX' and pA'. The curve passes through the origin O, and the positive weights are measured on OX, while negative weights are measured on OX'. The asymptote AA' lies at a distance from YY' equal to $\frac{a}{2}$; which signifies that if the hand were lifted by a weight equal to that of half the arm, it could be held out for an infinite time, without being tired, as is evident.

The portion of the curve with which we are concerned lies between OX and OY.

The ordinate (or useful effect) attains a maximum mm', when the weight Om' is equal to half the weight of the arm; and the point n corresponding to On' equal to the weight of

the arm, is a point of inflexion on the curve where the curvature changes sign, and the curve becomes asymptotic to the line OX.

The following are the values of the abscissæ and ordinates at these important points:—

$$Om' = \frac{a}{2} \qquad On' = a$$

$$mm' = \frac{A}{2a} \qquad nn' = \frac{4A}{9a} \tag{132}$$

The tangent drawn at the point of inflexion cuts off an abscissa on the axis OX equal to $4a$. The readiest mode of approximating to the best numerical values of A and a, is by comparing the co-ordinates of the maximum effect (m) with the equations (132).

In Fig. (106) I have drawn, on a larger scale, the portion of the cuspidal cubic (131) corresponding to positive values of w and t, and have also shown the actual observations by means of the small circles. The agreement between theory and observation is very satisfactory.

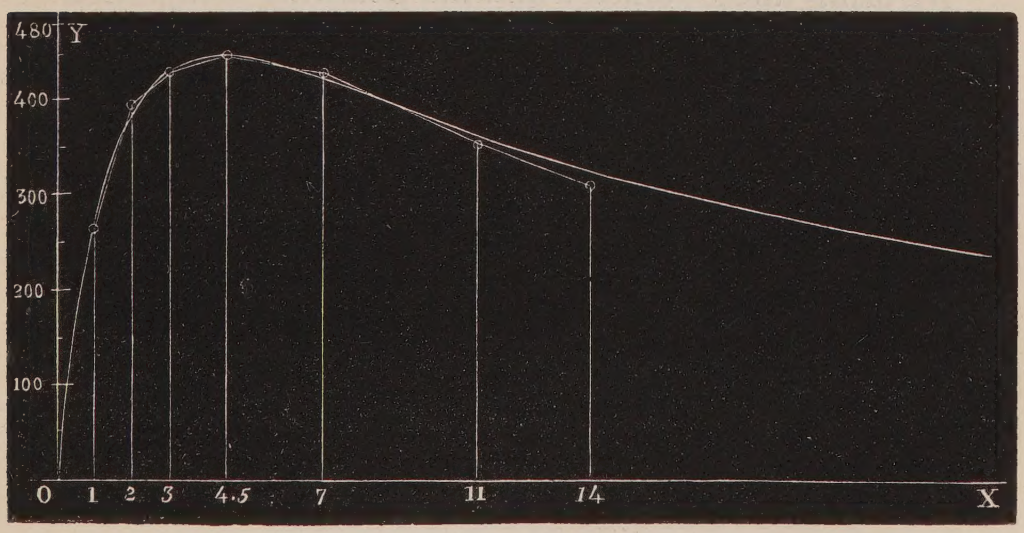


Fig. 106.

Fig. 106.—*Dr. Haughton's Experiments—(Holding Arm horizontal)—Useful Effect.*

$$\left\{ wt = \frac{Aw}{\left(w + \frac{a}{2}\right)^2} \qquad \begin{matrix} A = 7730. \\ a = 8.5 \text{ lbs.} \end{matrix} \right\}$$

A valuable series of observations of the same kind have been made by Mr. Jevons,* of which the following are the results :—

Mr. Jevons' Experiments—(Holding Arms horizontal).

Weight.	Time.	Useful Effect.
1 lb	321 2 secs.	321
2 ,,	218.9 ,,	438
4 ,,	147.9 ,,	592
7 ,,	87.4 ,,	612
10 ,,	60 3 ,,	603
14 ,,	32.5 ,,	455

I found equations (130) and (131) to represent the time useful effect best, when

$$A = 13340.$$
$$a = 10.9 \text{ lbs.}$$

By using these values I find the following comparison of theory and observation.

No. 1.—*Mr. Jevons' Experiments—(Time of holding Arm horizontal).*

w.	t (observed).	t (calculated).	Difference.
1	321 2 secs.	320.7 secs.	+ 0.5
2	218.9 ,,	240.3 ,,	- 21.4
4	147.9 ,,	149.4 ,,	− 1.5
7	87.4 ,,	86.1 ,,	+ 1.3
10	60.3 ,,	55.9 ,,	+ 4 4
14	32.5 ,,	35.2 ,,	− 2.7

* *Nature*, June 30, 1870, page 158.

No. 2.—*Mr. Jevons' Experiments—(Useful Effect)—(Holding Arms horizontal).*

w	Useful Effect (observed).	Useful Effect (calculated).	Difference.
1	321	320.7	+ 0.3
2	438	480.7	− 42.7
4	592	597·5	− 5·5
7	612	602.5	+ 9·5
10	603	558.9	+ 44.1
14	455	493 7	− 38.7

The maximum useful effect corresponds to a weight 5·45 lbs., equal to half the weight of the arm, and its value is (132)

$$\text{Maximum useful effect} = \frac{A}{2a} = 612.$$

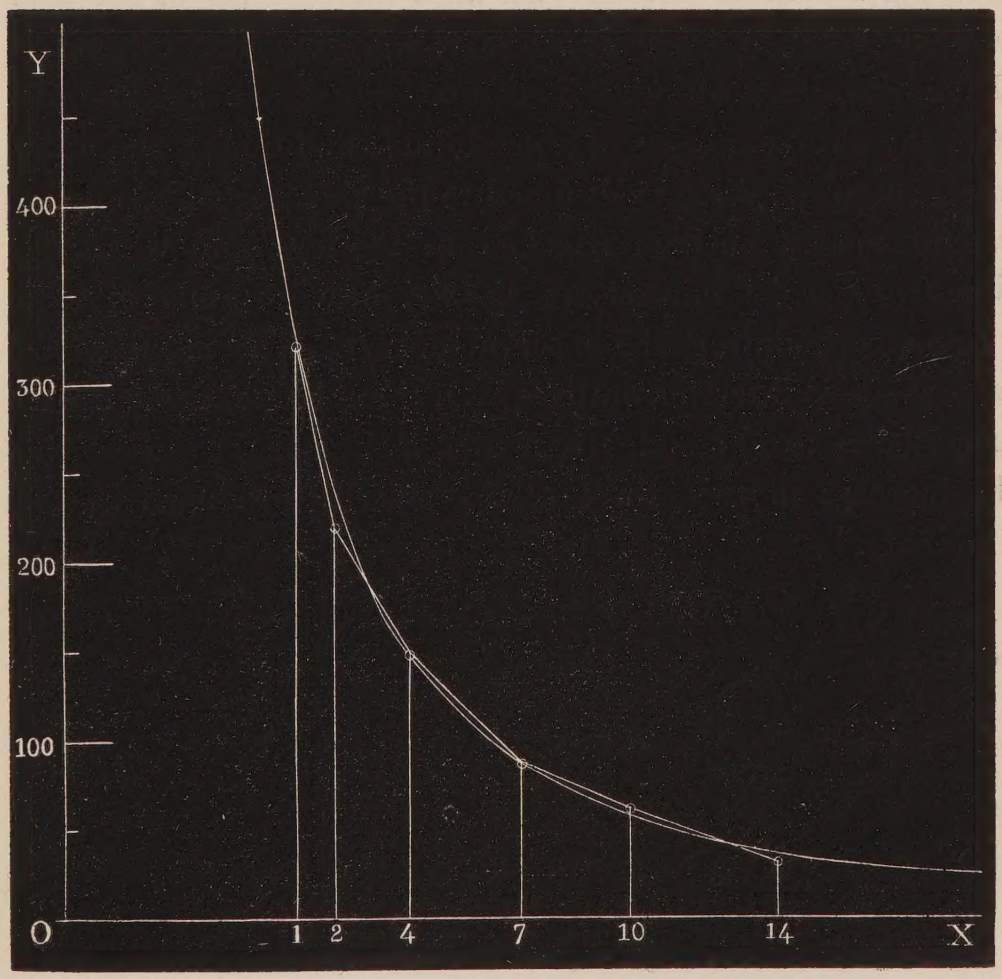


Fig. 107.

In order to show the agreement of theory and observation, I have drawn in Fig. (107) the cubical hyperbola (130), and have represented Mr. Jevons' observations with small circles.

Fig. 107.—*Mr. Jevons' Experiments—(Holding Arms horizontal).*

$$\left\{ t = \frac{A}{\left(w + \frac{a}{2}\right)^2} \qquad \begin{matrix} A = 13340 \\ a = 10.9 \text{ lbs.} \end{matrix} \right\}$$

Third Illustration of Law of Fatigue.—I shall take the third illustration of the Law of Fatigue from the experiments performed by Mr. Nipher.*

Mr. Nipher lifted a weight (w) through a height (h) in the time (t), a number of times (n), until the muscles of the arm became fatigued. The arm was raised from the vertical to the horizontal position. The lifts were made isochronous with the alternate beats of a pendulum, and during the other alternate beats, the arm dropped to the side, the weight being caught upon a cushion attached to the leg; and when the time (t) exceeded one second, the weight was supported upon a framework during the interval of rest. The experiments were made at intervals not less than three hours, and were so arranged as to eliminate from each series, as far as possible, the increase in the power of the muscles produced by the "training."

The following Table contains the results of one set of experiments:—

w = kilograms, h = 0.74 metre, t = 1.164 secs.

* Iowa School Laboratory, vol. i., p. 108 (1871).

Mr. Nipher's Experiments—(Raising Weights at given rate).

w.	n (right arm).	n (left arm).
1 kil.	255	213
2 ,,	97	75
3 ,,	61	50
4 ,,	37·7	31·7
5 ,,	29·3	24·7
6 ,,	21·5	17·8
7 ,,	15·8	12·8
8 ,,	12·8	9·5

In this case, the work done involves the raising of the arm as well as the raising of the weight; let a represent the weight of the arm, then we have, supposing the arm to be an uniform cylinder,

$$\text{Total work done} = \left(w + \frac{a}{2}\right) hn,$$

$$\text{Rate of work} = \frac{\left(w + \frac{a}{2}\right) h}{t}$$

Hence, by the *Law of Fatigue*,

$$\frac{\left(w + \frac{a}{2}\right)^2 h^2 n}{t} = \text{constant},$$

or, since h and t are constant,

$$\left(w + \frac{a}{2}\right)^2 n = A. \qquad (133)$$

This, as before, represents a cubical hyperbola similar to equation (130).

I find, by trial, the following values of the constants :—

Right arm.	Left arm.
$A = 1000.$	$A = 850.$
$a = 2.0$	$a = 2.2.$

Calculating by these constants from equation (133) I obtain the following comparison of Theory and Observation.

Mr. Nipher's Experiments—(*Raising Weights at same rate*).

h = 0.74 metre. t = 1.164 sec.

	Right Arm.			Left Arm.		
w.	n (observed).	n (calculated).	Difference.	n (observed).	n (calculated).	Difference.
1 kil.	255	250	+ 5.0	213	209.4	+ 3.6
2 ,,	97	111	− 14.0	75	88 4	− 13.4
3 ,,	61	62.5	− 1.5	50	50.6	− 0.6
4 ,,	37·7	39·9	− 2.2	31·7	32.7	− 1.0
5 ,,	29·3	27.8	+ 1.5	24.7	22 9	+ 1.8
6 ,,	21.5	20.4	+ 1.1	17.8	16.9	+ 0.9
7 ,,	15.8	15.6	+ 0 2	12.8	13 0	− 0.2
8 ,,	12.8	12.4	+ 0.4	9 5	10.2	− 0.7

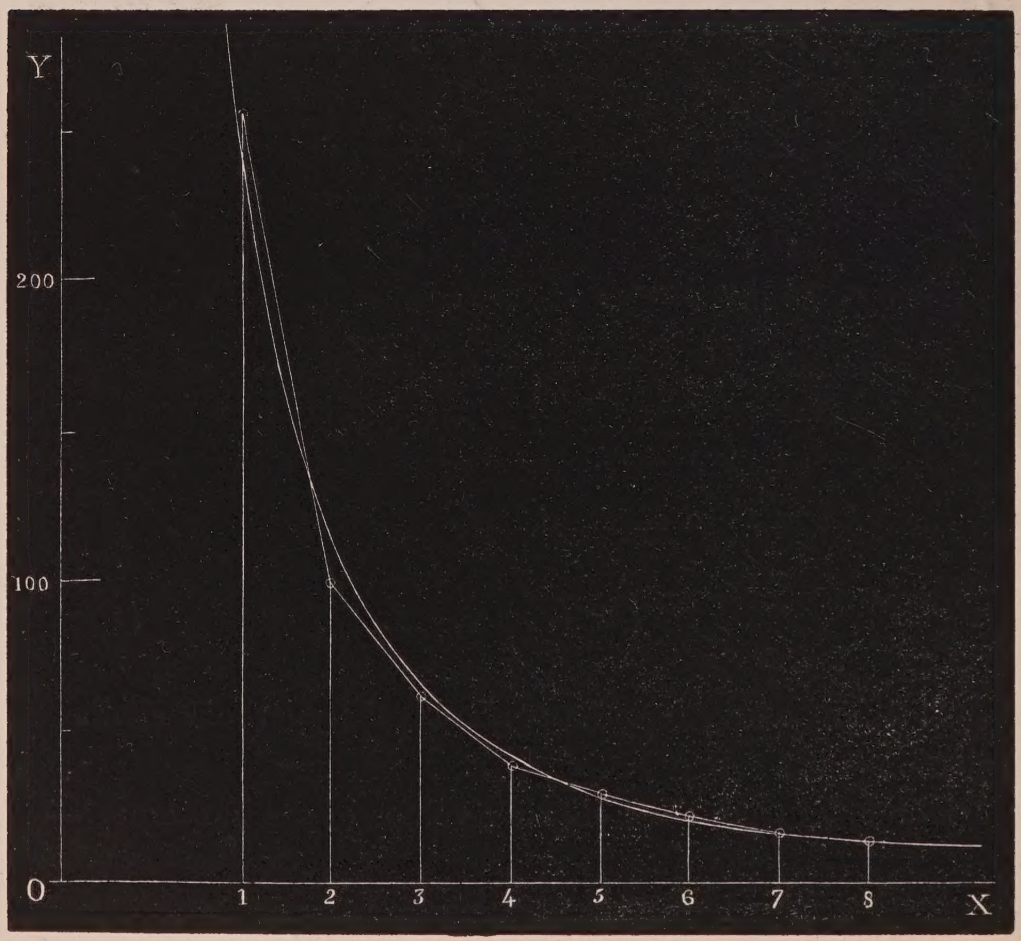


Fig. 108.

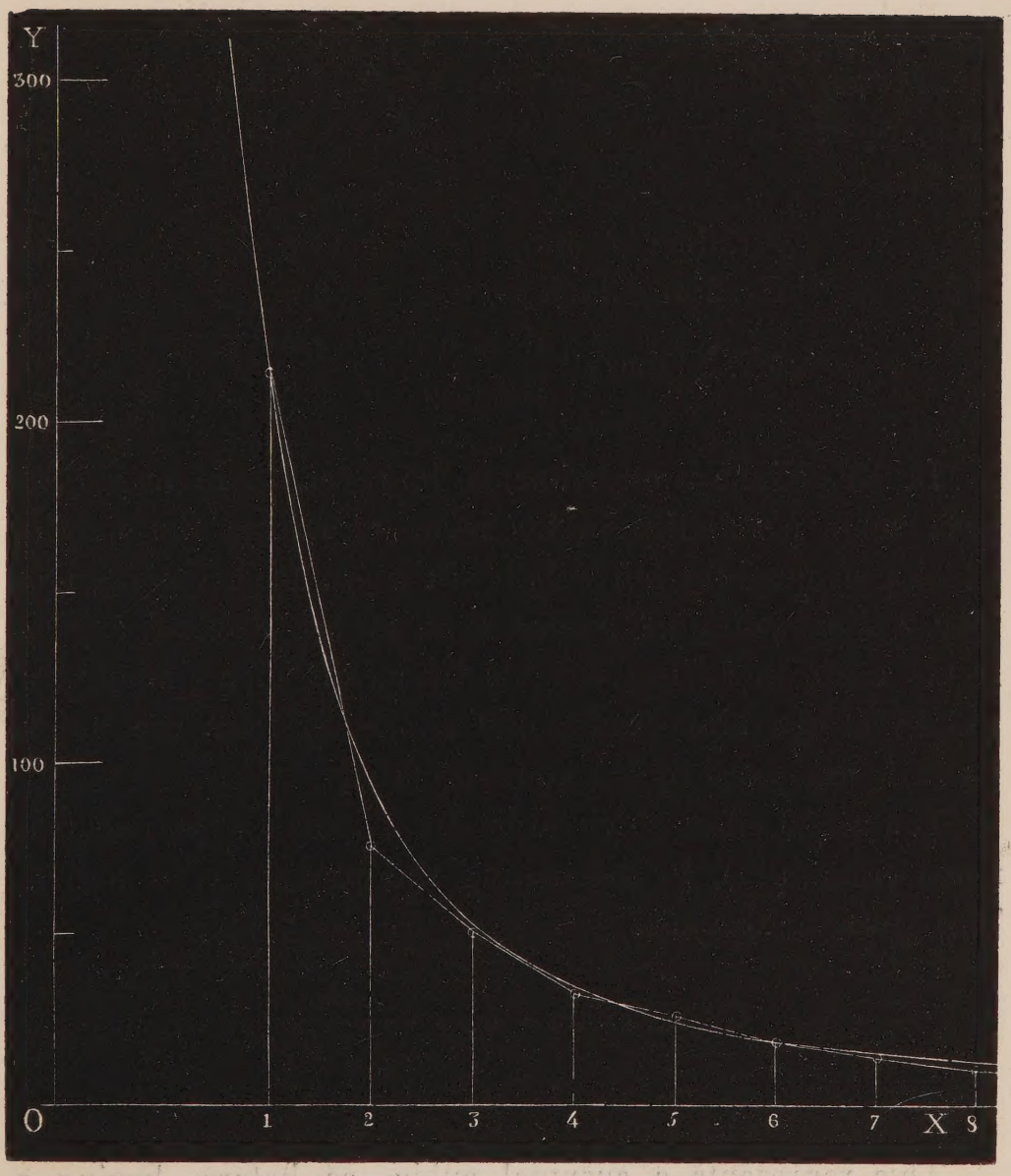


Fig. 109

In Figs. 108 and 109 I have constructed the Cubical Hyperbolas for each arm, and shown the actual observations by small circles. The agreement between theory and observation is as close as can be expected.

Mr Nipher himself represents the observations empirically, by means of an Equilateral Hyperbola, which, for the portion of the curves under observation, does not differ materially in shape from the Cubical Hyperbola, indicated by the Law of Fatigue as the true form of the curve.

On comparing these experiments with those made by Mr. Jevons and myself, the small value of the weight of the arm appears striking. Converting kilograms into pounds, we find—

Mr. Jevons' arm		= 10.9	lbs.
Dr. Haughton's arm		= 8.5	,,
Mr. Nipher's	right arm	= 4.41	,,
	left arm	= 4.85	,,

In Mr. Nipher's experiments, however, there may have been some peculiarity in his mode of raising the weight, which prevented the full weight of the arm from coming into play. We have already seen that, in Mr. Jevons' experiments made in raising weights by means of a pulley, the weight of the arm disappeared altogether from the result, which is expressed by the equation (124).

The useful effect obtained by Mr. Nipher's experiments is found from (133), by calculating the product of w and n. This gives the formula,

$$\text{Useful effect} = wn = \frac{Aw}{\left(w + \frac{a}{2}\right)^2}. \qquad (134)$$

This represents a cuspidal cubic, as before, having a maximum ordinate corresponding to the weight,

$$w = \frac{a}{2}.$$

I find, by using the numerical constants already given, the following comparison between theory and observation :—

No. 1.—*Mr. Nipher's Experiments (Useful Effect)—Right Arm (raising Weights at given rate).*

w.	Useful Effect (observed).	Useful Effect (calculated).	Difference.
0 kil.	0	0	0
1 ,,	255	250	+ 5
2 ,,	194	222	− 28
3 ,,	183	187	− 4
4 ,,	150.8	160	− 9.2
5 ,,	146.5	139	− 7.5
6 ,,	129	123	+ 6
7 ,,	110.6	110	+ 0.6
8 ,,	102.4	99	+ 3.4

No. 2.—*Mr. Nipher's Experiments (Useful Effect)—Left Arm (raising Weights at given rate).*

w.	Useful Effect (observed).	Useful Effect (calculated).	Difference.
0 kil.	0	0	0
1 ,,	213	193	+ 20
2 ,,	150	176	− 26
3 ,,	150	151	− 1
4 ,,	126.8	131	− 4.2
5 ,,	123.5	122	+ 1.5
6 ,,	106.8	102	+ 4.8
7 ,,	89.6	91	− 1.4
8 ,,	76	82	− 6

The maximum effects, calculated from (132), are

$$\text{Maximum effect} = \frac{A}{2a} = \left\{ \begin{matrix} 250 & \text{(Right arm)} \\ 193.2 & \text{(Left arm)} \end{matrix} \right\}.$$

Fourth Illustration of Law of Fatigue.—Another, and very interesting, method of testing the Law of Fatigue consists in raising weights held in the hand or hands, from the vertical

to the horizontal position, at a given rate, until fatigue sets in, and then repeating the experiments with the same weight, at a different rate. In every case, the time of raising the weight is made equal to the interval of rest that elapses until the next lift.

The following observations were made by me on Dr. Alexander Macalister. The weight used was 14 lbs., and the height through which it was lifted was 30 inches. One arm only (the right) was used in the experiments.

In the following Table, I have compared the results, observed with the results calculated from the *Law of Fatigue*, and the agreement is highly satisfactory.

n represents the number of lifts before fatigue;
t represents the time occupied by the lift, which is made equal to the time of repose.

Dr. Macalister (*Raising Weights at varying rates*).

w = 14 lbs. h = 30 inches.

	t.	n (observed).	n (calculated).	Difference.
1.	0.65 secs.	36.0	36.1	− 0.1
2.	0.75 ,,	39.0	39.7	− 0.6
3.	1.18 ,,	51.0	49.5	+ 1.5
4.	1.40 ,,	51.7	51.4	+ 0.3
5.	2.00 ,,	53.0	50.6	+ 2.4
6.	2.84 ,,	44.5	44.5	0.0
7.	3.68 ,,	37.8	38.1	− 0.3
8.	5.09 ,,	27.5	29.8	− 2.3

The column of calculated values of n was found as follows :—In raising the arm and weight from the vertical to the horizontal position in the time t, the muscles perform two distinct kinds of work, dynamical and statical; the dynamical work being measured by weight multiplied by height lifted, and the statical work being measured in a manner similar to

the work of holding out the loaded arms horizontally. In other words, the arm and weight must be not only *lifted* but *sustained* during the time t.

Using the same notation as before, we have

$$\begin{aligned} &\text{Dynamical work} && = (w + a)\, xn\,; \\ &\text{Rate of Dynamical work} && \frac{(w + a)\, x}{t}. \end{aligned} \qquad (135)$$

If the arm were held in the horizontal position during the time t, the statical work would be represented by

$$\omega\ (w + a)\ xt,$$

and the total statical work would be

$$\omega\ (w + a)\ xnt.$$

But the arm is not held horizontally, but sustained for equal times in every position of the quadrant from vertical to horizontal, while the arm and weight are passing from the vertical to the horizontal position.

Hence, the statical work done bears a certain fixed proportion to the statical work of holding the arm and weight horizontally, and

$$\begin{aligned} &\text{Statical work} && = \omega\ (w + a)\ xnt \times K; \\ &\text{Rate of statical work} && = \omega\ (w + a)\ x \times K. \end{aligned} \qquad (136)$$

The coefficient K is thus found: let θ denote the angle between the vertical and the axis of the arm in any position; and let the whole time t be divided into m parts; and let the loaded arm rest at the angle θ during the mth part of the time t. The work done in the position θ is (resolving the weight along the tangent to the circle),

$$\omega\ (w + a)\ xn \times \sin\theta \times \frac{t}{m}.$$

Let us now divide the quadrant into m equal parts, and

suppose that the arm in rising from the vertical to the horizontal position rests at each division of the quadrant, for the mth part of the time t. The angle θ in our last equation will have, successively, the following value:—

$$\frac{\pi}{2m}, \quad \frac{2\pi}{2m}, \quad \frac{3\pi}{2m}, \quad \ldots \quad \frac{m\pi}{2m}.$$

Let us write

$$\phi = \frac{\pi}{2m};$$

then the total work done will be

$$\omega\,(w + a)\,xn\frac{t}{m}\,(\sin\phi + \sin 2\phi + \&c. + \sin m\phi).$$

Comparing this with equation (136), we find

$$K = \frac{1}{m}\,(\sin\phi + \sin 2\phi + \&c. + \sin m\phi), \qquad (137)$$

where m is to be made indefinitely large; but

$$\sin\phi + \sin 2\phi + \&c. + \sin m\phi = \sin\frac{\left(\frac{(m+1)\,\phi}{2}\right)\sin\left(\frac{m\phi}{2}\right)}{\sin\frac{\phi}{2}},$$

or,

$$K = \frac{\sin\frac{(m+1)\,\phi}{2}\,\sin\frac{m\phi}{2}}{m\sin\frac{\phi}{2}}.$$

Substituting, in this equation, $\frac{\pi}{2m}$ for ϕ, we obtain

$$K = \frac{\sin\frac{(m+1)\,\pi}{4m}\,\sin\frac{\pi}{4}}{m\sin\frac{\pi}{4m}},$$

and, finally, making m very large, we find

$$K = \frac{4 \sin^2 \frac{\pi}{4}}{\pi} = \frac{2}{\pi}. \qquad (138)$$

Substituting this value of K in equations (136), we obtain

$$\begin{aligned} \text{Statical work} &= \frac{2\omega}{\pi} (w + a)\, xnt\,; \\ \text{Rate of statical work} &= \frac{2\omega}{\pi} (w + a)\, x. \end{aligned} \qquad (139)$$

The two kinds of Work, Dynamical and Statical, just described, are performed by the muscles of the shoulder, until fatigue occurs. The *Law of Fatigue* applied to this case gives us the following result :—

Let

W = Total work of both kinds :
R = Rate of work of both kinds ;
W_1 = Total dynamical work ;
R_1 = Rate of dynamical work ;
W_2 = Total statical work ;
R_2 = Rate of statical work.

The *Law of Fatigue* gives the following equation :

$$WR = W_1R_1 + W_2R_2 = \text{constant}. \qquad (140)$$

Substituting, in this equation, the dynamical and statical work and rate of work given in equations (135) and (139), we obtain

$$(w + a)^2 x^2 \left\{ \frac{1}{t} + \left(\frac{2\omega}{\pi} \right)^2 t \right\} n = \text{constant}. \qquad (141)$$

In the present experiments $(w + a)\, x$ is a constant, and therefore (141) may be thus written,

$$n \left\{ \frac{1 + \left(\frac{2\omega}{\pi} \right)^2 t^2}{t} \right\} = A,$$

or,

$$n = \frac{At}{1 + \left(\frac{2\omega}{\pi}\right)^2 t^2}. \qquad (142)$$

The values of n in the experiments made on Dr. Macalister were calculated from (142), using the constants,

$$A = 64.5,$$

$$\frac{2\omega}{\pi} = 0.6221.$$

If n, t be made the ordinate and abscissa of a curve, equation (142) will represent one of the central cubics, having its centre at the origin, and the axis of x asymptotic at positive and negative infinity. The ordinate will have a maximum value for co-ordinates.

$$t = \pm \frac{\pi}{2\omega} = \pm 1.6075 \text{ secs.}$$

$$n = \pm \frac{A\pi}{4\omega} = \pm 51.84.$$

The curve (142), on the side of positive axes is represented in Fig. 110, and the actual observations are marked by small circles.

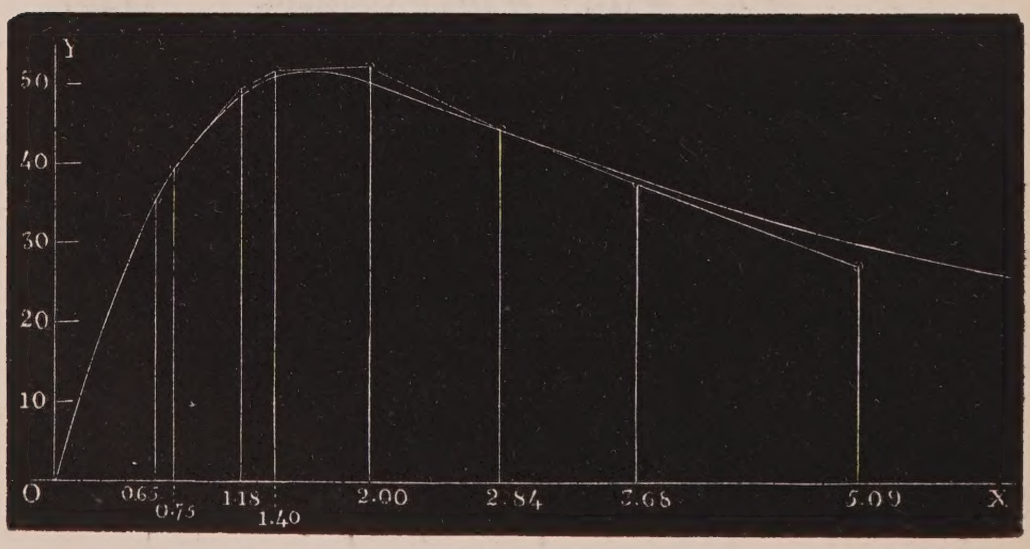


Fig. 110.

Fig. 110.—*Dr. Macalister* (*Raising Arm from vertical to horizontal position, at varying rate*).

$$\left\{ n = \frac{At}{1 + \left(\frac{2\omega}{\pi}\right)^2 t^2} \qquad \begin{matrix} A = 64.5 \\ \frac{2\omega}{\pi} = 0.622 \end{matrix} \right\}$$

The foregoing observations and theory establish the important fact, that there exists a certain rate of lifting the loaded arm to the horizontal position, which will give a maximum amount of work. This rate is

$$t = \frac{\pi}{2\omega} = 1.6075.$$

The observations also give us the value of the important constant ω, used in pp. 24–44; substituting for π, we find

$$\omega = 0.9772.$$

Having obtained the preceding results, I next made a series of observations on my son, Mr. J. Gilbert Haughton (æt. 21), who is accustomed to the use of 10 lb. dumb-bells. Taking a 10 lb. dumb-bell in each hand, he raised both arms to the horizontal position, with palms upward; the time of raising (t) being constant for each experiment, but varied from one experiment to another; the interval between each effort of raising was also made equal to the time t. The following results were obtained :—

Mr. J. Gilbert Haughton (Raising Weights at varying rates).

w = 10 lbs. h = length of arm.

	t.	n (observed).	n (calculated).	Difference.
1.	0.53 sec.	13.0	13.8	− 0.8
2.	1.195 „	21 5	21.0	+ 0 5
3.	2 143 „	21.0	20.9	+ 0.1
4.	3.00 „	17.0	18.0	− 1.0
5.	6.00 „	10.2	10.8	− 0.6

The calculated values of n were found from equation (142), using the constants

$$A = 27.5.$$

$$\frac{2\omega}{\pi} = 0.630.$$

This curve is shown in Fig. 111, and the observations are marked by small circles.

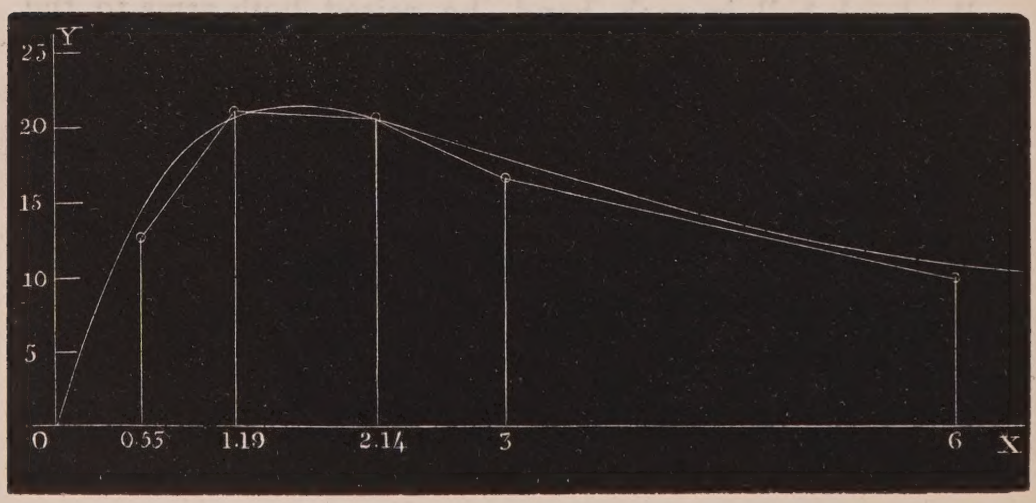


Fig. 111.

Fig. 111.—*Mr. J. Gilbert Haughton* (*Raising both Arms loaded, from vertical to horizontal position, at varying rates*).

$$\left\{ n = \frac{At}{1 + \left(\frac{2\omega}{\pi}\right)^2 t^2} \qquad \begin{matrix} A = 27.5. \\ \frac{2\omega}{\pi} = 0.630 \end{matrix} \right\}$$

The maximum work possible, and the time of lift corresponding to it, are found as before, viz. :

$$n = \frac{A\pi}{4\omega} = 21.83.$$

$$t = \frac{\pi}{2\omega} = 1.5873 \text{ secs.}$$

The value of ω found from the last of these equations is

$$\omega = 0.9896.$$

Mr. Nipher's experiments on raising the loaded arm from the vertical to the horizontal position, at varying rates, gave him the following results:—

No. 1.—*Mr. Nipher* (*Raising Weight with Right Arm, at varying rates*).

$w = 7$ kilos. $h = 0.74$ metre.

	t.	n (observed).	n (calculated)	Difference.
1.	1.164 secs.	15.8	22.2	− 6.4
2.	1.50 ,,	22.8	22.8	0
3.	2.0 ,,	18.5	21.7	− 3.2
4.	3.0 ,,	17 3	18.2	− 0.9
5.	4.0 ,,	15.3	15.5	− 0.2
6.	4.5 ,,	15.0	13.8	+ 1.2
7.	5.0 ,,	14.3	12.6	+ 1.7
8.	6.0 ,,	12.8	9.4	+ 3.4

I have calculated the column for n, from equation (142), using the constants

$$A = 30\,4.$$

$$\frac{2\omega}{\pi} = 0.666.$$

The maximum value of n, and the time of lift corresponding to it, are

$$n = \frac{A\pi}{4\omega} = 22.82$$

$$t = \frac{\pi}{2\omega} = 1.50 \text{ secs.}$$

and the value of ω deduced from the above is

$$\omega = 1.0472.$$

Mr. Nipher's results, from experiments made with his left arm, are given in the following table:—

No. 2.—*Mr. Nipher* (*Raising weights with Left Arm, at varying rates*).

$w = 7$ kilos. $h = 0.74$ metre.

	t.	n (observed).	n (calculated).	Difference.
1.	1.164 secs.	12.8	19.1	− 6.3
2.	1.50 ,,	19.7	19.6	+ 0.1
3.	2.0 ,,	16.2	18.8	− 2.6
4.	3.0 ,,	13.5	15.7	− 2.2
5.	4.0 ,,	11.7	12.9	− 1.2
6.	4.5 ,,	10.8	11.8	− 1.0
7.	5.0 ,,	10.8	10.8	0
8.	6.0 ,,	9.7	9.2	+ 0. 5

I have calculated the column for n, from equation (142), using the constants

$$A = 26.2$$

$$\frac{2\omega}{\pi} = 0.666.$$

The maximum value of n, and the time of lift corresponding to it, are

$$n = \frac{A\pi}{4\omega} = 19.65.$$

$$t = \frac{\pi}{4\omega} = 1.50 \text{ secs.}$$

The value of ω is the same as in the experiments made on the right arm, viz.:—

$$\omega = 1.0472.$$

Combining together all the values of ω, we obtain the mean value of this important constant.

Value of ω.

1. Dr. Macalister, ...	0.9772
2. Mr. J. Gilbert Haughton, .	0.9896
3. Mr. Nipher, ...	1.0472
Mean, ...	1.0046

The angular velocity ω, turns out, therefore, to be equal to unity, so that the arc described in a second would be equal to the radius, and the time of describing the quadrant would be

$$t = \frac{\pi}{2} = 1.5708 \text{ sec.},$$

and the maximum effect would be produced when the arms are raised to the horizontal position at the rate corresponding to an angular velocity equal to unity.

Law of Refreshment.—In considering the Law of Fatigue of muscles, an interesting question arises as to the law by means of which the fatigued muscles are restored to a condition in which they are able to resume work as before. This law may be called the *Law of Refreshment*, and its investigation requires the solution of the following problem:—At what rate does fresh arterial blood supplied to muscles restore to them the power of again giving out work, of which they have been deprived by fatigue?

In the case of the heart, we have a muscle that never tires during life, and we may therefore assume that during each cycle of work and rest, a supply of fresh arterial blood is given to the heart, sufficient to enable it to commence again, quite refreshed, a second cycle, and so on for ever, or at least for eighty or ninety years of a long life. If we define the *Coefficient of Refreshment* to be the work restored to the muscle in foot pounds per ounce per second, we obtain the coefficient of the heart's refreshment from the following data, already given (pp. 144–5).

Work done by heart in one cycle, equal to 3 ounces lifted through 9.92 feet $= \frac{3}{16} \times 9.92$ ft. lbs.

Assuming the heart to beat 72 times in one minute, the

work done by heart in one second $= \frac{72}{60} \times \frac{3}{16} \times 9.92$ ft. lbs.

The weight of the heart is 9.39 oz.

Hence,

The work done by the heart per ounce per second is

equal to $$\frac{72 \times 3 \times 9.92}{60 \times 16 \times 9.39} = 0.2377 \text{ ft. lb.}$$

This amount of work per ounce per second must be given

back to the heart by the arterial blood, because the heart never tires; hence

Coefficient of Refreshment of human heart = 0.2377 ft. lb.

Next to the action of the heart, the best example we have of muscles continuing to act for a long time without fatigue, is the case of the muscles used in the act of walking. Let us examine the *Coefficient of Refreshment* for these muscles. A man walking at the most convenient pace for a long day's work walks at the rate of one mile per $17\frac{1}{2}$ minutes. Hence the distance travelled in one second will be

$$\frac{5280}{17.5 \times 60}.$$

The work done (p. 54) will be, at this pace, $\frac{1}{23.03}^{rd}$ part of the weight, which may be taken at 150 lbs., lifted through the above space. Hence, the work done in one second by the walking muscles, is

$$\frac{5280 \times 150}{17.5 \times 60 \times 23.03} = 32.752 \text{ ft. lbs.}$$

The muscles more or less employed in the act of walking are those of the hip, knee, and ankle-joints. Taking the weights of these muscles from the tables (pp. 401, 404), we obtain for each side 124.27 oz. This weight doubled, or 248.54 oz., is the total weight of muscles employed in doing the work just calculated. Hence we obtain

$$\textit{Coefficient of Refreshment} \text{ of walking muscles} = \frac{32.752}{248.54} = 0.1318 \text{ ft. lb.}$$

The following experiments were undertaken in order to obtain additional data to determine the Coefficient of Refreshment. Dr. Macalister, holding 10 lbs. in each hand, at a

given signal, raised both arms to the horizontal position, and kept them in that position until compelled by fatigue to lower the weights. After a fixed interval of rest, the operation was repeated, and so on; the successive times of holding up the arms were noted, and formed, as may be well supposed, a series which diminished rapidly at the commencement, and afterwards more slowly. On plotting the experiments, taking the number of the observation as the x, and the time of holding up the arms as the y of the curve, it became apparent that the form of the curve was hyperbolic, and convex towards the axis of x. We can find the horizontal asymptote of this curve, and so determine the time of holding out the arms, during which the fatigue incurred is exactly counterbalanced by the refreshment supplied in each cycle of rest and work. When this condition is attained, the shoulder muscles can continue to work for hours without sensible fatigue, as in the case of the muscles employed in walking.

Experiments No. I.—Interval of Rest = 60 *seconds.*

1st Effort,	.	60 seconds	. . (mean).
6th ,,	. .	25 ,,	. . ,,
31st ,,	. .	16 ,,	. . ,,
56th ,,	. .	14 ,,	. . ,,

These results are the means of many observations made on different days; and I selected the 6th, 31st, and 55th observations in order to simplify the calculations. It is evident that the hyperbola which ultimately coincides with the curve of observations must have for its equation,

$$(x + \alpha)(y - \beta) = k^2; \qquad (143)$$

and we are to determine α, β, and k^2, from assuming any

three points on the curve to be known. Expanding (143), we find

$$xy + y\alpha - x\beta = k^2 + \alpha\beta.$$

If we substitute in this equation the corresponding co-ordinates

$$x = 5 \qquad x = 30 \qquad x = 55$$
$$y = 25 \qquad y = 16 \qquad y = 14$$

we obtain three linear equations to determine α, β, and k^2.

The values of these constants are

$$\beta = 10.85$$
$$\alpha = 9.29$$
$$k^2 = 202.67.$$

We have now to determine the work done, when the condition of dynamical equilibrium is attained, and the refreshment restores the work during each cycle. By the equations which lead to (127), the work done is

$$\text{Work} = \omega (w + a) xt$$

where

$\omega = 1$.

w = weight held in hand.

a = weight of arm.

x = distance of centre of gravity of loaded arm from centre of glenoid cavity.

In addition to the work done during the time t, which denotes the time of holding the arm horizontally, we must take account of the work done in lifting the loaded arms from the vertical into the horizontal position, which is evidently

$$(w + a) x.$$

Hence

$$\text{Total work} = (w + a) x (t + 1). \qquad (144)$$

If we assume the weight of the arm to be 10 lbs., and its length 2 feet, we have

$$w = 10 \text{ lbs.}$$
$$a = 10 \text{ lbs.}$$
$$x = 1.5 \text{ ft.};$$

and equation (144) gives us

$$\text{Total work} = 30\,(t + 1).$$

Substituting for t, its value $10.85 = \beta$, we have

$$\text{Total work} = 30 \times 11.85 \text{ ft. lbs.}$$

To find the Coefficient of Refreshment, we must divide this work by the number of ounces in the muscles in action, and by the whole time of the cycle of rest and work.

The muscles more or less engaged in lifting and holding the loaded arm horizontally are

1. Upper half of Trapezius, 2. Cleido-mastoid, . . 3. Levator anguli scapulæ, .	Holding up scapular arch.
4. Deltoid, . . . 5. Supraspinatus, . .	Holding up arm.
6. Biceps humeri, . . 7. Brachiæus, . . . 8. Supinator radii longus, .	Guarding the forearm from forcible extension.
9. The flexors of the wrist and fingers, . .	Guarding the hand from forcible extension.

From the Tables (pp. 406, 408), we may calculate the total weight of these muscles at 35.41 oz. Hence as the time of the cycle of rest and work is

$$60 + 10.85 = 70.85;$$

we have

$$\text{Coefficient of Refreshment} = \frac{30 \times 11.85}{35.41 \times 70.85} = 0.1417.$$

Experiments No. II.—Interval of rest = 30 seconds.

1st Effort,	. .	60 seconds.	. . (mean).
41st ,,	. .	10.47 ,,	. . ,,
79th ,,	. .	6.58 ,,	. . ,,
99th ,,	. .	6.00 ,,	. . ,,

From these data we have the following pairs of co-ordinates, to determine from equation (143) the distance β of the horizontal asymptote from the axis of x.

$x = 40$	$x = 78$	$x = 98.$
$y = 10.47$	$y = 6.58$	$y = 6.00.$

The constants of the hyperbola turn out to be

$$\beta = 4.23.$$
$$a = -1.83.$$
$$k^2 = 171.74.$$

From equation (144) we have

$$\text{Work done} = 30(t + 1) = 30 \times 5.23 \text{ ft. lbs.},$$

and, since the period of the cycle is $30 + 4.23 = 34.23$; we find as before

$$\text{Coefficient of Refreshment} = \frac{30 \times 5.23}{35.41 \times 34.23} = 0.1296.$$

Experiments No. III.—Interval of rest = 15 seconds.

1st Effort,	. .	60 seconds,	. . (mean).
8th ,,	. .	10.64 ,,	. .
43rd ,,	. .	5.93 ,,	. . ,,
92nd ,,	. .	4.02 ,,	. . ,,

From these data we find the following pairs of co-ordinates :—

$x = 7$	$x = 42$	$x = 91.$
$y = 10.64$	$y = 5.93$	$y = 4.02.$

Substituting in equation (143), we obtain, finally, for the constants of the hyperbola,

$$\beta = 1.323$$
$$a = 39.2$$
$$k^2 = 430.58.$$

Hence, since the period of the cycle is

$$15 + 1.323 = 16.323,$$

we find the Coefficient of Refreshment

$$= \frac{30 \times 2.323}{35.41 \times 16.323} = 0.1206.$$

Collecting together the results obtained from the walking muscles, and from the muscles of the arms we find

Coefficient of Refreshment of Voluntary Muscles.

	ft. lbs. per oz. per sec.
1. Walking Muscles, . . .	0.1318
2. Muscles of Arm (60 secs. rest),	0.1417
3. „ „ (30 „),	0.1296
3. „ „ (15 „),	0.1206
Mean, . .	**0.1309**

On comparing the mean Coefficient of Refreshment of the voluntary muscles with the Coefficient of Refreshment of the heart, viz., 0.2377, we see that the heart receives double the refreshment in the same time. This interesting result is quite in accordance with the views of those anatomists who believe that the heart receives double the supply of arterial blood that any other muscle receives, in consequence of the

semi-lunar valves, during life, not closing the openings of the coronary arteries during systole; for if the heart receives twice the supply of arterial blood, it ought to possess a Coefficient of Refreshment double that of any other muscle.

Conclusion.—I bring my work to a close with some regret, as it has afforded me many pleasant hours of thought and research. None of my readers can see its defects more clearly than I myself perceive them, for I have not been able to use more than a tenth part of the materials at my disposal, and I feel that I have failed to convey by my words much of the interest that attaches to the problems which I have endeavoured to solve.

This much, however, I can guarantee; all the dissections, weighings, and observations have been made by my own hands, with every precaution of which I could think, to ensure accuracy. My observations have also been made without preconceived hypotheses to guide them; and many of my most interesting results have been forced upon my notice by the facts placed before my eyes in the Dissecting-room and Laboratory.

I have done my best to travel *non quo itur, sed quo eundum est;* and if I have erred in attaching too much importance to Geometry, as the Queen and Mistress of all the Sciences, I have at least the sanction of Plato for my error, who, when asked how the Divine Being spent His time, replied, γεωμετρεῖν τὸν Θεόν.

"Errare mehercule malo cum Platone, quàm—"

I leave my objectors to finish the quotation.

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